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Designing of gradient measurements in the atmospheric surface layer*

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The problem of estimating the parameters of the atmospheric surface layer from the data of gradient observations of wind velocity and temperature is considered. The numerical method of analyzing measurements and sequential design of experiment is proposed. For the temperature–stratified surface layer the numerical calculations of optimal location of observations are carried out.

1. Introduction

In the problem of determining the characteristics of turbulence in the atmospheric surface layer: the friction velocity u_*, the vertical eddy heat flux $\overline{w'T'}$ and moisture flux $\overline{w'q'}$, by using the indirect methods, for example, from measurements of profile gradients, errors arise due to random measurement errors in mean values of meteorological elements. If suitable models, that correspond to physical conditions of observations, are applied for approximating the measured wind speed and temperature profiles, various criterions of the accuracy of the estimates of u_*, and the temperature scale T_*, and the Monin–Obukhov length L can be formulated by the use of the method of least squares (MLS). Since the quantities measured are one of the different physical nature, the problem of many-criterion estimating the surface-layer parameters arises. In this case, for estimating u_* the calculational techniques can use only the measured wind velocity data and T_* only the measured temperature gradients [1, 2]. In the method presented in this work for estimating u_*, T_*, L the unified criterion is used which takes account of the variances of the measurement errors of both wind velocity and temperature as the weight factors [3].

For decreasing the effect of random measurement errors of these quantities on the quality of the estimates, the question of the optimal allocation for the measuring heights is considered. According to [4], the problem of constructing the experimental plan is phrased for the case, when some quantities are measured at the same levels and in the same time.

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2. Mathematical formulation of the estimation problem for the surface layer parameters

We consider the stationary, horizontally-homogeneous and stratified surface layer in the atmosphere. Then, according to the similarity theory, the vertical profiles of mean values of wind velocity, potential temperature, specific humidity can be presented in the form [5–7]

$$\begin{aligned} u(z) &= \frac{u_*}{æ}\left[f_u\left(\frac{z}{L}\right) - f_u\left(\frac{z_0}{L}\right)\right], \\ T(z) &= T_0 + T_*\left[f_T\left(\frac{z}{L}\right) - f_T\left(\frac{z_0}{L}\right)\right], \\ q(z) &= q_0 + q_*\left[f_q\left(\frac{z}{L}\right) - f_q\left(\frac{z_0}{L}\right)\right]. \end{aligned} \tag{1}$$

Here u is the horizontal component of the fluid velocity, T is the potential temperature, q is the specific humidity, æ is the Karman constant, q_* is the specific humidity scale, f_u, f_T, f_q are the continuous universal functions, z_0 is the roughness length. The Monin–Obukhov length L is defined by

$$L = \frac{u_*^2}{æ^2 \lambda T_*}, \tag{2}$$

where λ is the buoyancy parameter.

Further, without the loss in generality, we will consider the problem of determining both the momentum and heat flows. Let, for the evaluation of the unknown parameter vector $\vec{\theta} = (u_*, T_*, L, T_0, z_0)^T$, the measured data of wind velocity and temperature be available at N levels:

$$\begin{aligned} &u_i = u(z_i, \vec{\theta}) + \xi_1^{(i)}, \quad T_i = T(z_i, \vec{\theta}) + \xi_2^{(i)}, \quad z_i \in [\hat{z},\ h], \\ &E\left[\xi_j^{(i)}\right] = 0, \quad E\left[\xi_j^{(i)} \cdot \xi_j^{(i_1)}\right] = \delta_{ii_1}\sigma_j^2, \quad i, i_1 = \overline{1,N}, \ j = \overline{1,2}, \end{aligned} \tag{3}$$

where E denotes a probabilistic averaging, δ_{ii_1} denotes Kronecker's delta, $\hat{z}$ is the lower level of measurements, h is the upper level of observations.

The parameter vector $\vec{\theta}$ is searched for so that the functional

$$\begin{aligned} I(\vec{\theta}) = \sum_{i=1}^{N}\Big[&\sigma_1^{-2}\big[u_i - u(\hat{z}) - u(z_i, \vec{\theta}) + u(\hat{z}, \vec{\theta})\big]^2 + \\ &\sigma_2^{-2}\big[T_i - T(\hat{z}) - T(z_i, \vec{\theta}) + T(\hat{z}, \vec{\theta})\big]^2\Big], \end{aligned} \tag{4}$$

is minimized, where the weight factors σ_1, σ_2 are mean root square errors of measurements of wind velocity and temperature, respectively. This form has been chosen to eliminate the dependence on z_0 and θ_0 from (4).

For minimizing the criterion function of (4), we rewrite relations (1) as follows:

$$u(z) = \omega_1 f_u(z/L) + \omega_2, \qquad T(z) = \omega_3 f_T(z/L) + \omega_4,$$
$$\omega_1 = \frac{u_*}{æ}, \quad \omega_2 = -\frac{u_*}{æ} f_u(z_0/L), \quad \omega_3 = T_*, \quad \omega_4 = T_0 - T_* f_T(z_0/L). \tag{5}$$

By the linearity of the functions of (5) with respect to ω_i, $i = \overline{1,4}$, and the necessary conditions of the minimum of functional (4), it is not difficult to derive the explicit representations for the coefficients ω_i at the fixed value of L. Substituting the derived expressions for $u_*(L)$ and $T_*(L)$ in (2), we can determine L from the resulting equation, for example, by the bisection method. Then, the remaining parameters are calculated by the known value of L.

3. Designing of gradient measurements

Consider the problem of optimal allocation of measuring heights for wind velocity and temperature gradients, that is, the experimental design is constructed according to some criterion of optimality. The D-criterion of optimality is used since the D-optimal design corresponds to the minimum value of the determinant of the variance-covariance matrix of estimated parameters and by the equivalency theorem [4], this plan turns out to be the G-optimal design simultaneously. The maximum of the dispersion of the response function is minimized by using the G-optimal design.

By the nonlinear dependence of the regression functions of (1) on $\vec{\theta}$, the priori computation of the optimal design is, generally speaking, impossible. The application of the sequential procedure of the analysis and the designing of observations is the most convenient for the case of simultaneous measuring of some quantities [4]. This formulation of the design problem is appropriate for the stationary processes on the sufficiently large time interval.

The construction of the local D-optimal design is realized according to the following iterative procedure.

Step 1. The arbitrary original plan ε_0 is chosen under the condition of nondegeneration of the information matrix

$$|M(\varepsilon_0, \vec{\theta})| = \left| \sum_{i=1}^{N_0} F(z_i, \vec{\theta}) \cdot F^T(z_i, \vec{\theta}) \right| \neq 0,$$

where

$$F(z_i, \vec{\theta}) = \|\vec{f}_1(z_i, \vec{\theta}), \vec{f}_2(z_i, \vec{\theta})\|,$$
$$\vec{f}_1^{\,T} = \left(\frac{1}{\sigma_1} \frac{\partial u}{\partial u_*}, \frac{1}{\sigma_1} \frac{\partial u}{\partial L} \right), \quad \vec{f}_2^{\,T} = \left(\frac{1}{\sigma_2} \frac{\partial T}{\partial u_*}, \frac{1}{\sigma_2} \frac{\partial T}{\partial L} \right).$$

Step 2. MLS-estimate $\hat{\vec{\theta}}_0$ and the matrix $M(\varepsilon_0, \hat{\vec{\theta}}_0)$ are calculated.

Step 3. The point z^* is determined that corresponds to

$$\max_{z\in[\hat{z},h]} \mathrm{Sp}[d(z,\varepsilon_0,\hat{\vec{\theta}}_0)],$$

where

$$d(z,\varepsilon_0,\vec{\theta}) = F^T(z,\vec{\theta})\cdot M^{-1}(\varepsilon_0,\vec{\theta})\cdot F(z,\vec{\theta}).$$

Step 4. The plan ε_N is constructed as

$$\varepsilon_N = \varepsilon_{N_0+1} = \Big(1 - \frac{1}{N_0+1}\Big)\varepsilon_0 + \frac{1}{N_0+1}\varepsilon(z^*),$$

according to which Steps 2–3 are repeated.

If $|M^{-1}(\varepsilon_N, \hat{\vec{\theta}}_N)|/N$ is small enough, then stop, else set $N = N+1$ and go to Step 2.

4. Numerical results

Consider the examples of numerical constructing the local D-optimal design for the variable upper level of measurements and the known parameter vector $\vec{\theta}$ which determines different conditions of turbulence for the stratified surface layer. The Monin–Obukhov length L was chosen between -50 and -10 and from 10 to 50. These ranges characterize two distinctive regimes of turbulence in the atmospheric surface layer. The friction velocity u_* is set equal 0.5 m/s both in stable conditions and for the labile stratification. Thus, the essential role of the temperature irregularity is underlined in the analysis of the turbulent flux in the surface layer. Since, the approximation of vertical profiles of both wind velocity and temperature, according to (1), (2), proposes the availability of the original level of measurements $\hat{z}$ (the thickness of displacement layer), so, for calculations, $\hat{z}$ is set equal 1 m for the labile stratification which corresponds, for example, to the level of herbage of the wheat field [8] and for the case of the stable–stratified surface layer $\hat{z} = 0.5$ m.

The optimal plans of measurements presented in the table were obtained by using the iterative procedure 1–4. The analysis of the labile stratification ($L < 0$) shows that, with a rise of nonstability of turbulent fluid, for determining the parameter estimates, the three-point plans of measurements are preferred. These plans contain the following levels: $\hat{z} = 1$ m, $z_1 \approx 2$ m, $z_2 = 4$ m. In this case the weight of the midpoint of the design is reduced with the decrease of the value of the parameter of L. With increasing the height of the upper level of gradient measurements to 10 m, the design also

The local *D*-optimal plans of measured gradients of wind speed and temperature for the friction velocity 0.5 m/s (measuring levels z_1, z_2 are given in meters, p_1, p_2 are observation weights)

L, m	Plan points for $\hat{z} = 1$ m		L, m	Plan points for $\hat{z} = 0.5$ m	
	z_1/p_1	z_2/p_2		z_1/p_1	z_2/p_2
−10	1.96/0.50 2.62/0.45	4/0.5 10/0.55	10	1.07/0.50	2/0.5
−30	2.14/0.29 2.98/0.42	4/0.71 10/0.58	30	1.13/0.50	2/0.5
−50	3.70/0.26	4/1.0 10/0.74	50		2/1.0

remains the three-point one with the height of the mid-level $z_1 \approx 3$ m. In weak nonstability the number of the design points may be reduced to two points depending on the height of the upper level of measurements. A similar situation takes place for weak stability. Thus, in near-neutral conditions, for determining the parameter estimates, the measurements at two levels are sufficiently used. Under high and moderate stability the derived three-point optimal designs are: $\hat{z} = 0.5$ m, $z_1 \approx 1$ m, $z_2 = 2$ m.

The performed numerical simulation of the optimal designs suggests that, for estimating the parameters of the temperature–irregular surface layer some redundant number of levels are required. Apparently, this circumstance is not accidental because the developed optimal designs correspond also to the recommendations presented in the works [8, 9].

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