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A numerical model of density currents in estuaries of the siberian rivers*

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Abstract. A numerical model for studying the dynamic mixing of the sea and river water in the estuarial area is proposed. Computations are based on the two-dimensional longitudinal vertical stratified fluid mechanics equations and the equation of transport of salt. The model focuses on the reproduction of local density currents at the mouth of arms branched deltas of the rivers of Siberia. The results of numerical experiments are given, the dynamic structure of the flow and salinity profiles are compared to the observational data.

Keywords: numerical simulation, turbulence, gradient-density flows, river flow, sea area, transport of salt

1. Introduction

The problem of the influence of river flows on the sea water is attracting increasing attention of domestic and foreign researchers because of the economic development of the marginal seas of the Siberian Arctic. The removal of substances from freshwater fluvial systems into the sea is accompanied by complex phenomena due to the stratification and difference in water densities through mineralization [1]. The observations reveal that in the process of mixing, the sea water forms a thin layer of salt on the bottom of the channel, extending upstream, while the brackish river water moves on top to the mouth. The result is a water front, i.e., a narrow contact boundary between the river and sea water, and the flow is a divergent bundle of two greatly different in their characteristics layers: the upper fresh and the underlying cold salty. A zone of a dynamic rupture is formed because of a large density gradient on the waterfront line, the vertical exchange between the layers is blocked, thereby isolating and stabilizing the salt wedge.

The inflow of saline water into the channel significantly impairs the consumer and hydro-biological quality of water making it unsuitable for drinking and industrial needs. For a waterfront, the concentration of dissolved oxygen decreases, the normal functioning of the river ecosystems is complicated and hydro-chemical composition of the river water changes. The estuarial flow, in turn, has a significant impact on the water balance of receiving marine water, thus causing the desalination of the upper layers, changing the thermal and ice conditions of the ocean water.

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Water mixing is exclusively mobile, and the configuration and position of the waterfront depends on many factors: the water flow in the river, the force and direction of wind, stratification, etc., and changes in the diurnal and seasonal time scales. Detecting the boundaries of penetration of salt water into the channel and the influence of rivers on the marine area of water is of great scientific and practical interest for the development of monitoring systems and forecasting of hydrological processes in the sub-Arctic regions.

By now, patterns of the interaction of water flows at the mouths of the northern rivers have not been sufficiently studied in terms of theory. The first mathematical models of estuaries were formulated based on simple one-dimensional equations that describe processes of longitudinal displacement of the sea water into the channel under the influence of the pileup effects [2]. More meaningful 2D models from the standpoint of physics have already contained the vertical detailing of interaction processes [3], [4], however the description of the turbulent exchange was based on relatively simple postulates of the turbulence scale.

The nature of estuarial processes largely depends on the morphometry features of the channel and occurs differently in the mouth of each. Most earlier performed theoretical studies of the Siberian rivers concerned large estuarial areas such as those of the rivers Ob and Yenisei with characteristic scales of processes of about 100 *km*.

Gradient density flows are evolving in estuarial areas of other morphological types such as branched river deltas. An example of such a river system is the river Lena delta. The main channel of the river is divided into a number of channels, which are then divided into a dense net of branches flowing into the Laptev Sea. Each river sleeve forms a local zone of water mixing at its mouth, the whole set of such zones having the total determining influence on the flow field near to the coast as a whole. Note that in recent years the research into the delta of the Lena River has received impressive support. In particular, a laboratory and test sites with modern equipment have been formed that are intended for expeditions on the coast of the Laptev Sea [5].

For obtaining integral estimations of the influence of the continental runoff on the mouth coast, it is necessary to consider local features of flows with spatial scales of individual branches of river deltas. According to the above-said we define the objective of this study as constructing a numerical model of an estuarial area, based on stratified fluid hydrodynamic equations, aimed at investigation of local density currents through mixing the sea and river water.

2. Statement of the problem

The use of a three-dimensional mathematical model for describing the above-mentioned processes as applied to watercourses in northern latitudes is not optimal due to the lack of morphometry data needed for spatial detalization. In addition, it seems important to study the mechanisms of mixing water in estuarial areas of simple geometry. For this reason, the schematization of a longitudinal vertical flow is taken as basis, which has shown satisfactory results in a number of studies [3], [6].

For obtaining the basic equations we start with a system describing the flow in a stratified fluid [7]. Let $y = b(x, z)$ be a variable width of a channel flow, the axis x is directed horizontally along the main flow, the axis z is directed vertically upwards. The averaged over cross-section (along the coordinate y) equations of motion and continuity, as well as the transport of salinity in the longitudinal vertical plane will take the form [8]

$$\begin{aligned} &\frac{\partial bu}{\partial t} + \frac{\partial buu}{\partial x} + \frac{\partial buw}{\partial z} \\ &\qquad = -gb\frac{\partial \zeta}{\partial x} - \frac{gb}{\overline{\rho}} \cdot \frac{\partial}{\partial x}\int_z^\zeta \rho' dz + \frac{\partial}{\partial x}\Big(bK_x\frac{\partial u}{\partial x}\Big) + \frac{\partial}{\partial z}\Big(bK_z\frac{\partial u}{\partial z}\Big), \\ &\frac{\partial bu}{\partial x} + \frac{\partial bw}{\partial z} = 0, \\ &\frac{\partial bs}{\partial t} + \frac{\partial bus}{\partial x} + \frac{\partial bws}{\partial z} = \frac{1}{Sc}\Big[\frac{\partial}{\partial x}\Big(bK_x\frac{\partial s}{\partial x}\Big) + \frac{\partial}{\partial z}\Big(bK_z\frac{\partial s}{\partial z}\Big)\Big], \end{aligned} \tag{1}$$

where u, w are the horizontal and vertical velocity components, $z = \zeta(x, t)$ is the equation of the free surface, $\overline{\rho}$ is the average density of water, ρ' is the density perturbations due to mineralization, K_x, K_z are the coefficients of the horizontal and vertical turbulent exchange, s is the concentration of the sea salinity, Sc is the Schmidt number for the salt solution. In (1), the hydrostatic approximation conventional for natural watercourses is taken as suggesting a relative smallness of the vertical velocities and accelerations.

System (1) is supplemented by the equation of state that relates density with salt concentration

$$\rho = \rho(s). \tag{2}$$

System (1) is closed by relation (2), which is used to calculate buoyancy forces and variations of the density field in the equation of motion. The temperature effect on the dynamic behavior of estuarial water in a cold climate is insignificant [9], therefore the thermal factors are not taken into account in (1), (2).

Let us denote the equation of bottom surface by $z = z_b(x)$, the vertical flow of salt by $P_z = b\Big(ws - K_z\frac{\partial s}{\partial z}\Big)$, and formulate the boundary conditions. On the bottom line let us set

$$K_z\frac{\partial u}{\partial z} = c_d \cdot |u| \cdot u, \quad w = \frac{\partial z_b}{\partial x}u, \quad P_z = 0 \quad \text{at } z = \zeta_b, \tag{3}$$

where c_d is the coefficient of the bottom resistance. On the free surface, the surface boundary conditions are of the form

$$K_z \frac{\partial u}{\partial z} = 0, \quad \frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} = w, \quad P_z = 0 \quad \text{at } z = \zeta. \tag{4}$$

The calculation of parameters of the vertical turbulent exchange is based on the balance equation of turbulent kinetic energy e and the equation for its dissipation rate ε, which according to [10] have the form

$$\frac{\partial\, be}{\partial t} + \frac{\partial\, bue}{\partial x} + \frac{\partial\, bwe}{\partial z} = \alpha_e \Big(\frac{\partial}{\partial x} b K_x \frac{\partial e}{\partial x} + \frac{\partial}{\partial z} b K_z \frac{\partial e}{\partial z} \Big) + b K_z J - b\varepsilon,$$
$$\frac{\partial\, b\varepsilon}{\partial t} + \frac{\partial\, bu\varepsilon}{\partial x} + \frac{\partial\, bw\varepsilon}{\partial z} = \alpha_\varepsilon \Big(\frac{\partial}{\partial x} b K_x \frac{\partial \varepsilon}{\partial x} + \frac{\partial}{\partial z} b K_z \frac{\partial \varepsilon}{\partial z} \Big) + c_2 b \frac{\varepsilon}{e} K_z J - c_3 b \frac{\varepsilon^2}{e},$$
$$K_z = c_\mu \frac{e^2}{\varepsilon}, \tag{5}$$

where α_e, c_μ, c_2, c_3 are empirical constants, $J = u_z^2 + \frac{g}{\rho}\frac{\partial \rho'}{\partial z}$ is the rate of replenishment of the turbulence energy.

Before proceeding to the boundary conditions for the fields of turbulence, let us note the following. In some publications [3], the following ratios on the free surface are used:

$$\frac{\partial e}{\partial z} = 0, \quad \frac{\partial \varepsilon}{\partial z} = 0 \quad \text{at } z = \zeta.$$

These conditions do not provide the damping of K_z on the free surface, which is typical of the flow in the absence of wind. On the contrary, from system (5) it follows that under these conditions $\frac{\partial K_z}{\partial z}$ holds, which means that a monotonic increase in the coefficient of turbulence with a distance from the bottom and reaching a maximum at the surface. Such a profile as K_z contradicts the observed vertical structure of the flow, in which the shift rate is zero, and the flow of turbulent energy is decreasing towards the free surface. Consequently, the above-written conditions for e and ε are physically incorrect and one should use other conditions, such as

$$e = 0, \quad \frac{\partial \varepsilon}{\partial z} = 0 \quad \text{at } z = \zeta. \tag{6}$$

On the bottom, let us set the following boundary conditions

$$\frac{\partial e}{\partial z} = 0, \quad \varepsilon = c_\varepsilon \frac{e^{3/2}}{z_{sb}} \quad \text{at } z = z_b, \tag{7}$$

where z_{sb} is the roughness of the bottom, c_ε is an empirical constant.

When setting boundary conditions on the upstream section (in the channel of the river) there is assumed to be no significant heterogeneity of a flow, for example, due to a sharp change of the bottom topography at the input alignment. As consequence, in equations (1), (5) the horizontal variations of desired fields can be neglected, and it appears possible to consider the stationary one-dimensional (with respect to z) problem:

$$
\begin{aligned}
& - g\Big[b\frac{\partial\zeta}{\partial x}\Big]_{x=x_1} + \frac{\partial}{\partial z}\Big(bK_z\frac{\partial u}{\partial z}\Big) = 0,\\
& \frac{\partial}{\partial z}bK_z\frac{\partial e}{\partial z} + bK_zJ - b\varepsilon = 0,\\
& \alpha\frac{\partial}{\partial z}bK_z\frac{\partial \varepsilon}{\partial z} + c_2b\frac{\varepsilon}{e}KJ - c_3b\frac{\varepsilon^2}{e} = 0,\\
& K_z = c_\mu\frac{e^2}{\varepsilon}.
\end{aligned}
\tag{8}
$$

For equations (8), the boundary conditions (2), (3), (6) are set. The functions u_1, e_1, ε_1, obtained with one-dimensional model are boundary values in the boundary condition on the section $x = x_1$:

$$
u = u_1, \quad s = 0, \quad e = e_1, \quad \varepsilon = \varepsilon_1 \quad \text{at} \quad x = x_1. \tag{9}
$$

At the exit alignment of the section $x = x_1$ (the mouth seashore) set the conditions

$$
\frac{\partial u}{\partial x} = 0, \quad s = s_2, \quad \frac{\partial e}{\partial x} = 0, \quad \frac{\partial \varepsilon}{\partial x} = 0 \quad \text{at} \quad x = x_2, \tag{10}
$$

where s_2 is the sea water salinity.

The system of equations for determining the shape of the free surface is formed from the equation of motion and the continuity equation in (1), integrating with respect to z in terms of boundary conditions. The integration is carried out over a discrete space. The equation of motion produces a ratio for the total momentum $Q_i = \sum_{k=1}^{K} b_{ik}u_{ik}\Delta z$, and the continuity equation implies an unsteady relation between Q_i and $\frac{\partial\zeta_i}{\partial t}$. With excluding Q_i from the system we obtain a one-dimensional finite difference equation for the level ζ, whose differential analog can be presented as

$$
\frac{\partial^2\zeta}{\partial t^2} = \frac{\partial}{\partial x}gh\frac{\partial\zeta}{\partial x} + \frac{\partial}{\partial y}gh\frac{\partial\zeta}{\partial y} + F_\zeta, \tag{11}
$$

where F_ζ contains combinations of the terms, integrated by z, of the equations of motion. Equation (11) is hyperbolic and describes non-stationary wave oscillations of the free surface. The equation is solved by the factorization method with the boundary conditions

$$
Q = Q_1 \ \text{at} \ x = x_1, \quad \zeta = \zeta_2 \ \text{at} \ x = x_2, \tag{12}
$$

where Q_1 is a given flow of water in the channel, ζ_2 is a certain level of the sea.

Numerical methods for solving the problem are based on implicit algorithms using conservative schemes. Finite difference analogs of the numerical model are derived from the energy relations and preserve the total mass of liquid at each time, as well as the balance of kinetic and potential energy (exact execution of the Bernoulli integral in without vortex flow) [11]. When integrating the transport equation of concentration the monotonic TVD-scheme of the second order of accuracy is used. Bottom topography is taken into account by the transition to the curvilinear sigma coordinates. The spatial resolution is characterized by the grid steps $\Delta x = 100$ m horizontally and $\Delta z = 0.1 \div 0.5$ m vertically.

3. The calculation results

The solution domain in the vertical plane is shown in Figure 1 and is a river channel passing into the estuarial coast. The origin of coordinates is compatible with the "river–sea" boundary. The slope of decreasing bottom we define to be $0.5 \cdot 10^{-3}$, which is close to characteristic values of the slope of the coastal zone of the Laptev Sea [12], and the slope of the free surface of a uniform river flow $\dfrac{\partial \zeta}{\partial x}$ in system (8) is chosen so that the unperturbed velocity be 0.25 m/s at a maximum. The depth of the flow is assumed to be 5 m. In the river part of the stream, the channel width b is assumed to be constant, and in the marine waters it increases linearly with the angle of 32°, which is observed in the process of spreading the river flow [13].

Water flows and velocities in the sleeves of deltas widely vary and can differ in either direction from that presented above [5]. However, at high velocities of a river flow, the counterclaims sea waters are replaced by the runoff current, and the intrusion is insignificant and the effects of advancing the salt wedge into the channel are weakly expressed. The accepted values of parameters make possible to distinctly reflect in the solution the most interesting details and distinguish characteristic features of the stream.

The total length of the computational domain we define to be equal to 70 km, of which 20 km stretch of the river belong to the river part. The initial distribution of the longitudinal velocity and fields of the turbulence model is given by an ensemble of the vertical profiles, calculated by system (8) at each node of the horizontal grid. At the initial moment, the salinity s is taken to equal zero in the river part ($x < 0$) and to equal $s_{\max}$ in the sea water ($x > 0$), where $s_{\max} = 30$ ‰ is the concentration of salt on the Laptev Sea coast [12].

The equations were integrated with respect to the time with a step $\Delta t = 30$ s before attaining the steady state. In the course of integration, the initial salinity field varied with forming the internal and surface waves. Since the sea water is heavier than the river water, the salt water descends and spreads above the bottom layer thus forming a mineralized "tongue", which penetrates into the channel. The salt water intrusion into the sea is accompanied by the formation of a deep oncoming flow directed to the shore.

The circulation in the area of interaction is shown in Figure 1a presenting the calculated steady flow streamlines.

The streamlines reflect the presence of two dynamic structures in the stream: the basic runoff flow directed to the sea, and the compensating countercurrent, formed a result of leakage of the saline water from the bottom into the channel. The interface between the sea and river waters coincides with isotach of zero velocity, which determines the hydro-front position and develops the jump salinity. In Figure 1a, the hydro-front (a dotted line) starts on the bottom of the channel at the point that is about 6 km apart from the estuarial alignment then it rises into the flow interior and in the marine waters it gradually merges with the water table.

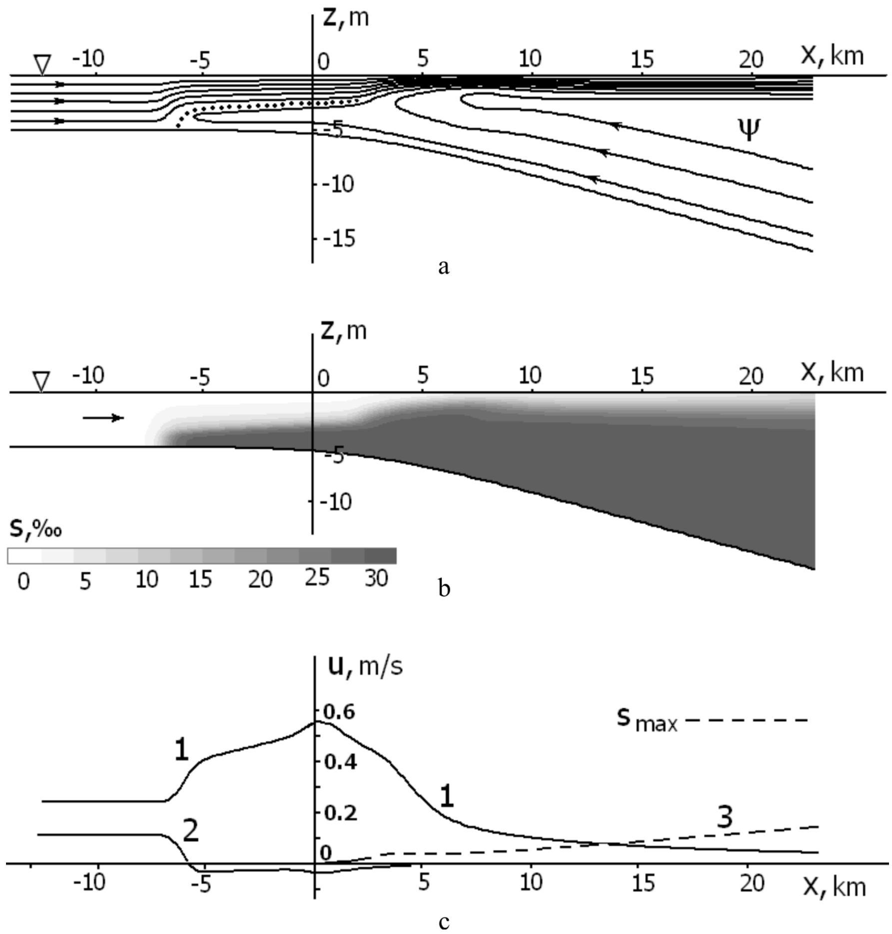

Figure 1. Calculation results: a) contours of the stream function; b) the salinity field in the estuarial area; c) the distribution of the longitudinal velocity on the surface and on the bottom (Curves 1, 2) and the surface salt concentration (Curve 3)

The interface between water masses is distinct when analyzing the salinity field in the solution obtained (Figure 1b). In this case, the hydro-front can be identified by rather a sharp color boundary between the dark color (salt water) and the light color (fresh water) areas. On a part of the estuarial sea coast, the interface gradually blurs ($x > 7$ km) at the expense of the turbulent mixing of the layers. The salt water is flagged for the upper part of the watercourse, on which salt is factored back into the sea. The surface salt content slowly increases downstream as shown by Curve 3 in Figure 1c (the absolute values s can be identified by the value $s_{\max}$ given in the figure).

An undisturbed flow in the upper reaches is only fresh water. A river stream is pushed up when impinging the hydro -front, a fresh layer becomes thinner thus resulting in an increase of velocity. This is illustrated by Curve 1 in Figure 1c, which shows the value of the longitudinal velocity on the free surface. In the mainstream, the velocity from the value 0.25 m/s has increased up to 0.57 m/s at the mouth ($x = 0$) i.e., more than doubled (Figure 1c). Further, on the sea section ($x > 0$) the velocity falls that is mainly due to the planned divergence of the current lines when extending the jet in the receiving water body. A maximum of the vertical velocity field w in the calculation was 1.2 mm/s.

The longitudinal velocity in the lower part of the flow is negative and directed against the main stream, its amplitude being relatively low not exceeding modulo 0.08 m/s, vanishing down the stream. Curve 2 in Figure 1c gives an idea about values of the bottom velocity.

A density gradient is an important inner force when forming the hydro-front, which forces the salt wedge move toward the river flow. The translational motion of the front terminates and the flow structure is stabilized when the density gradient is balanced by the force of inertia, i.e., the dynamic pressure of the incoming flow becomes comparable in magnitude with the term of density difference. In this case, a part of the kinetic energy of water is spent on overcoming the resistance generated by the interface in its flow around.

The described model structure of the flow in mouth area is qualitatively confirmed in the observations made [1, 2]. A difference between the results proposed from the known model structures for the Arctic rivers consists in the scales of processes under study. Thus, from the observations it follows that the length of the halocline in the Ob Bay is a few hundred kilometers [4]. Such a large size of salt water leaking can be associated with low flow velocities, which decrease when leaving the channel into the estuary with rather a large cross-section. A weak dynamic mode cannot provide a necessary resistance to salinity gradients in estuary and the hydro-front extends unhindered deep into the inland waters. In the mouths of estuarial delta ducts, the estuarial extension is absent and the flow velocities are not extinguished that is essential for the generation of blocking mechanisms of the hydro-front.

Figure 2 presents a vertical profile of salinity (Curve 1) measured at the mouth seashore in the inflow sleeves of the Lena River delta into the Laptev Sea [12]. Curve 2 is built according to the model field salinity on the outflow boundary of the computational domain at $x = 50$ km.

Note the qualitative and quantitative proximity of the profiles compared in Figure 2, which indirectly indicates to the adequacy of the model. Certain differences can be explained by insufficiently accurate input data about parameters of the river water flows in the delta and natural environment in the process of measurements. The slightly expressed upper mixed layer in the model profile (Curve 2) is probably due to incompleteness of the model of the vertical turbulence exchange.

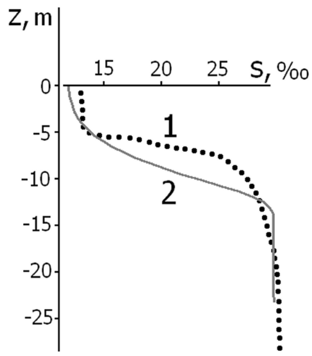

Figure 2. Real and calculated salinity profiles (Curves 1, 2) at the exit outlet of the area

The vertical distribution of salinity essentially determines the density stratification of the upper layers in the northern seas. As Figure 2 shows, a layer of the density jump at the mouth seashore is located in the surface layers. The power and depth of the jump affect the efficiency of the oxygen supply of the lower layers, thus regulating the aquatic life of hydro-bionts in the deep horizons. Therefore it is important to correctly describe the transformation processes of the river water at the mouth seashore and to define parameters of the salinity fields of the coastal area.

References

[1] Samolubov B.I. Density Flow and Diffusion of Impurities. — Moscow: LKI, 2007.

[2] Ivanov V.V., Svyatskiy A.Z. Numerical modeling of the sea water intrusion in estuaries in the seasonal time scale // Water Resources. — 1987. — No. 5. — P. 17–31.

[3] Vasiliev J.F. Dumnov S.V. A two-dimensional model for salt water intrusion in a estuary // Proc. XX IAHR Congress. — Moscow, 1983. — Vol. 2. — P. 10–16.

[4] Doronin J.P. Simulation of the vertical structure of the river mouth area with the sea halocline // Meteorology and hydrology. — 1992. — No. 8. — P. 76–83.

[5] System of the Laptev Sea and the Adjacent Arctic seas. The Current State and History of Development. — Moscow: MGU, 2009.

[6] Ovchinnikova T.E., Bocharov O.B. The effect of mineralized warm water inflow on the development of spring and summer convection in a deep lake // Computer Applications. — 2006. — Vol. 11, No. 1. — P. 63–72.

[7] Marchuk G.I., Kochergin V.P., Sarkician A.S., et al. Mathematical Models of Ocean Circulation. — Novosibirsk: Nauka, 1980.

[8] Vasiliev O.F., Voevodin A.F., Nikiforovskaya V.S. Numerical modeling of the stratified flows in the systems of open channels and water bodies of branched structure // Computer Applications. — 2004. — Vol. 9, No. 2. — P. 26–41.

[9] Molchanov V.N. Numerical model of the ocean circulation on the estuarine seaside with the effects of the liquid, heat and ion flows // Proc. Arctic and Antarctic Research Institute. — 1976. — Vol. 314. — P. 36–434.

[10] Rodi W. Turbulence Models and their Application in Hydraulics. A State of the Art Review. — Delft: International Association for Hydraulics Research (IAHR), 1980.

[11] Shlychkov V.A. Numerical model for the shallow water equations on a curvilinear grid with the preservation of the Bernoulli integral // Computational Mathematics and Mathematical Physics. — 2012. — Vol. 52, No. 7. — P. 1072–1078.

[12] Kassens H. Russian-German Cooperation in the Siberian Shelf Seas: Geo-System Laptev-Sea. — Alfred-Wegener-Institut fur Polar und Meeresforschung, 1994.

[13] Simonov A.J. Hydrology and hydrochemistry of estuarine waters of seaside in the seas no tides. — Moscow: Hydrometeoizdat, 1969.