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Dynamic two-phase flows in natural systems*

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Abstract. The paper considers applications of a two-velocity model of hydrodynamics for a two-phase medium for describing natural geological systems, such as movement of magma melts in magma channels, the flow of a river and the erosion of an unfixed sandy river bottom. The study deals with a test problem of spreading a square area with a high content of a dispersed phase.

Non-stationary problems are solved numerically on the basis of a thermodynamically consistent model for two-phase medium which takes into account a wide class of dissipative phenomena. The numerical analysis is based on the application of a completely implicit control volume method for a rectangular uniform grid with a shift in the calculation nodes for velocities.

Keywords: two-phase media, two-velocity hydrodynamics, conservation laws method, control-volume method

Introduction

Modeling the non-stationary dynamics in natural geological systems makes it relevant to develop mathematical models of heterophase media suitable for taking into account various dissipative processes and phenomena in a wide range of thermodynamic parameters. The paper considers applications of a two-velocity hydrodynamic model for a two-phase medium for describing various types of natural geological systems, such as the intrusion of magmas into magma channels, the river flow and erosion of a sandy bottom of the river, the dynamics of granular media and suspensions. Numerical modeling of magmatic melts intrusion in permeable zones of the lithosphere and evolution of emerging magmatic and ore-magmatic systems is one of the urgent tasks in this field. The dynamics of forming the ore-bearing intrusions in the trap formation is characterized by the following processes: intrusion of basic melts into a layered platform cover; the structural inhomogeneities development in a heterophase nonisothermal magmatic flow in an emerging intrusive body; differentiation of the melt while moving along the channel and halting the pressure magma flow in the intrusion-produced channel. The solution of this class of problems is based on a non-stationary model of heat and mass transfer for heterophase media obtained by a method that ensures its physical correctness. As test problems, the problems of the square area spreading and the erosion of an unfixed sandy river bottom has been considered.

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1. Mathematical model

The model equations are derived on the basis of the conservation laws method [1, 2] which makes it possible to create the thermodynamically consistent systems of continuum dynamics equations. A two-phase suspension model is built on the assumption that the phases are in temperature equilibrium and there is no equilibrium by pressure in the phases. The pressure difference in the phases is associated both with the presence of surface tension forces and with the possible direct interaction of dispersed particles. The elementary volume of a suspension is characterized by partial densities ρ_1, ρ_2 and velocities u, v of the dispersed and continuous phases, impurity density ρ_a, the number of particles in the dispersed phase n, and entropy S. In the gravitational field, the equations taking into account the energy dissipation take the form [3]:

$$\frac{\partial \rho_1}{\partial t} + \operatorname{div}(\rho_1 \boldsymbol{u}_1) = 0, \quad \frac{\partial \rho_2}{\partial t} + \operatorname{div}(\rho_2 \boldsymbol{u}_2) = 0, \quad \frac{\partial n}{\partial t} + \operatorname{div}(n \boldsymbol{u}_1) = 0, \tag{1}$$

$$\frac{\partial \rho_a}{\partial t} + \operatorname{div}\left(c\boldsymbol{j} - DT\nabla\left(\frac{\mu_a}{T}\right)\right) = 0, \tag{2}$$

$$\frac{\partial j_i}{\partial t} + \partial_k(\rho_1 u_{1i} u_{1k} + \rho_2 u_{2i} u_{2k} + (p + q\rho_1)\delta_{ik} - \eta_1 u_{1ik} - \eta_2 u_{2ik}) = \rho g_i, \tag{3}$$

$$\begin{aligned} \frac{\partial u_{2i}}{\partial t} + (\boldsymbol{u}_2, \nabla) u_{2i} = & -\frac{1}{\rho}\partial_i p + \frac{\rho_1}{\rho}\partial_i q + \frac{n}{\rho}\varsigma \partial_i \sigma + \\ & \frac{\rho_1}{2\rho}\partial_i w^2 + b\, w_i + \frac{1}{\rho_2}\partial_k(\eta_2 u_{2ik}) + g_i, \end{aligned} \tag{4}$$

$$\frac{\partial S}{\partial t} + \operatorname{div}\left(S\frac{\boldsymbol{j}}{\rho} - \kappa \frac{1}{T^2}\nabla T + D\mu_a \nabla\left(\frac{\mu_a}{T}\right)\right) = \frac{1}{T}R, \tag{5}$$

$$\begin{aligned} R = & \rho_2 b w^2 + \kappa\left(\frac{\nabla T}{T}\right)^2 + DT^2\left(\nabla\left(\frac{\mu_a}{T}\right)\right)^2 + \\ & \frac{1}{2}\eta_1 u_{1ik} u_{1ik} + \frac{1}{2}\eta_2 u_{2ik} u_{2ik}. \end{aligned} \tag{6}$$

Here $\rho = \rho_1 + \rho_2$, $\boldsymbol{j} = \rho_1 \boldsymbol{u}_1 + \rho_2 \boldsymbol{u}_2$ are the density and momentum of a two-phase medium; p is the pressure; q is the interfacial interaction parameter determined by pressure in the dispersed phase; σ is the surface tension tensor; ς is the specific surface of the dispersed phase; μ_a is the chemical potential of the impurity; $c = \rho_a/\rho$ is the impurity mass concentration; S is the entropy density of a two-phase medium, T is the temperature; $\boldsymbol{g}$ is the gravity acceleration. The kinetic coefficients of interfacial friction b, phase shear viscosity η_1, η_2, thermal conductivity of a two-phase medium κ, diffusion D are functions of appropriate thermodynamic parameters. Here

R is a dissipative function. The strain rate tensors u_{1ik}, u_{2ik} are determined by the relations

$$u_{1ik} = \partial_i u_{1k} + \partial_k u_{1i} - \frac{1}{3}\partial_l u_{1l}\delta_{ik}, \quad u_{2ik} = \partial_i u_{2k} + \partial_k u_{2i} - \frac{1}{3}\partial_l u_{2l}\delta_{ik}.$$

The equation of a two-phase medium state is assumed to be linear: $\delta\rho = \rho\alpha\,\delta p - \rho\beta\,\delta T$, $\delta s = c_p\delta T/T - \beta\delta p/\rho$, where $s = S/\rho$. Coefficients of volumetric compression α, thermal expansion β, specific heat capacity of c_p a two-phase medium are additive in phases.

The impurity is taken into account in the ideal solution approximation: $\mu_a = d_1 p + d_2 T + \bar{R}T\ln c$, where $\bar{R}$ is the universal gas constant. The dissipative impurity flow in equation (2) in this case takes the form

$$Dd_1\nabla p + Dd_2\nabla T + D\bar{R}Tc^{-1}\nabla c. \tag{7}$$

The coefficients before the gradients of pressure, temperature, and impurity concentration determine the effects of barodiffusion, thermal diffusion, and diffusion. Due to the dependence of the chemical potential on the gravitational potential in the impurity flow, a term is also added to the gravity field (which determines the effect of gravidiffusion $Dd_1\rho_a g_k$).

The surface tension is determined by the Shishkovsky relation:

$$\sigma = \sigma_r\frac{T_c - T}{T_c - T_{\text{ref}}} - \sigma_1\ln(1 + ac). \tag{8}$$

The parameter σ_r generally depends on the chemical composition of the phases. This dependence in the model was given by the expression $\sigma_r = \sigma_0(\rho_1 - \rho_2)^m$. Here T_c, T_{ref}, a, m are environment parameters.

The mathematical model of two-velocity media for a suspension with an admixture takes into account the phase compressibility, temperature and surface effects, as well as different dissipative phenomena (interfacial friction, thermal conductivity, viscosity, diffusion, thermal diffusion, barodiffusion, and gravity diffusion). The difference approximation for two-velocity hydrodynamics equations is carried out for a complete nonlinear non-isotropic system of equations for a mixture of compressible liquid media and it is based on the control volume method. The discretization of equations was carried out on a rectangular uniform grid with a shift of the computational nodes for the velocity vectors components. A completely implicit scheme is used here. When approximating the convective terms (for calculating the flows through the faces of the control volumes), the second-order HLPA scheme is implemented. To calculate the pressure fields consistent with the flow field, an iterative IPSA procedure was implemented. For the numerical solution of the SLAE for discrete analogues of main equations and the correction equation for pressure, the alternating direction method and the PARDISO solver from the Intel MKL library are used.

2. Model problems of two-velocity dynamics

2.1. Spreading of a square "drop" with a high solids content in pure liquid. To verify this model, we consider a problem of evolution for a high solids suspension zone placed into a low-solids suspension. Initially, the "drop" is in the center of the computational domain. The parameters of the two-phase medium were set corresponding to the parameters of water (dispersed phase) and sand (dispersed phase). The initial thermodynamic parameters corresponded to normal conditions. The simulations were carried out for the non-dissipative case and different values of the dissipative coefficients. In the absence of dissipation, there is no spreading and no change in the "drop" contour. The addition of dissipative processes and surface stress leads to deformation of this square region into a round one with an increase in the "drop" radius. The observed pattern is shown in Figure 1 and qualitatively agrees with the calculations in [4]. The difference is related to a different choice of the composition for two-phase medium: in the cited work, a mixture of methane and decane was considered at normal temperature and pressure of $1.6 \cdot 10^7$ Pa.

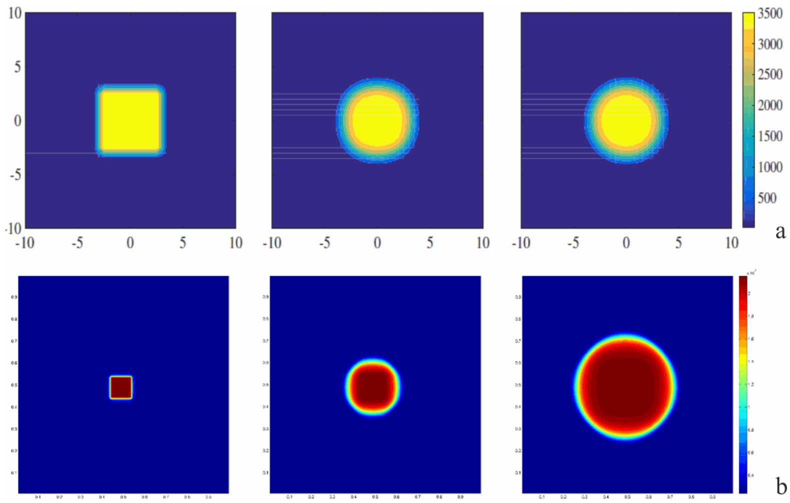

Figure 1. Distribution of the solids content in the mixture for different time points: (a) data from [4], (b) data from this paper

2.2. Layered suspension flows in an open channel. To verify the model, a problem of inhomogeneous suspension flow was also considered using the example of deformation of the channel bottom occurring due to impact from the low-solids suspension flow. The problem geometry is shown in Figure 2. The process of erosion of the loose bottom is shown in Figure 3.

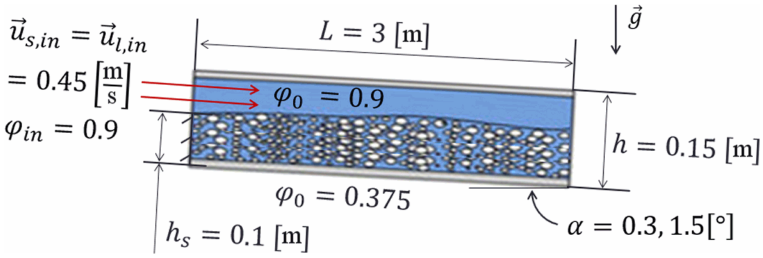

Figure 2. Geometry of the problem of erosion of non-cohesive channel bottom

Figure 3. Dynamics of erosion of the interface between the low-solids and high-solids areas for a geometry with the channel inclination angle of 1.5°

In papers devoted to this problem based on 1D stationary models, the calculated data fail to agree with experimental data without additional assumptions (Figure 4a). The reason is usually associated with the unsteadiness of the flow. To match the calculated and experimental data, the so-called calibration of the channel bottom erosion depth is introduced:

$$\zeta(t, x) = \zeta_0(t, x)\left(1 - \exp\left(-\frac{x}{1 + at}\right)\right).$$

In this paper, we use a non-stationary model without additional assumptions about the boundary erosion process. The calculation results demonstrate a behavior corresponding to stationary models without extra calibration (Figure 4b). Thus, the unsteady flow is not a physical reason requiring an extra calibration as a tool for simulation correction. This aspect requires further research.

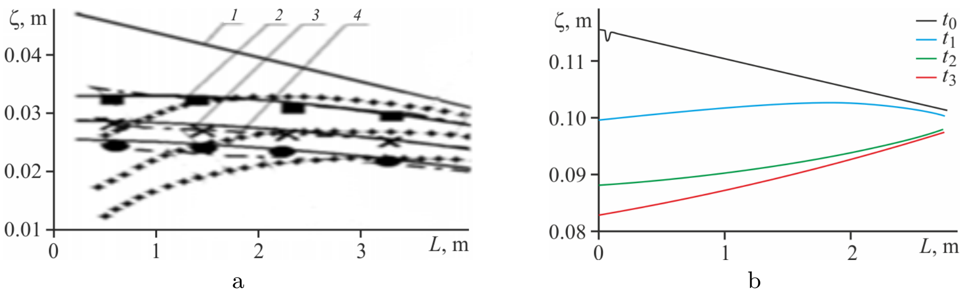

Figure 4. Blurring of the interface between regions with high solids concentration and low solids concentration: (a) squares — experimental data [5], dotted lines — data [6] without calibration, solid lines — data [6] with calibration; (b) data from this paper

3. High viscosity flow in an inclined channel

The use of a suspension model for studying the geological systems requires setting thermodynamic and kinematic parameters in a wide range of values. In this paper, a non-stationary non-isothermal model was used to analyze the magma flow in vertical and inclined magma channels and dikes. The physical parameters of the phases of such a medium were taken as follows: solid phase — density 2600 kg/m^3, bulk modulus $3.2 \cdot 10^{-13}$ Pa^{-1}, thermal expansion coefficient $7.0 \cdot 10^{-5}$ K^{-1}, dynamic viscosity took values from 10^{-1} up to 10^5 kg/(m·s). The input data for liquid phase: density 550 kg/m^3, bulk modulus $9.5 \cdot 10^{-10}$ Pa^{-1}, thermal expansion coefficient $1.8 \cdot 10^{-4}$ K^{-1}, dynamic viscosity $4.5 \cdot 10^{-5}$ kg/(m·s); the thermal diffusivity of a two-phase melt was assumed to be $7.7 \cdot 10^{-7}$ m^2/s. The pressure drop was set to 100 Pa.

A study was made for a flow pattern as a function of continuous phase viscosity. The calculation results are shown in Figure 5 for a dike with a 25° inclination. The flow regime while varying the viscosity of the dispersed phase changes significantly due to contribution to the effective viscosity of the two-phase suspension. At low values, active mixing is observed (Figure 5b), but which weakens with increasing the viscosity (Figures 5c, 5d) up to its complete disappearance at viscosities above 10^5 kg/(m·s) (Figure 5e).

Changing the channel inclination up to 70° leads to a change in the melt flow regime. The melt flow pattern with a viscosity below 10 kg/(m·s) changes from non-stationary mixing (for horizontal channel orientation) to the mixing flow with approaching a steady laminated flow (vertical channel orientation) (Figure 6). For the viscosity above 10^5 kg/(m·s), the two-phase medium flow pattern changes from zero mixing in the horizontal channel to the displacement of liquid phase with solid phase (the process of settling is observed): this generates and almost uniform steady flow in the vertical channel (Figure 7).

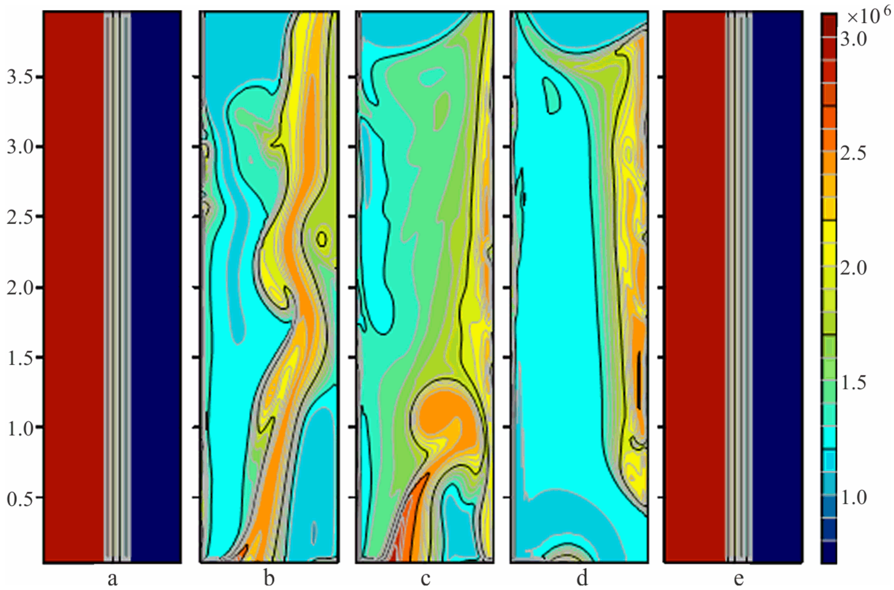

Figure 5. Distribution of the solids content for different values of the viscosity of the dispersed phase: (b) 10^{-1}, (c) 10^{1}, (d) 10^{3}, (e) 10^{5} kg/(m·s); graph (a) corresponds to the initial distribution of particles (m^{-3})

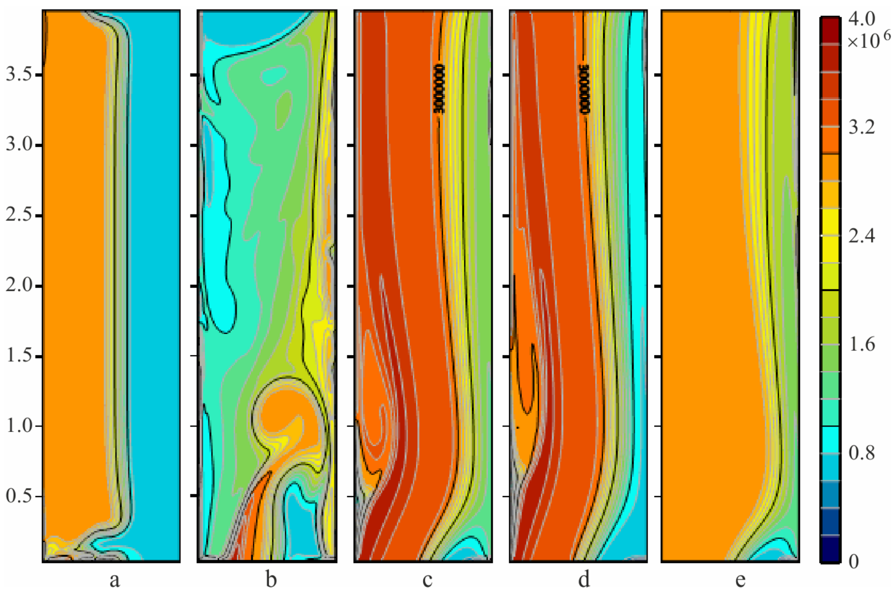

Figure 6. Distribution of the content of dispersed particles for different channel inclination angles: (b) 0°, (c) 25°, (d) 45°, (e) 70°; graph (a) — initial distribution (m^3); viscosity — 10 kg/(m·s)

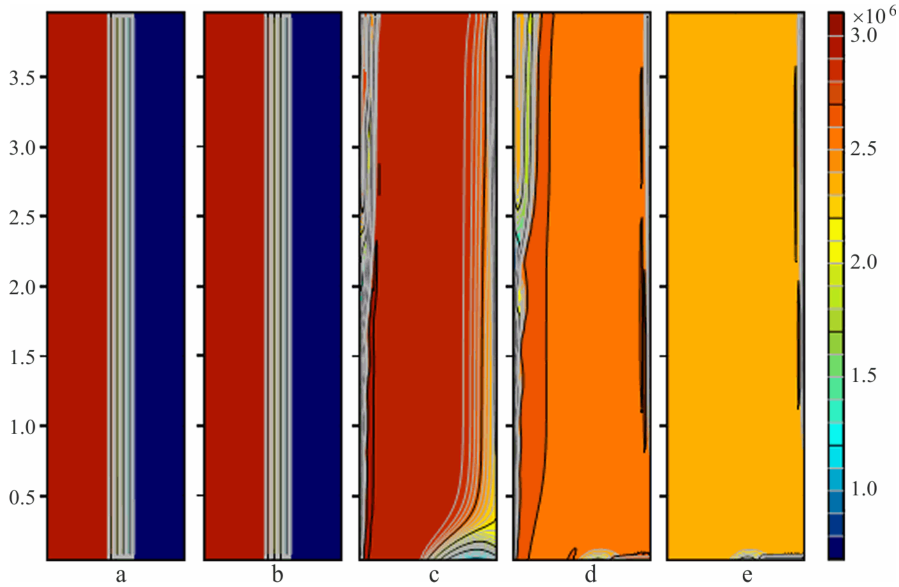

Figure 7. Distribution of the content of dispersed particles for different channel inclination angles: (b) 0°, (c) 25°, (d) 45°, (e) 70°; graph (a) — initial distribution (m^3); viscosity — 10^5 kg/(m·s)

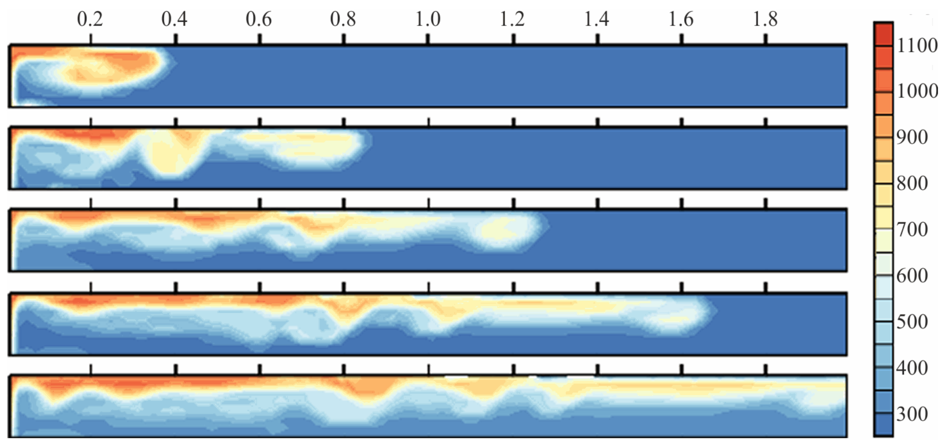

Figure 8. Temperature distribution of a two-phase mixture for different times, the viscosity of the dispersed phase is 10 kg/(m·s)

The change in temperature distribution during intrusion of a high-temperature melt into a horizontal channel filled with a low-temperature mixture of the same mineral composition (for the case of a low-viscosity dispersed phase) is shown in Figure 8.

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