Bull. Nov. Comp. Center, Math. Model. in Geoph., 8 (2003), 127–138

Some questions of representation and analysis of travel time curves

Vladimir I. Znak

The object of the research is a travel time curve (the second term is a hodograph) $t_i = f(s_i)$ which is obtained by profiling. In this paper, an attempt is made to offer a model of the hodograph as well as some ways of its analysis. The model of the above curve is reduced, in the ideal case, to a broken line $f^{(0)}(s_i)$, where the segments linearly or quasi-linearly depend on time. At the same time the function values are fixed at discrete moments of time t_i $(i = \overline{1,n})$. To that broken line a function, given on a set of breakdowns, is compared. We consider the process of transition to a set of breakdowns with attracting the concept of threshold detection. In case of distortion of the above line by additive errors, we investigate the function $f^{(0)}(s_i) + \xi_i = f^{(\xi)}(s_i)$, and the problem of reducing the latter to an ideal form is transformed to the problem of minimization of the appropriate functional $|f^{(0)}(s_i) - f^{(\xi)}(s_i)|$.

1. Introduction

In point of fact, a travel time curve is a curve, which reflects the arrival time of an elastic wave as a function of the distance that it passes up to the recording point $t_i = f(s_i)$, where the function values are fixed at discrete moments of time t_i $(i = \overline{1,n})$. Essentially, a travel time curve depends on a number of factors beyond the distance: structure, characteristics, and state of a medium. The indicated factors determine velocities and/or slowness of the wave.

In the given work, a travel time curve is understood as dependence obtained under the condition of only one fixed source, and receivers (or one receiver at different moments of time) are offset along some line. In the future we will use a simplified notation $f(s)$ if there is no need for indexing separate function values (in the VSP experiments we use the depth notation H instead of distance).

The problem of the paper is to propose a method of decreasing distinctions between the practical dependence $f(s)$ and some true process, and, also, the evaluation of the measure of their correspondence. The situation is explained by the fact that in practice a travel time curve is obtained by means of the analysis of records recorded in field conditions. As in the process of a field experiment and in analysis of its results one or the other type of errors takes place. Some of errors are in the "hardware" part of the research, in the accuracy of restoration of real signals (records), while the others are due to the essence of evaluation (by the formal aspect) of the concept "arrival time". We will consider them in more detail.

As a matter of fact, the information about the state of a medium in field experiments comes from a geophone to a recording device. The record is made in the digital form and with a given time of the quantization period. Already at this stage, as was revealed in paper [1], the quality of records depends on the character of a geophone-ground coupling. On the other hand, as follows from the research [2, 3], the characteristics and parameters of a wave are also a function of distance. That is, even in conditions of the ideally exact restoration of records we should speak about a limited accuracy of evaluation of arrival times. Thus, when building-up the empirical dependence $f(s)$ both during the recording, restoration of records, and during their analysis or estimations of arrival times there take place various sorts of distortions and errors having a casual character, for example: unbiased error of transformation of a signal from the analog to the digital form (the quantization or the round-off error); the biased error due to the amplifier drift in geophones; the interdependent error of estimation of the arrival time due to the waveform as a function of distance and/or the presence of microseisms, and due to other deformations.

To the techniques and algorithms of processing and analysis of seismic data was given a great attention. Herewith it is possible to distinguish the following directions of the research. First, taking into account the fact, that field seismorecords is primary source of information, these are the techniques of analysis and processing of frequency processes. This methods are based, as a rule, on one or the other frequency filtration, and in the last few years with attracting robust filters (see, for example, [4–6]). Second, these are the questions of increasing the quality of representation of a travel time curve as a functionally dependent set of observed variables. In this case the preference is given either to interactive algorithms, as for example, in papers [5, 7], or to the formal maximum-likelihood estimation as in paper [8]. Finally, the third direction of the research is evaluation of the quantitative conformity of the available data in regard to real data before or after their processing as is presented in papers [7, 9, 10].

Let us dwell on the papers concerning the latter direction in some detail. The authors mentioned above give the main attention in their works to the accuracy of evaluation of velocities. However, the quality of representation of a time curve is not limited to the evaluation of only one parameter. Of interest is also an other parameter, that is, the layer thickness or layouts of strata, i.e., the information about depths, stretches, etc. Moreover, the wave velocity and the attenuation depend upon pressure, the kind of layering as was determined in paper [11] and upon movement and distribution of fluid in strata, according to the research [12]. In this connection, in the general case it is reasonable to carry out a complex research of the time curve for the quantitative conformity of the available data in regard to the source process.

Let us now note that the evaluation of such a correspondence is complicated in addition to the above-mentioned by the following factors: our imperfect understanding of the source process, the limitation of *a priori* information and by dependence of real data on particular conditions, for example, on weather conditions when carrying out the field experiment. All these make the method of maximum-likelihood estimation to be attractive. However, this approach does not give an answer to the question about the degree of closeness of a set of *a posteriori* data in regard to the ideal values. It only indicates to the direction of movement.

In this paper, an attempt is undertaken to offer the definition of a criterion of the quality of representing a travel time curve, and to formulate some problems arising in the course of improving the practical data to some original or ideal kind. The offered approach allows the geophysicist to verify (by means of the quality criterion) and to correct (by means of sorting out the filtering procedures) a process, which is different from those mentioned above. In this way the approach is, to a certain degree, heuristic that make it more flexible and acceptable in the more range of *signal-to-noise ratio*.

2. Statement of the problem

Let us turn to the definition of a hodograph as a functional dependence of the type $t = f(s)$. As a model of this dependence we take a broken line, where the segments linearly or quasi-linearly depend on time. Here, moments of breakdowns (beginning/end points of segments) correspond to the media interfaces, and the inclination of segments is determined by the appropriate travel times. The quasi-linearity is understood in the sense that the corresponding segment is approximated by the line without essential loss of accuracy. Let us note the case of an ideal broken line as function of the form

$$t = f^{(0)}(s). \tag{1}$$

The dependence of this kind is represented by the hypothetic broken line in Figure 1. It is typical of the vertical seismic profiling (VSP) and includes three segments with the distribution of velocities in segments, respectively: $v_1 = 1600$ m/s, $v_2 = 2100$ m/s, $v_3 = 2700$ m/s.

As was already marked, in the process of practical building-up dependence (1) there take place various sorts of distortions and errors, which we designate by θ (for example, curve (1) in Figure 2 is curve (1) from Figure 1 plus white noise with a *signal-to-noise ratio* that is about three). This enables us to consider a hodograph as a random function of the form

$$t = f^{(0)}(s) + \theta = f^{(\theta)}(s). \tag{2}$$

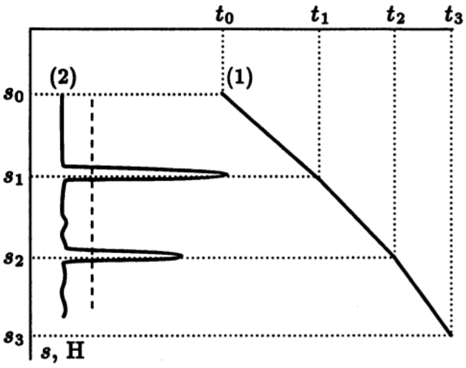


Figure 1. The model of the VSP hodograph without distortion: (1) the travel time curve; (2) the distribution of values of the segmentation criterion; the dash line represents the threshold value

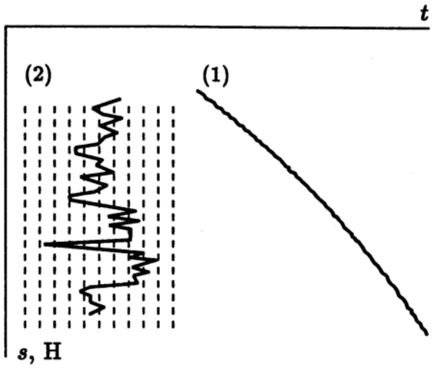


Figure 2. The model of the noisy VSP hodograph: (1) the noisy travel time curve; (2) the distribution of values of the segmentation criterion; the dash lines are the threshold values

As it is possible to see, the model, represented here by expression (2), has the same origin, as expressions (1), (2), represented in paper [8]. In an informal sense it is required to bring nearer the experimental data to the initial or to the ideal kind of a travel time curve both as in the above-mentioned sense of paper [8] and in our case. But the formal statement of the problem in this case is reduced to a problem of minimization of the functional

$$|f^{(0)}(s) - f^{(\theta)}(s)|, \tag{3}$$

that represents the module of difference of data for the "ideal" and for the real hodographs.

In this paper, we dwell on the transition from a problem of minimization of functional (3) to a problem of minimization of some object function, as well as on some related tasks.

3. Discussion of segmentation

The procedure of segmentation allows us to go from the representation of a hodograph in the form of function (1) or (2) to the representation on a set Q of its critical points or breakdowns q_i, $i = \overline{1,n}$:

$$f(s) \longrightarrow Q, \tag{4}$$

where

$$Q = \{q_0, q_1, \ldots, q_n\} = \{(t,s)_0, (t,s)_1, \ldots, (t,s)_n\}. \tag{5}$$

Here each moment of the breakdown is represented by a pair of "time-distance" $q = (t,s)$: $(t,s)_0$ is the beginning of the first segment, $(t,s)_n$ is the end of the latter, and n is the number of segments. Let $z_i = \{q_{i-1}, q_i\}$ be the i-th segment of a hodograph, $i = \overline{1,n}$. Then

$$Q = \bigcup_{i=1}^{n} z_i. \tag{6}$$

We will discuss the process of segmentation (or localization of breakdowns) as a probability or a stochastic process.

A number of papers, for example, [8, 13–19] etc. with different techniques of segmentation of the curves of interest are offered. With all the distinctions of the presented modes and segmentation criteria in these papers, the statistical approach was used. In our opinion, in papers [15, 18] this feature – the statistical direction in the research – is most pronounced and clear. According to the proposals stated the corresponding process includes two stages: 1) building-up the function $S^{(\theta)}(s)$ of distribution of values of the segmentation criterion on the dependence $f^{(\theta)}(s)$; 2) analyzing the function $S^{(\theta)}(s)$ or executing the process of detection.

In this paper, as operator for building-up the function of distribution of values of the segmentation criterion we will use a standard deviation of a signal from the line with a some basis L of the signal values:

$$\sigma(L) = \sqrt{\sum_{i \in L} (x_i - M(x))^2}. \tag{7}$$

Let the basis L be running along the signal values from the beginning to the end of the signal. After performing the operator (7) we obtain the distribution of values of the segmentation criterion which, as a rule, has the form of a pulsing function, for example, curves (2) in Figures 1 and 2. In the ideal case – in the absence of any distortions – the number of pulses of such a function (here $S^{(0)}(s)$) are determined by the number of breakdowns of a time curve, and the size of pulses is determined by appropriate angular factors: $f^{(0)}(s) \to Q^{(0)}$. In practice, the function of criterion depends – in addition to the marked parameters – on the character and size of distortions, as well as on parameters of the segmentation operator. Curve (2) in Figure 2 is an example of the distribution of values of the segmentation criterion for a noised travel time curve for $L = 5$. We will discuss the stage of detection in more detail in the next paragraph. Here we only underline that, by virtue of a random nature of values of a real hodograph, none of the known techniques of segmentation can ensure authentic results in a practical case.

The casual character of distribution of values of the segmentation criterion allow us to set the problem of processing of the corresponding function as an independent one: it is required to bring the distribution of the segmentation criterion $S^{(\theta)}$ closer to the initial form $S^{(0)}$, i.e., to the distribution which would correspond to the ideal dependence $f^{(0)}$.

There exist four situations which precede the detection:

Filtration of function $f^{(\theta)}$	No	Yes	No	Yes
Filtration of function $S^{(\theta)}$	No	No	Yes	Yes

If $f^{(\theta)}$ or $S^{(\theta)}$ is a casual function, then any its filtration will be a result of this casual function. We can speak only about a modification of *signal-to-noise ratio*. Further we will suppose that we have a set of operators of filtration $G = \{g_1, \dots, g_k\}$, which can be used in one way or another for processing the functions $f^{(\theta)}$ and $S^{(\theta)}$. Before we present the list of such filters (it is not compete, because it demands a special attention) to substantiate the conclusion: a set of filtration operators will lead to an assemblage $\Im$ of functions of distribution of values of the segmentation criterion:

$$f^{(\theta)}(s), S^{(\theta)}(s) \longrightarrow \Im = \{S^{(l)},\ l = \overline{1, N}\} \tag{8}$$

4. Estimation of the threshold index

For the analysis of values of the distribution of the segmentation criterion there is involved, to a certain degree, a concept of the threshold index (or, simply, a threshold), that we designate as γ). Generally, the threshold is considered as the user's parameter. Metha *et al.* [8], in particular, indicate to close to the second order inverse dependence between γ and the density of critical points. In other words, with a decrease in a threshold value the number of segments grows rapidly enough. A part of them with a certain accuracy reflects the structure of a real medium. The remaining part has not a proper covering, that is, the corresponding critical points (or moments of breakdowns) imply false events. In view of the importance of the threshold index concept we should consider its essence in more detail.

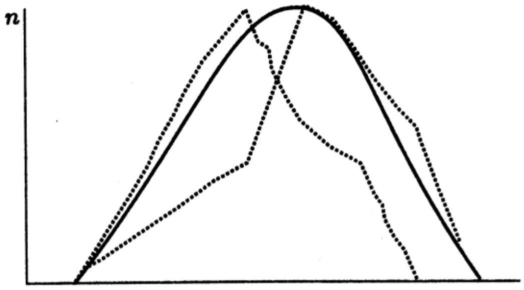
If a hodograph as a time curve is the initial function, i.e., the distribution of values of the segmentation criterion has the ideally precise presentation $S^{(0)}$, which was obtained using $f^{(0)}$, the range of a threshold lies within the limits

$$\eta \leq \gamma < \sigma_{\min}, \tag{9}$$

where η is any small, positive definite value and $\sigma_{\min}$ is the value of a minimum pulse, for example, the point (t_2, s_2) of curve (2) in Figure 1, and the dash line represents a threshold.

A natural requirement to a threshold value is the following: to detect such a number $m^{(l)}, l = \overline{1, N}$, of breakdowns on a given distribution $S^{(l)} \in \Im$, which would be equal to the true value $n^{(0)}$ or which would be close to it. At the same time the detected critical points should be localized inside their ranges of definition, limited from left to right by the centres of true segments:

$$m^{(l)}(\gamma) \simeq n_{(0)}, \quad l = \overline{1, N}. \tag{10}$$



γ	n	γ	n
0	0	0	0
0.152	1	0.180	2
0.304	2	0.361	5
0.457	3	0.541	7
0.609	7	0.721	4
0.761	6	0.901	3
0.913	4	1.08	0
1.07	0	1.26	0

Figure 3. The model of the noisy VSP hodograph from Figure 2: the dotted lines are the distributions of repetition rate (right-hand tables); the solid line is an average of dotted lines

However then for each distribution $S^{(l)} \in \Im$ (by virtue of its specific features) its own threshold of segmentation will be required:

$$\gamma = \gamma(l) \mid (m^l \simeq n^{(0)}), \tag{11}$$

where symbol "$\mid$" has the meaning "provided that".

We can assume that the repetition rate distribution $m(\gamma)$ for each $S^{(l)} \in \Im$ tends to the normal distribution

$$f(\gamma_l) = \frac{1}{\sigma_l\sqrt{2\pi}} \exp\left(-\frac{(\gamma_l - \bar{m}^{(l)})^2}{2\sigma_l^2}\right). \tag{12}$$

This assumption is confirmed by appropriate trials (Figure 3). Then as threshold index we can use such value which would be equal to the mathematical expectation

$$\bar{m}^{(l)} = \int \gamma_l f(\gamma_l)\, d\gamma_l. \tag{13}$$

Now we can present the mean value of the number of breakdowns for the assemblage $\Im$ of functions of distribution of values of the segmentation criterion in the form

$$\tilde{n} = \left[\frac{1}{N}\sum_{l\in\Im} \bar{m}^{(l)}\right], \tag{14}$$

where the notation $[x]$ has the meaning "the integer nearest to x".

5. Criterion of the quality of a travel time curve

We make an assumption that for a assemblage $\Im$ exists some number, denoted as $\tilde{N}$, of such functions for which the condition

$$\tilde{N} \mid (m^{(l)} = \tilde{n}) \tag{15}$$

holds. We will denote the corresponding subset as $\tilde{\Im}$, $(S^{(l)} \in \tilde{\Im} \subset \Im)$. Now we can calculate the relation

$$\mu = \frac{\tilde{N}}{N}, \tag{16}$$

which will be used as index of the quality of segmentation $0 \leq \mu \leq 1$. For a subset $\tilde{\Im}$ we determine a common set of critical points as

$$\bar{Q} = \{\bar{q}_0, \bar{q}_1, \ldots, \bar{q}_n\} = \{(\bar{t}, \bar{s})_0, (\bar{t}, \bar{s})_1, \ldots, (\bar{t}, \bar{s})_n\}, \tag{17}$$

where

$$\bar{t}_i = \frac{1}{\tilde{N}} \sum_{j \in \tilde{\Im}} t_i^{(j)}, \quad \bar{s}_i = \frac{1}{\tilde{N}} \sum_{j \in \tilde{\Im}} s_i^{(j)}.$$

If within a separate stratum the velocity of wave propagation is constant, then the appropriate i-th segment of a hodograph can be approximated by an segment of the straight line with certain values of the distances s_i, s_{i-1} and the moments of time t_i, t_{i-1} $(i = \overline{1,n})$ on its ends. Now for the l-th distribution of $S_s^{(l)} \in \tilde{\Im}$ $(l = \overline{1,\tilde{n}})$ it is possible to find evaluations of strata velocities as

$$V_{i(l)} = \frac{s_i^{(l)} - s_{i-1}^{(l)}}{t_i^{(l)} - t_{i-1}^{(l)}}, \quad l = \overline{1,\tilde{N}}, \quad i = \overline{1,\tilde{n}}, \tag{18}$$

and to require the identity of instant velocities within an individual stratum $z_i^{(l)}$:

$$v_j^{(i)} \equiv v_{j-1}^{(i)} \quad \text{if} \quad j(i), j(i) - 1 \in z_i^{(l)}, \tag{19}$$

where $z_i^{(l)} \in \bar{Q}^{(l)}$ is the i-th segment of the l-th set of breakdowns which corresponds to the distribution $S^{(l)} \in \tilde{\Im}$ $(l = \overline{1,\tilde{N}})$, and $s_i^{(l)}$, $s_{i-1}^{(l)}$, $t_i^{(l)}$, $t_{i-1}^{(l)}$ are the values at the ends of the segments; $j(i) = \overline{1,k(i)}$; $i = \overline{1,\tilde{n}}$. For a subset of "exact" segmentation $\tilde{\Im}$ expressions (18), (19) enable us to calculate the average error of estimations for instant velocities as

$$\aleph_v = \frac{1}{\tilde{n}\tilde{N}} \sum_{l=1}^{\tilde{N}} \sum_{i=1}^{\tilde{n}} \sum_{j \in z_i} \frac{v_{i(l)}^{(0)} - v_{j(i)}^{(l)}}{k(i)} \tag{20}$$

and a standard deviation

$$\sigma_v = \mathrm{Var}(v). \tag{21}$$

In a similar way we can calculate the average error of segmentation:

$$\Delta = \frac{1}{\tilde{n}\tilde{N}} \sum_{i \in \bar{Q}} \sum_{l=1}^{\tilde{N}} (\bar{\Delta}_i - \Delta_i^{(l)}), \tag{22}$$

where

$$\bar{\Delta}_i = \frac{1}{\tilde{N}} \sum_{l=1}^{\tilde{N}} \Delta_i^{(l)} = \frac{1}{\tilde{N}} \sum_{l=1}^{\tilde{N}} \sqrt{\left(t_i^{(l)} - t_{i-1}^{(l)}\right)^2 + \left(s_i^{(l)} - s_{i-1}^{(l)}\right)^2} \tag{23}$$

and the corresponding standard deviation

$$\sigma_\Delta = \mathrm{Var}(\Delta). \tag{24}$$

Here $\Delta_i^{(l)}$ is the distance between the boundaries of a stratum obtained as a result of approximation of the i-th segment of the l-th distribution $S^{(l)}$ by the straight line.

Let us return to the question of the representation quality of a hodograph. If a set of operators of filtration $G = \{g_1, \dots, g_k\}$ is such that $\tilde{Q} = Q^{(0)}$, then the problem of minimization of functional (3) can be examined as problem of minimization of the target function $(\aleph_v(g), \sigma_v(g), \Delta(g), \sigma_\Delta g)$ on a set of the filtration operators $g \in G$ under the condition of maximization of the value of the index of quality of segmentation (16) or

$$|f_0(s) - f_\theta(Z)| \Longrightarrow f\Big(\aleph_v(g), \sigma_v(g), \Delta(g), \sigma_\Delta(g)\Big) = \min \mid \mu = \max. \tag{25}$$

6. Conclusion

Thus, the improvement of the quality of representation of a travel time curve is considered as a process of its handling with the intent of bringing closer the structure of an experimental curve $f^{(\theta)}$ to some ideal form $f^{(0)}$. As a model of a hodograph a broken line is used, where the segments linearly or quasi-linearly depend on time.

In its basis the problem of improving the quality of a hodograph can be considered as the problem of minimization of distances between the connected items of curves $f^{(\theta)}$ and $f^{(0)}$ on a two-dimensional plane "time-distance". In the present paper, with invoking a set of filtering operators G, the transition from an experimental curve $f^{(\theta)}$ to a set of distribution values of the segmentation criterion. For the latter we build up a common set of critical points (breakdowns) $\tilde{Q}$ with the quality index of segmentation $0 \le \mu \le 1$. With a set $\tilde{Q}$ the error of estimations of velocities $\aleph_v$ with standard deviation σ_v is connected, and, also, the error of segmentation Δ with standard deviation σ_Δ. In these conditions the problem of minimization of the functional $|f^{(0)}(s) - f^{(\theta)}(s)|$ is reduced to the problem of minimization of the target function $(\aleph_v(g), \sigma_v(g), \Delta(g), \sigma_\Delta(g))$ provided maximization of the quality index of segmentation μ.

A specific feature of the target function $(\aleph_v(g), \sigma_v(g), \Delta(g), \sigma_\Delta(g)) =$ *min* is in that with an increase of the ***signal-to-noise ratio*** (excess of angular factors in the places of breakdowns of a hodograph against the background noise) the index μ tends to 1. Thus, the index of quality of segmentation of a hodograph as measure of sufficiency of the statistics used for the estimation of parameters of the functions $(\aleph_v(g), \sigma_v(g), \Delta(g), \sigma_\Delta(g))$.

Finally, we give some examples of filtration operators which can be used as processing operators for a hodograph. In this paper we take no assumptions concerning the character of distortions of a signal and concerning a set of processing procedures. Here it is possible to make the following remarks. In the case when a signal is deformed by the Gaussian noise and a set of the processing operators is limited to the linear ones, the given problem can be solved in the explicit form. However, a distortion of a travel time curve has, as a rule, rather a complex character, and the average size of a deviation of the considered function from its true values can vary even within the limits of one object of research. At the same time the processing procedures eliminating distortions, should save the information basis of a hodograph, i.e., the moments of its breakdowns and inclination of segments. In such a case, of great practical interest are nonlinear operators, in particular, robust filters as operators of processing. Such questions demand, on the whole, a separate attention. Due to these reasons the following does not aspire to the completeness and is not accompanied with comments:

- classic median filters proposed by Tuky [20];
- modified trimmed mean filters offered by Lee and Cassam [21];
- center-weighted median filters considered by Ko and Lee [22];
- percentile filters with $\alpha \neq 0.5$ taken as a principal weighted-order statistics of a general type considered by Znak [23];
- robust Kalman filters proposed by Tom'as and Rosario [24] for time series analysis;
- filtration with attracting Markov Chains [25, 26].

For this purpose it is possible to use approximations of one or an other degree as well as various spline-functions. The investigation of specific features of the presented methods is also beyond the frame of this paper.

References

[1] Maxwell P.W., Edelman H.A.K., Faber K. Recording reliability in seismic exploration as influenced by geophone-ground coupling // 56th EAEG meeting, Extended Abstracts. – Vienna, 1994. – B014.

[2] Bickel S.H. Velocity-depth ambiguity of reflection traveltimes // Geophysics. – 1990. – Vol. 55. – P. 266–276.

[3] Ross W.S. The velocity-depth ambiguity in seismic traveltime data // Geophysics. – 1990. – Vol. 59. – P. 830–843.

[4] Duncan G., Beresford G. Median filter behaviour with seismic data // Geophysical Prospecting. – 1995. – Vol. 43. – P. 329–345.

[5] Znak V.I., Panov S.M. Algorithm of interactive processing of vertical seismic profiling data // Geology and Geophysics. – 1992. – № 8. – P. 121–127.

[6] Znak V.I. Weighted median filter and VSP result processing // Problemno-orientirovannye vychislitel'nye kompleksy. – Novosibirsk, 1992. – P. 78–85 (in Russian).

[7] Glosovskii V.M., Langman S.L. New methods of interactive determination of layered subsurface model // Russian Geophysical Seminar. – Selangor, 1992. – P. 1–190.

[8] Metha C.H., Radhakrisnan S., Srikanth G. Segmentation of well logs by maximum-likelihood estimation // Mathematical Geology. – 1990. – Vol. 22. – P. 853–869.

[9] Kirlin R.L. The relationship between semblance and eigenstructure velocity estimators // Geophysics. – 1992. – Vol. 57. – P. 1027–1033.

[10] O'Konnel R.J., Rai Ch.S. Method for characterizing velocity estimates: Patent 4964086 USA, G 01 V 1/30, Amoco Corp, Nb. 424430, 1990.

[11] Best A. The effect of pressure on seismic velocity and attenuation in reservoir rocks // 56th EAEG meeting, Extended Abstracts. – 1994. – P. 1042.

[12] Gurevich B., Lopatnikov S.L. Effect of fine layering on the velocity and attenuation in fluidsaturated rocks // 56th EAEG meeting, Extended Abstracts. – Vienna, 1994. – P. 109.

[13] Chen H., Fang J.H. A Heuristic search method for optimal zonation of well logs // Mathematical Geology. – 1986. – Vol. 18. – P. 489–500.

[14] Hawkings D.M., Merrian D.F. Optimal zonation of digitized sequential data // Mathematical Geology. – 1973. – Vol. 5. – P. 389–395.

[15] Kerzer M.G. Image processing in well log analysis. – Reidel Publishers Co., 1986.

[16] Kulinkovich A.Ye., Sokhraniv N.N., Churinova I.M. Utilization of digital computers to distinguish boundaries of beds and identity sandstones from electric log data // International Geology Reviews. – 1966. – Vol. 8. – P. 416–420.

[17] Testerman J.D. A statistical reservoir-zonation technique // J. Petroleum Technology. – Aug, 1962. – P. 889–893.

[18] Webster R. Automatic soil boundary location from transect data // Mathematical Geology. – 1973. – Vol. 5. – P. 27–37.

[19] Wu X., Nyland E. Automated stratigraphic interpretation of well-log data // Geophysics. – 1987. – Vol. 52. – P. 1665–1676.

[20] Tuky J.W. Exploratory Data Analysis. – Preliminary Edition, 1971. – Addison-Wesly, Reading, MA, 1977.

[21] Lee Y.H., Kassam S.A. Generalized median filtering and related nonlinear filtering techniques // IEEE Transactions on Acoust., Speech, Signal Processing. – 1985. – Vol. 33. – P. 672–683.

[22] Ko S.-J., Lee Y.H. Center weighted median filters and their applications to image enhancement // IEEE Transactions on Circuits and Systems. – Sept, 1991. – Vol. 38. – P. 984–993.

[23] Znak V.I. Some models of noise signals and heuristic search for weighted-order statistics // Math. Comput. Modelling. – 1993. – Vol. 18. – № 7. – P. 1–7.

[24] Tom'as C., Rosario R. Robust Kalman filter and its application in time analysis // Kybernetika. – 1991. – Vol. 27. – № 6. – P. 487–494.

[25] Godfry R., Moir F., Rocca F. Modelling seismic impedance with Markov Chains // Geophysics. – 1980. – Vol. 45. – P. 1351–1372.

[26] Chournard P.N., Paulson K. V. A Markov-Gauss algorithm for blocking well logs // Geophysics. – 1988. – Vol. 53. – P. 1118–1121.