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Particle-in-Cell method with adaptive mass*

T.V. Snytnikova

1. Introduction

Particle-in-Cell methods (PIC) [1–4] are widely used in numerical simulations. The media under study in these methods are represented with a sufficiently large number of model particles with definite characteristics such as mass, charge, velocity. The evolution of a system of particles at each time step is evaluated in two steps. First, the Euler step computes the impact of particles on the medium with fixed particles in grid values. Second, the Lagrange step computes positions and velocities of particles from the equation of motion with the right-hand side computed at the Euler stage.

Since it is possible to trace trajectories of model particles, the PIC method allows studying the evolution of a medium. But it is necessary to keep in mind that model particles do not exactly correspond to real prototypes. Thus, it would be a mistake to directly compare them. The number of the model and the real particles significantly differ. A model particle represents tens of millions real particles but not a single physical atom or an ion.

The PIC method has a relatively low precision due to several sources of errors. One of them is interpolation of forces from the nodes of the Euler grid into the position of particles as well as approximation of grid functions. Another source of errors is the so-called self-force. It is the impact of the particle field on the particle itself through the spatial grid. To decrease errors of the above two given sorts various particle form-factors are used [1, 3, 5, 6].

One more source of errors is statistical fluctuations and noises that arise due to a difference of model particles and real particles. It was proved [4] that the precision of computation depends not only on the time and the the spatial steps, but also on the number of model particles. Thus, the only universal way to reduce noises is to increase the number of particles. But it is not always possible, especially for the 2D and the 3D computations. Moreover, it greatly increases computer costs.

Nevertheless, the PIC method is capable of simulating many physical effects that are unreachable for other computational schemes.

Due to this reason, new methods are being developed that enable us to control the number of particles in a cell: to add new particles if their number is lower than some definite number, or to remove particles if their

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number is too high. The main characteristics such as density, momentum, the center of mass and energy in a cell, must not be changed in the course of adding or removing model particles. At this point, model particles start having different masses. This may improve precision of computation when the process involves masses or numbers of atoms less than the size of one model particle. This situation may take place in the following problems: gas dynamics at the boundary with vacuum, chemical processes with a low concentration of reagents, multistream flows.

Methods of controlling the number of particles (decreasing or increasing) are introduced in [7, 8]. Pairwise coalescence [7] of particles has been previously utilized to manage the number of model particles. The method searches for particles that have matching momenta to a specified tolerance in a given cell. The two particles are combined into a single particle with a mass-averaged position and momentum. The global charge, mass, and momentum are exactly conserved. The energy conservation is only as good as specified tolerance.

The adaptive particle management (APM) method [8] can also be used to reduce or to increase the number of particles in each cell. It computes masses and velocities from arbitrary positions of new particles using the conservation laws. But it allows the particles with negative mass appear. This leads to non-physical results. Such a situation mY takes place when a spatial distribution after using the method is not equal to spatial distribution before using it. If a process under simulation is not sufficiently studied, then more computations would be necessary to attain a good spatial distribution. The particles velocities are calculated conserving not only a global momentum and energy in a cell, but also local ones (in the grid nodes), the velocity distribution is conserved for one-stream flows. However, it is not sufficient for multi-stream flows. That is why this method is not uniform.

In this paper, the PIC method with an adaptive mass is introduced. This enables us to conserve the velocity distribution, and not only density, momentum, the center of mass, energy in a cell. This is important for simulating multi-stream flows. That is why this modification was tested on the Riemann problem for the ion density.

In Section 2, the model problem statement and methods of solution are described. In Section 3, the PIC Method with an adaptive mass is presented. In Section 4, computing experiments for PIC modifications with a constant and adaptive masses are given and their results are compared.

2. Statement of the problem

Let us consider an example of a model 1D Riemann problem for the ion density in a dispersive medium [9]. The medium is a non-isothermal rarified plasma with the Boltzmann electron distribution.

The problem has one dimension along the spatial coordinate x. The following system of equations is the basis:

$$\begin{aligned}
&\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} - \frac{e}{m_i}\frac{\partial \phi}{\partial x}\frac{\partial f}{\partial u} = 0,\\
&\frac{\partial^2 \phi}{\partial x^2} = 4\pi e\left[n_0 \exp\Big(\frac{e\phi}{T_e}\Big) - \int_{-\infty}^{\infty} f\partial u\right],\\
&E(x) = -\frac{\partial \phi}{\partial x}.
\end{aligned} \tag{1}$$

Here f is the ion distribution function, u is the ion velocity, m_i is the ion mass, ϕ is the potential, $E(x)$ is an electric field, n_0 is a non-perturbed plasma density (plasma is assumed to be quasi-neutral), T_e is the electron temperature.

After transition to dimensionless variables and to a finite computation domain the system of equations has the form

$$\frac{\partial x_j}{\partial t} = u_j, \qquad \frac{\partial u}{\partial t} = E(x_j), \qquad \beta\frac{\partial^2 \phi}{\partial x^2} = \exp(\phi) - \rho. \tag{2}$$

Here j is the particle number, the potential is measured in the units of T_e/e, the velocity u is measured in the units of ion sound $c_s = T_e/m_i$, $\beta = (D/L)^2$, $D = (4\pi n_0 e^2/T_e)^{-1/2}$ is the Debye length, $\rho = \int\limits_0^L f\partial u$ is the density, L is the size of computation domain.

Let us set the following initial and boundary conditions:

$$\rho(x,0) = \begin{cases} C, & 0 \le x \le x_0,\\ 1, & x_0 \le x \le L, \end{cases} \qquad u(x,0) = 0; \tag{3}$$

$$u(0,t) = u(L,t) = 0, \qquad \phi(0,t) = \ln C, \qquad \phi(L,t) = 0, \tag{4}$$

where C is the shear of ion density, x_0 is the position of the shear. The boundary conditions are set under assumption that the wave does not reach the domain boundaries, thus the velocity and the potential at the boundaries do not change.

System of equations (2) with the initial and boundary conditions (3), (4) satisfies the conservation laws for mass, impulse and energy [4]:

$$\begin{aligned}
&\int_0^L \rho\, dx = \text{const},\\
&P = \int_0^L \rho u\, dx = (e^{\phi_0} - 1)t,\\
&W = \int_0^L \Big[\frac{1}{2}\rho u^2 + \frac{1}{2}\beta\phi_x^2 + e^{\phi_0}(\phi - 1)\Big]\, dx = \text{const}\,.
\end{aligned} \tag{5}$$

The computation domain is split into N_c equal cells by the nodes with the coordinates $x_k = kh$ $(k = 0, 1, \ldots, N_c)$; $h = L/N_c$ is the splitting step. At the initial moment of time N_k, immobile particles are placed into each cell $[x_{k-1}, x_k]$. The potential ϕ_k and the density ρ_k are computed in the grid nodes and the electric field $E_{k-1/2}$ — in the centers of cells $[x_{k-1}, x_k]$.

The density ρ_k is computed in the grid nodes with the PIC form-factor:

$$\bar{R}(x, x') = \begin{cases} h^{-1}(1 - |x - x'|/h), & |x - x'| \le h, \\ 0, & |x - x'| > h, \end{cases}$$

with the formula

$$\rho_k = \sum_j m_j \bar{R}(x_k, x_j).$$

The electric field is interpolated into the position x_j where the particle is placed with the formula

$$E(x_j) = \frac{1}{h}\big[(x_{k+1/2} - x_j)E_{k-1/2} + (x_j - x_{k-1/2})E_{k+1/2}\big],$$

$x_j \in [x_{k-1/2}, x_{k+1/2}]$.

A computational loop at each time step τ is implemented in the following way.

Lagrange stage. For each particle with number j, the position x_j^{n+1} and the velocity u_j^{n+1} for the moment of time $t = (n+1)\tau$ are computed from the values at the previous moment of time $t = n\tau$ as follows:

$$\frac{u_j^{n+1} - u_j^n}{\tau} = E(x_j^n) \qquad \frac{x_j^{n+1} - {x_j}^n}{\tau} = u_j^{n+1}.$$

Euler stage. Values of the potential in the grid nodes ϕ_k are computed as limit of the sequence ϕ^s of solutions to the linear equation

$$\beta\frac{\phi_{k-1}^{s+1} - 2\phi_k^{s+1} + \phi_{k+1}^{s+1}}{h^2} = \exp(\phi_k^s)(1 + \phi_k^{s+1} - \phi_k^s) - \rho_k,$$
$$\phi_0^{s+1} = \ln C, \qquad \phi_{N_c}^{s+1} = 0.$$

This equation is solved with the sweep method. The value of the potential obtained at the previous time step is set as an initial approximation. The iteration process goes until the condition $\|\phi^{s+1} - \phi^s\| < \epsilon = 10^{-8}$ is satisfied. Then the electric field is computed with the formula

$$E_{k-1/2} = \frac{\phi_{k-1} - \phi_k}{h}.$$

3. Definition of the PIC method with adaptive mass

It is known [4] that the precision of calculation for the PIC method depends on the number of model particles in a cell. Therefore to solve equations, it is necessary to know the number of cells, in which the number of particles is reduced below the threshold. In this case, it is necessary to add particles to a cell for reaching the given level N^*. In addition, one should decide how mass, velocity and position of new particles are determined. The conservation laws and velocity distribution should be satisfied. The latter condition is necessary for a multi-stream velocity distribution to be undestroyed.

For keeping this the following items are introduced:

- The velocity step $h_u = 2v_{\max}/N_v$, here N_v is a positive integer, $v_{\max}$ satisfies the condition $\forall j, 1 \le j \le J_m, |u_j| \le v_{\max}$ at any time step.
- A set of particle indices $N_i = \{j : v_i < u_j \le v_{i+1}\}$, here $v_i = ih_u - v_{\max}$. Let us note that $N_i \cap N_j = \emptyset$ and $\bigcup N_i = N$.
- The total mass of particles belonging to N_j,

$$M_i = \sum_{j \in N_i} m_j. \tag{6}$$

- The total energy

$$Q_i = \sum_{j \in N_i} m_j u_j^2/2. \tag{7}$$

- The weighed average velocity

$$V_i = \begin{cases} \sum_{j \in N_i} \frac{m_j u_j}{M_i}, & N_i \neq \emptyset, \\ 0, & N_i = \emptyset. \end{cases} \tag{8}$$

- The centers of mass

$$X_i = \begin{cases} \sum_{j \in N_i} \frac{m_j x_j}{M_i} & N_i \neq \emptyset, \\ 0, & N_i = \emptyset. \end{cases} \tag{9}$$

Now, the procedure of particle adding is split into the following steps for every N_i':

1. Calculating the number of particles k_i;
2. Calculating the mass m_j' so that $M_i' = M_i$;
3. Calculating the velocity u_j' so that $V_i' = V_i$ and $Q_i' = Q_i$;
4. Calculating the position x_j' so that $X_i' = X_i$.

Each of them may be performed in several ways.

3.1. Calculating the particles number and masses. One of possible methods to calculate the number of particles is an equal distribution between all non-empty sets N_i, that is

$$k_i = \begin{cases} K, & N_i \neq \emptyset, \\ 0, & N_i = \emptyset, \end{cases}$$

where $K = N^* / \sum_{\{i:N_i \neq \emptyset\}} 1$. This is the same as

$$k_i = \begin{cases} K, & M_i \neq 0, \\ 0, & M_i = 0. \end{cases}$$

Another method is a probabilistic one:

$$k_i = \begin{cases} \max\Big(\frac{N^* M_i}{\sum M_j},\ 1\Big), & M_i \neq 0, \\ 0, & M_i = 0. \end{cases}$$

The first condition provides a greater number of particles in the sets N_j, where the total mass is greater. The second condition prevents the loss of particles when the total mass of a set is less than the mass of one particle. Also, one needs to be sure that $\sum k_i \leq N^*$.

For both methods

$$m'_j = M_i / k_i. \tag{10}$$

When the first method is used, the particles with the near zero mass can appear. This leads to incorrect results, because the velocities are too high for such particles. Also, particles with a mass greater than the average mass can appear. This leads to an increase in the noise of density computation.

With a probabilistic method, these problems do not arise. But non-uniform distributions of particles into the sets N'_i can occur. The probabilistic method is chosen for carrying out experiments.

3.2. Velocity computation. To calculate the velocities of particles in a set N'_i the following conditions should be satisfied:

1. Momentum conservation in a cell $\sum m'_j u'_j = \sum m_j u_j$;
2. Energy conservation $Q'_i = Q_i$;
3. The condition of belonging to N'_i: $v_i < u'_j \leq v_{i+1}$.

Let us set velocities of the new particles in the following way:

$$u'_j = \begin{cases} V_i - \Delta, & j < [\frac{k+1}{2}], \\ V_i, & j = [\frac{k+1}{2}], \\ V_i + \Delta, & j > [\frac{k+1}{2}], \end{cases} \tag{11}$$

where

$$\Delta^2 = \begin{cases} \dfrac{2Q_i}{M_i} - V_i^2, & k_i = 2p, \\ \dfrac{k_i}{k_i - 1}\Big(\dfrac{2Q_i}{M_i} - V_i^2\Big), & k_i = 2p+1. \end{cases} \tag{12}$$

Let us check that the momentum is conserved after adding particles. To do this, let us substitute the mass (10) into the left-hand side of the momentum conservation equation

$$\sum m'_j u'_j = \sum (M_i/k_i) u'_i = M_i/k_i \sum_{j \le k_i} u'_j.$$

After substituting the velocity (11), for even $k_i = 2p$, we have

$$\sum m'_j u'_j = M_i/k_i \Big[\sum_{j \le p} (V_i - \Delta) + \sum_{p < j \le 2p} (V_i + \Delta) \Big] = M_i V_i,$$

and, for odd $k_i = 2p+1$, we derive

$$\sum m'_j u'_j = M_i/k_i \Big[\sum_{j \le p} (V_i - \Delta) + V_i + \sum_{p+1 < j \le 2p+1} (V_i + \Delta) \Big] = M_i V_i.$$

After substituting the weighed average velocity (8), we conclude

$$\sum m'_j u'_j = M_i V_i = M_i \frac{\sum m_j u_j}{M_i} = \sum m_j u_j.$$

This proves that the momentum in a cell is conserved after adding particles.

Now, let us check the energy conservation. We substitute mass (10) and velocity (11) into the left-hand side of the energy conservation equality and apply (12). For even $k_i = 2p$, we have

$$\begin{aligned} Q'_i &= \sum_{j \in [1, k_i]} \frac{m'_j {u'_j}^2}{2} = \sum_{j \in [1,p]} \frac{M_i (V_i - \Delta)^2}{2k_i} + \sum_{j \in [p+1, k_i]} \frac{M_i (V_i + \Delta)^2}{2k_i} \\ &= \frac{pM_i}{2k_i} \big[(V_i - \Delta)^2 + (V_i + \Delta)^2 \big] = \frac{M_i}{2} (V_i^2 + \Delta^2) \\ &= \frac{M_i}{2} \Big(V_i^2 + \frac{2Q_i}{M_i} - V_i^2 \Big) = Q_i. \end{aligned}$$

For odd $k_i = 2p + 1$, we derive

$$Q'_i = \sum_{j\in[1,k_i]} \frac{m'_j {u'_j}^2}{2} = \sum_{j\in[1,p]} \frac{M_i(V_i-\Delta)^2}{2k_i} + \frac{M_i V_i^2}{2k_i} + \sum_{j\in[p+1,k_i]} \frac{M_i(V_i+\Delta)^2}{2k_i}$$
$$= \frac{M_i}{2k_i}\left[p(V_i-\Delta)^2 + {V_i}^2 + p(V_i+\Delta)^2\right] = \frac{M_i}{2k_i}\left[k_i V_i^2 + (k_i-1)\Delta^2\right]$$
$$= \frac{M_i}{2k_i}\left[k_i V_i^2 + \frac{(k_i-1)k_i}{k_i-1}\left(\frac{2Q_i}{M_i} - V_i^2\right)\right] = Q_i.$$

Thus, the energy conservation low is valid.

Let us note that Δ is the standard deviation of the velocity. So, it can be expected that the velocity is within the given boundaries.

3.3. Position computation. The position of particles is given by the following way: $x'_j = X_i + \delta_j$, where δ_j satisfies the following conditions: $h(l-1) < X_i + \delta \le hl$ and $\sum \delta_j = 0$, here l is the number of a cell.

The first condition provides that particles are in the l-th cell. The second condition provides that the position of the mass center is conserved. This means that the density in grid nodes does not change.

4. Numerical experiments

The following modifications of the PIC method are used to solve the problem:

PIC-1 with constant mass: particles have the same constant mass. At the initial moment they are distributed with some density shear.

PIC-2 with an adaptive mass: at the initial moment, particles are uniformly distributed. The density shear is set by different masses of particles. If the number of particles in a cell is lower than a given level, then new particles with mass recomputation for each particle in this cell are added.

For both modifications the computations were conducted on the grid with 1001 nodes ($N_c = 10^3$ cells) and $N_p = 10^4$, 10^5 particles and on the grid with 10001 nodes ($N_c = 10^4$ cells) and $N_p = 10^5$ particles. Though the computational domain is equal to the interval $[0, 100]$, the results are shown for a smaller interval $[10, 60]$.

To compare the results obtained let us introduce the definition of noise. Noise is considered to be a relative change of density when a particle moves:

$$Sh = \max_j \frac{n(x_l,t) - n(x_l,t')}{n(x_l,t)} \cdot 100\,\%,$$

here j is a particle number, l is the number of a cell that contains this particle.

Because this value is very small for real plasma, a change in the model plasma density is non-physical. This is because model particles are larger than real ones.

4.1. The PIC method with a constant mass. Masses of all particles are equal and constant in time in this modification of the PIC method. At the initial moment, particles are distributed with some density shear. Because the shear is $C_n = 40$ and the number of particles is relatively low (10^4 for 10^3 cell), the number of particles in a cell is equal to:

- 44–45 in a high-density area ($0 < x < x_0 = 20$);
- 1–2 in a low-density area ($x_0 < x < L = 100$).

The distribution of particle velocities is shown at the top graphic of Figure 1. The density and the potential are shown at the bottom. Though a distance between the adjacent particles is equal at the initial moment of time, a density disturbance arises and causes the potential perturbation. In a low-density area, values vary from 0.98 to 1.08. This equals to 10 % of an average value in this area. The potential changes are from -0.007 to 0.013. This results in the electric field increase and affects particles. Consequently, density is affected. Later (see Figure 1) the density varies from 0.5 to 1.3. This equals 80% of an average value. The potential varies from -0.09 to

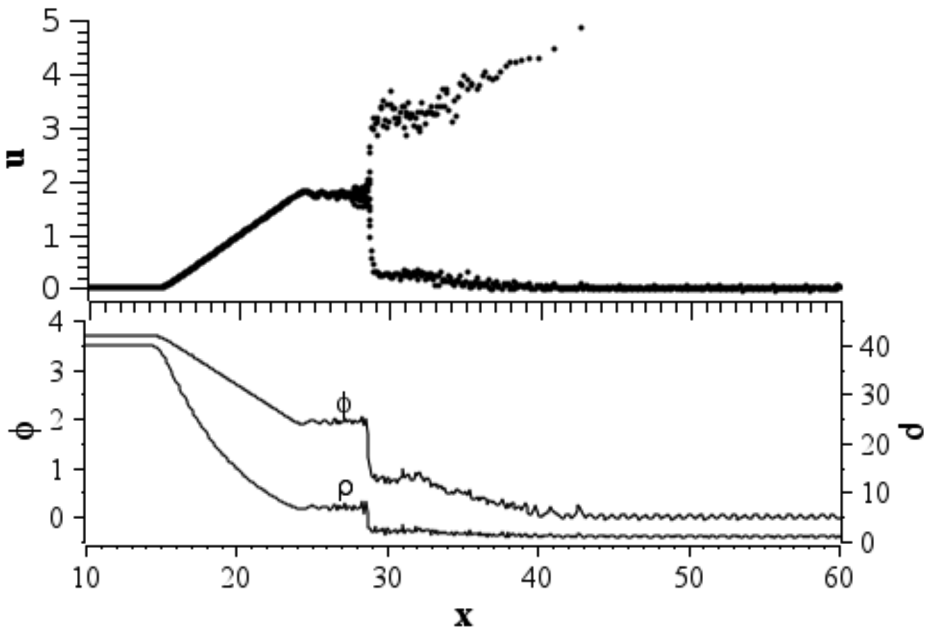

Figure 1. PIC-1 method for $N_c = 10^3$, $N_p = 10^4$ at $t = 5$

0.06. Particles perform oscillatory movements in the area, where the average potential equals 0.

Let us note that for computation with these values of the parameters N_p and N_c, noise attains 100 % in a low-density area. This greatly affects the potential: a potential jump is observed in the position of the first particle ($x = 43$) in the top stream (see Figure 1). To decrease the noise to 10 %, it is necessary to provide, at least, 10 particles per cell in a low-density area (about 400 particles per cell in a high-density area) at the initial moment. This means that the number of particles N_p should be equal to 10^5 for $N_c = 10^3$. And $N_p = 10^6$ (100/4000 particles per cell) to decrease the noise to 1 %. With increasing the number of particles in a cell, the initial fluctuation is also decreased to 1.5 and 0.0015 %, respectively.

Another feature of the solution is at the flow integration point ($x = 27$), because of an insufficient number of particles (due to their large size): particle movement is discrete, but not continuous. This feature is not distinctly expressed for $N_p = 10^5$.

But increasing the number of particles is not always possible because of increasing the number of resources that are necessary to solve problems. N_p is increased up to 10^{10} for 10 % noise level if a 2D problem is solved on a grid with 10^3 nodes per axes, and up to 10^{15} for a 3D problem. Let us note that for the noise level to be unchanged, one needs to increase the number of particles in the same extent as for the number of grid nodes.

4.2. The PIC method with an adaptive mass. In this modification, the density shear is set with different masses of particles. Particles are uniformly distributed among the cells. At the initial moment, each cell contains $N^* = 10$ particles ($N_c = 10^3$, $N_p = 10^4$). This distribution allows keeping off a density fluctuation and consequently a potential fluctuation.

At the initial moment, masses of particles from a high-density area ("heavy" particles) differ from the masses of particles from a low-density area ("light" particles) by C_n times. But after decreasing the number of particles that are lower than $0.5N^*$ in a cell, new particles with mass redistribution are added: "heavy" particles are broken up and "light" particles are consolidated. Figure 2 shows the particle mass distribution in an area of motion ($15 \le x \le 45$) at the moment $t = 5$. The steps between the values of masses of the "light" ($m = 0.1$) and "heavy" ($m = 4$) particles correspond to redistributions of mass while adding particles into a cell.

As for the previous modification, the velocities distribution, the density and the potential are given at the moment $t = 5$ in Figure 3. In a low-density area (the interval $[45, 100]$), there is no density fluctuation ($\delta\rho = 0$) and particles are at rest at the initial moment and later on. Also, noise is decreased in the first front of a wave. When a "light" particle moving, the density increases on 10 %. Particles flying from a high-density area to a low-

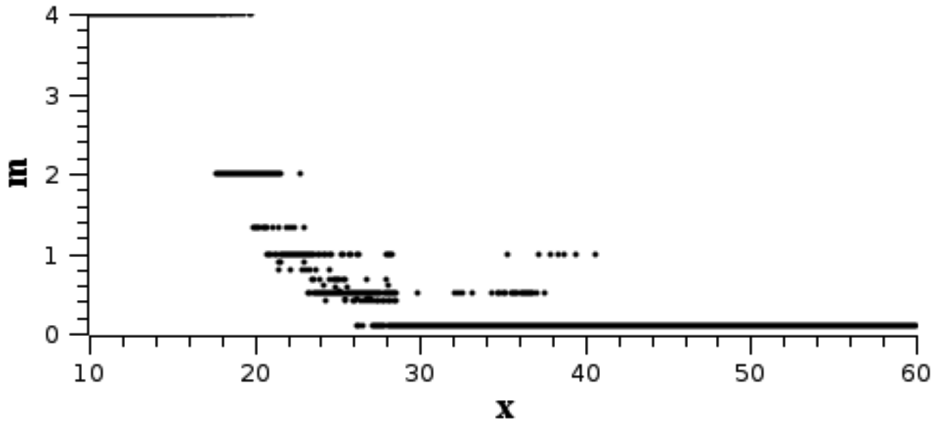

Figure 2. Distribution of particles by mass for $N_c = 10^3$, $N_p = 10^4$ at $t = 5$

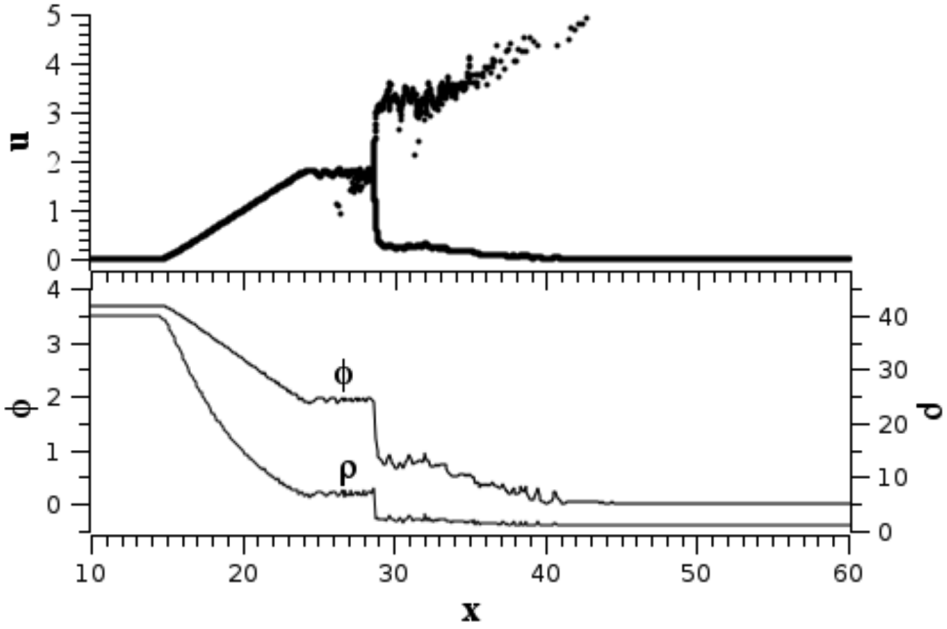

Figure 3. PIC-2 method for $N_c = 10^3$, $N_p = 10^4$ at $t = 5$

density area do not give more than 80 % of noise. In general, noise is less than in the previous modification, where noise is equal to 100 %. This is expressed by the absence of a potential jump at $x = 43$ and by a smooth decrease of the potential in the interval $[41, 45]$ ($\delta\phi = 0.01$) (Figure 3). Let us note that the particle movement is more continuous at the flow integration point ($x = 27$).

With an increase of the number of particles per cell to one hundred ($N_c = 10^3$, $N_p = 10^5$), noise is decreased to 1 and 6 %, respectively. This is much lower than in the previous modification.

It should be noted that the number of particles does not decrease below a given level ($0.5N^*$ for $N^* = 10$). Though the total number of particles is increased with time, it is equal to 10901 (less than 10 %) at $t = 5$ and to 13072 (about 30 %) at $t = 15$ when particles reach the right boundary.

There are some particles that have velocities that do not correspond to the common character. But this peculiarity occurs for the PIC modification with a constant mass for $N_c = 10^3$, $N_p = 10^5$ as well.

4.3. Comparison of modifications. In order to evaluate the obtained results, let us compare the fulfilment of conservation laws (5).

In the table, deviations from a constant value for the momentum δP, the energy δW, the density $\delta\rho$, and the noise level of density Sh are given for both modifications of the PIC method.

Method	δP	δW	$\delta\rho$	Sh, %
$N_c = 10^3$, $N_p = 10^4$				
PIC-1	$2.8 \cdot 10^{-2}$	$9.0 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$	100
PIC-2	$1.0 \cdot 10^{-3}$	$4.9 \cdot 10^{-4}$	$3.0 \cdot 10^{-11}$	10/80
$N_c = 10^3$, $N_p = 10^5$				
PIC-1	$7.5 \cdot 10^{-4}$	$3.7 \cdot 10^{-4}$	$6.5 \cdot 10^{-6}$	10
PIC-2	$7.5 \cdot 10^{-4}$	$4.3 \cdot 10^{-4}$	$2.0 \cdot 10^{-10}$	1/6
$N_c = 10^4$, $N_p = 10^5$				
PIC-1	$1.5 \cdot 10^{-4}$	$1.1 \cdot 10^{-5}$	$6.3 \cdot 10^{-5}$	100
PIC-2	$2.4 \cdot 10^{-5}$	$4.8 \cdot 10^{-6}$	$3.0 \cdot 10^{-11}$	10/30

For $N_p/N_c = 10$, the conservation laws are more accurately fulfilled using the PIC-2 method with adaptive mass.

If only the number of particles is increased, then, for the PIC-1 method with a constant mass, the accuracy significantly increases, but, for the PIC-2 modification, the accuracy does not essentially increase. Modifications are equivalent in the accuracy order, but the computer costs are lower for the PIC-1 modification.

On the other hand, the PIC-2 method for $N_p/N_c = 10$ is compared in accuracy with the PIC-1 method with $N_p/N_c = 100$ and is more accurate than the latter with the same parameters.

5. Conclusion

Two modifications of the PIC method were considered. Each of them has positive and negative features.

PIC-1 with the same constant mass:

- initial density fluctuation;
- a large number of particles is required for calculation a correct solution.

PIC-2 with an adaptive mass:

+ the absence of the initial fluctuation of density;
+ an equal distribution of particles by cells at the initial time;
+ the possibility of supporting the number of particles above the given level;
− increasing the total number of particles;
− increasing the computation time as compared to PIC-1 with the same parameters N_p and N_c.

Let us note that despite some negative features of the PIC-2 method with an adaptive mass, it gives better results than PIC-1 for a relatively low number of particles in cells ($N_p/N_c = 10$). This allows in increasing the number of grid nodes thus giving a significant increase of accuracy.

References

[1] Hockney R.W., Eastwood J.W. Computer Simulation Using Particles. — MacGraw-Hill, International Com., 1981.

[2] Birdsall C.K., Langdon A.B. Plasma Physics in Computer Simulation. — Adam-Hilger, 1991.

[3] Berezin Yu.A., Vshivkov V.A. Particle Method in Rarefied Plasma Dynamic. — Novosibirsk: Nauka, 1980 (In Russian).

[4] Grigoryev Yu.N., Vshivkov V.A., Fedoruk M.P. Numerical Particle-in-Cell Methods: Theory and Applications. — VSP, 2002.

[5] Vshivkov V.A., Romanov D.V., Snytnikov V.N. The problem of spontaneous heating of model plasma in the method of particles // Computational Technologies. — 1999. — Vol. 4, No. 3. — P. 62–72.

[6] Vshivkov V.A., Terekhov A.V. On the self-force in the particle-in-cell method // Numerical Methods and Programming. — 2008. — Vol. 9. — P. 48–57.

[7] Lapenta G., Brackbill J.U. Dynamic and selective control of the number of particles in kinetic plasma simulations // J. Computational Physics. — 1994. — Vol. 115. — P. 213–217.

[8] Welch D.R., Genoni T.C., Clark R.E., Rose D.V. Adaptive particle management in a particle-in-cell code // J. Computational Physics. — 2007. — Vol. 227. — P. 143–155.

[9] Sagdeev R.Z. Collective processes and shock wares in a rarefied plasma // Voprosy Teorii Plazmy. — Moscow: Atomizdat, 1964. — Vol. 4. — P. 20–80 (In Russian).