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Numerical simulation of orographic waves using an atmospheric model with artificial compressibility*

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A three-dimensional mathematical model of local climate based on the artificial compressibility approach is considered. No outer iterations are necessary in order to solve the diagnostic equation for the pressure in complex terrain. Higher order approximation schemes based on the splitting up method are used for the advective operators of the problem. The turbulence parameterization scheme is based on the concept of computation of the mixing length. The dry atmospheric waves over a hill have been simulated. The results of the numerical experiments are in qualitative agreement with the theory available.

1. Introduction

A number of mathematical-meteorological models have been developed during the last 20–30 years [1]. The knowledge of space and time distribution of meteorological variables is important for the solution of various problems of urban planning and environmental protection. It is difficult to detect large variations of these fields in irregular terrain only with the help of measurements. Mesoscale meteorological models have been developed which have become useful tools for contributing to the unknown information. The applications of mesoscale modeling include the simulation of air flow over an urban region and the associated dispersion and advection of pollutants, convective scale dynamics in mountainous regions, etc.

It was shown by the scale analysis of atmospheric dynamics that the applicability of the hydrostatic approximation for tropospheric motions breaks down when the resolvable horizontal and vertical scales are of the same order [2]. The treatment of non-hydrostatic dynamics is required, and this is often accomplished via the so-called anelastic approximation [3, 4, 5], where the sound waves are filtered out by a modification of the mass continuity equation.

An important property of the anelastic system is that the perturbation pressure must be such that the velocity field satisfies the continuity equation. The result is that the perturbation pressure can no longer be determined

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explicitly, but rather must satisfy an elliptic partial differential equation in complex terrain. It is necessary to solve this equation at each time step, in order to update the pressure field. Therefore, it is more difficult to work with the anelastic system numerically than with the set of equations which are obtained in the hydrostatic case [4]. The incorporation of orography by the use of the terrain-following coordinate transformation creates additional problems in solving this equation by a direct method. A popular approach is to solve the equation using the "outer iterations", that is, by an iterative application of a direct method [4].

In the last 15–20 years, the advancement of computational methods made it possible to develop non-hydrostatic models where sound waves are not filtered out (e.g. [6, 7, 8]). Various numerical techniques have been applied to solve these completely hyperbolic problems.

A three-dimensional mathematical model of local climate based on the artificial compressibility approach is considered in this paper. The artificial compressibility method proposed by Yanenko [9] and Chorin [10], has been successfully applied to various problems of fluid dynamics. Now descriptions of the method can be found in textbooks (e.g. [11]). In applying this method to a non-hydrostatic model, it is not necessary to use outer iterations in order to solve the diagnostic Poisson-type equation for the pressure in complex terrain [8].

The paper is organized as follows: Section 2 describes the model. In Section 3, the numerical algorithm for solving the model equations is given. Section 4 deals with the results of numerical simulations of atmospheric flow over a hill. Conclusions are given in Section 5.

2. The mathematical model

The basic equations for motion, heat and continuity in a terrain-following coordinate system are as follows:

$$\frac{dU}{dt} + \frac{\partial P}{\partial x} + \frac{\partial (G^{13}P)}{\partial \eta} = f_1(V - V_g) - f_2 W + R_u,$$

$$\frac{dV}{dt} + \frac{\partial P}{\partial y} + \frac{\partial (G^{23}P)}{\partial \eta} = -f_1(U - U_g) + R_v,$$

$$\frac{dW}{dt} + \frac{1}{G^{1/2}}\frac{\partial P}{\partial \eta} + \frac{gP}{C_s^2} = f_2 U + g\frac{G^{1/2}\bar{\rho}\theta'}{\bar{\theta}} + R_w,$$

$$\frac{d\theta}{dt} = R_\theta,$$

$$\frac{1}{C_s^2}\frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial}{\partial \eta}\Big(G^{13}U + G^{23}V + \frac{1}{G^{1/2}}W\Big) = \frac{\partial}{\partial t}\Big(\frac{G^{1/2}\bar{\rho}\theta'}{\bar{\theta}}\Big),$$

$U = \bar{\rho}G^{1/2}u$, $V = \bar{\rho}G^{1/2}v$, $W = \bar{\rho}G^{1/2}w$, $P = G^{1/2}p'$, where p', θ' are deviations from the basic state pressure $\bar{p}$ and potential temperature $\bar{\theta}$, C_s is the sound wave speed, u_g, v_g are components of the geostrophic wind representing the synoptic part of the pressure, η is the terrain following coordinate transformation:

$$\eta = \frac{H(z - z_s)}{(H - z_s)},$$

z_s is the surface height, H is the height of the top of the model domain. Here $H = \text{const}$, f_1 and f_2 are the Coriolis parameters equal to $2\Omega \sin \lambda$, $2\Omega \cos \lambda$, respectively, λ is the latitude,

$$G^{1/2} = 1 - z_s/H, \quad G^{13} = \frac{1}{G^{1/2}}\Big(\frac{\eta}{H} - 1\Big)\frac{\partial z_s}{\partial x}, \quad G^{23} = \frac{1}{G^{1/2}}\Big(\frac{\eta}{H} - 1\Big)\frac{\partial z_s}{\partial y}.$$

For an arbitrary function φ

$$\frac{d\varphi}{dt} = \frac{\partial \varphi}{\partial t} + \frac{\partial u\varphi}{\partial x} + \frac{\partial v\varphi}{\partial y} + \frac{\partial \omega\varphi}{\partial \eta},$$

where $\omega = \frac{1}{G^{1/2}}W + G^{13}u + G^{23}v$.

The terms R_u, R_v, R_ω, R_θ refer to the subgrid-scale processes. As a turbulence parameterization, we used a simple scheme based on the computation of Blackadar [12] mixing length (see, e.g. [1]). Conventional logarithmic wind profiles between the surface and the first layer in the atmosphere are evaluated. For the roughness length, we take 0.1 m.

For the upper boundary, we assume $\omega = 0$, $u = u_g$, $v = v_g$, θ is a constant value given by the basic state.

For the lower boundary, we assume $\omega = 0$, $\theta = \theta_s(x, y)$ is taken to be the same value as the basic state at the same level. The turbulent fluxes through the surface layer are determined from the Monin–Obukhov similarity theory.

For the lateral boundaries, the conditions of zero normal derivatives are used in the present study.

3. Numerical formulation of the model

The model is solved numerically by the conventional algorithm of splitting the problem into "advection–diffusion" and "adjustment" problems [13].

The advection processes are realized as follows: the first derivative approximation consists of central-difference operators in space:

$$\frac{d\varphi}{dx}\Big|_k = \sum_i \alpha_i D_i[\varphi]_k,$$

where $D_i[\varphi]_k = (\varphi_{k+i} - \varphi_{k-i})/2ih$.

This operator can be further split in space, provided the appropriate spatial filters are available [15].

Viscous and Coriolis terms are treated implicitly.

At the "adjustment" stage we proceed as follows: Let the appropriate system of equations with boundary conditions be

$$\frac{\underline{\varphi}^{j+1} - \tilde{\underline{\varphi}}}{\Delta t} + A\underline{\varphi}^{j+1} = \underline{f}$$

in real orography, where $\tilde{\underline{\varphi}}$ are vector fields obtained after the "advection – diffusion" stage, Δt is a time step interval. Rewriting the equation, we get

$$\frac{\underline{\varphi}^{j+1} - \tilde{\underline{\varphi}}}{\Delta t} + A_p\underline{\varphi}^{j+1} + (A - A_p)\underline{\varphi}^{j+1} = \underline{f},$$

where A_p is the operator A in plain orography [8]. We solve this problem at the "adjustment" stage provided the $(A - A_p)$ operator is realized at the "advection–diffusion" stage. The problem with A_p operator is easily solved by any standard method (Successive Over–Relaxation in this work).

The terms describing sound waves are treated by the "natural filter scheme" [13], that is, implicitly. The space filters of the Shapiro type [6] are used at each step of the computation.

4. Simulations of flow over a hill

The results of a simulation of flow over an isolated hill are presented here as an example. The bell-shaped hill with the height of 500 m is situated in the center of the 10 km × 10 km domain. The top of the domain is at 5 km. The geostrophic flow goes from the west, with $u_g = 5$ m/sec, $v_g = 0$.

As a basic state, the standard atmospheric stratification $d\theta/dz = 3.5$ K/km is assumed. An absorbing layer is situated above the height of approximately 1500 m. The computational grid consists of $31 \times 31 \times 16$ points, the horizontal grid size is $\Delta x = \Delta y = 333$ m, the vertical grid size Δz is variable, increasing with height. The hill is slowly inflated during the first 15 minutes of the computation.

Figures 1–6 show the potential temperature field θ at 45 min real time.

Figure 1 shows the $N - S$ cross-section through the center of the hill. The picture is rather symmetrical.

Figure 2 represents the $E - W$ cross-section through the center of the hill. At low heights in front of the hill, the field is lifted and it is sinked behind the hill.

Figures 3–6 are the surface plots at the levels from 20 m to 1200 m above ground reference. At the lower levels, the flow turns to the left, while at the upper levels the flow seems to be almost symmetrical.

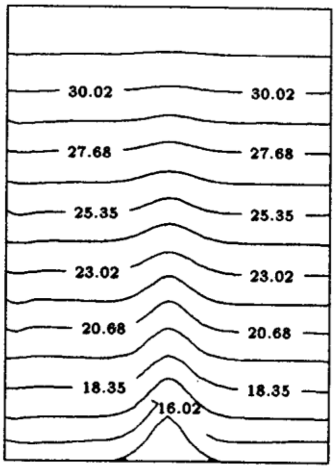

Figure 1. *N–S* cross-section through the center of the hill

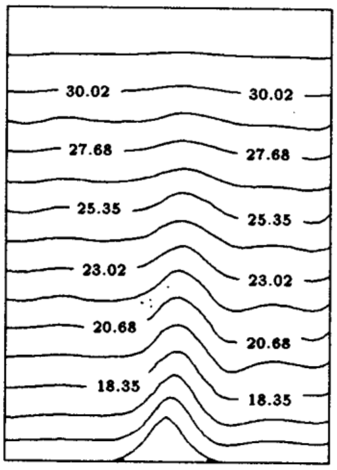

Figure 2. *E–W* cross-section through the center of the hill

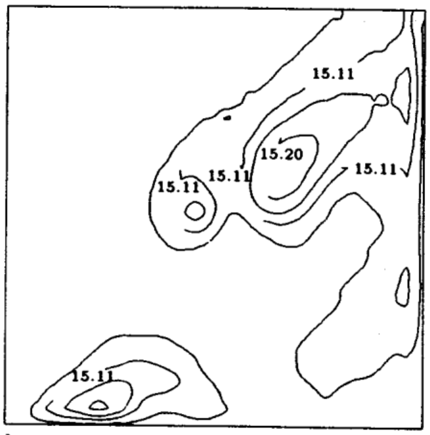

Figure 3. Surface plot at 20 m AGR (above ground reference)

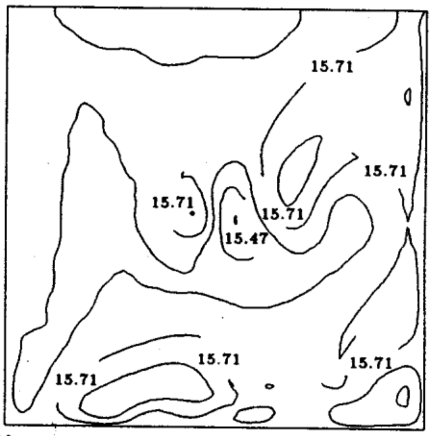

Figure 4. Surface plot at 100 m AGR

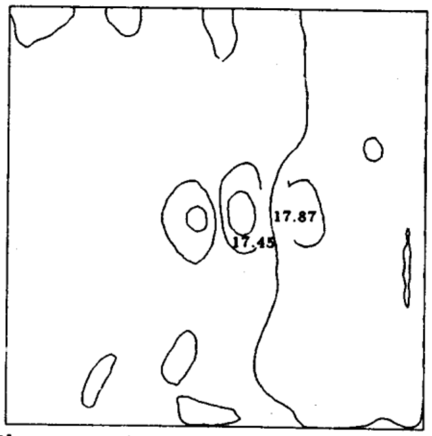

Figure 5. Surface plot at 400 m AGR

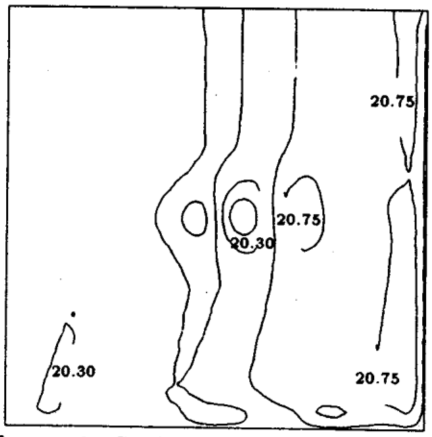

Figure 6. Surface plot at 1200 m AGR

The pictures described above are in qualitative agreement with the existing theory and observations (e.g. [16]).

5. Conclusions

A three-dimensional mathematical-meteorological model for the simulation of local climate was considered in the paper. The introduction of "artificial compressibility" into the model can greatly reduce the CPU time necessary for an effective solution of the pressure equation.

The results of the numerical experiments on the flows over an isolated hill are satisfactory and agree qualitatively with the existent knowledge of the behaviour of orographic waves.

The inclusion of moisture and soil processes into the model is now under way.

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