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The investigation of the sea–ice model thermodynamic regimes*

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The analysis of the dynamics of ice growth and melting over the sea water is presented by this paper. According to the system of equation proposed by Hibler [2] 3 thermal regimes are possible: ice growth due to atmospheric cooling, ice melting due to oceanic warming and ice melting caused by the heat income from both atmosphere and ocean. All these regimes are analyzed by means of the ice thermodynamics model that is introduced in the paper. The parameterizing procedure for ocean mixed-layer is also suggested here. 1D numerical modeling results indicates the ability of the model to reproduce basic ice thermodynamic processes adequately. All the results are considered as a preliminary ones on the way to build a complete 3D model of the ice dynamics over the Arctic Ocean.

Introduction

To investigate the Arctic ocean circulation the model of ice thermodynamics should be designed. This model is to reproduce the basic ice regimes. It should be also well correlated with the ocean mixed-layer parameterization and with the dynamics of the upper ocean currents. As a basis the model of Hibler [2] may be used. However, this model has some disadvanteges. The most significant one is that ice growth rate function is taken as a tabular function of ice thickness, ice compactness and the seasonal time. Thus it does not introduce any physical reasons, when simulating the ice growing or melting.

The world practic of the model construction is wide enough. The main features are the following:

- utilizing the heat fluxes from/into the atmosphere as the upper boundary condition is preferable instead of sea (ice) surface temperature;
- using linear distribution of ice temperature with vertical coordinate gives a quite simple and good approximation for most cases especially for the large scale circulation models [4];
- the snow lying on the ice may play a significant role in ice thermodynamics;

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- the salinity fluxes during ice growth and melting influence the stratification factor;
- the ice compactness may be continuosly reduced in the places of large shear deformation [3];

The wind stress is significantly rising up for the ridged ice, because of the growing wind resistance. Thus the whole momentum exchange between the atmosphere and ocean should increase in the case of floating ice. However, it should be neglidgible in the case of the packing ice that responds on the wind action by increase of the bulk and shear ice deformation stresses.

As a development of the Hibler concept differencing ice-covered and open surface thermodynamics, the approach may be considered according to which on each model time step the vertical processes are simulated differently for both types of the surfaces and then summerized with the weighting factors A – area part of the covered surface and $(1 - A)$ – open area part. This may be applied to the momentum exchange too. The resulting procedure may parameterize in some manner the polynia effects.

This paper represents the first steps on the way of building the sea ice model as a part of the Arctic Ocean circulation model that will be elaborated in Novosibirsk Computing Center. First section represents the dynamic part of the model. The second gives a description of the thermodynamic ice growth rate balance. The third explains the way the ocean mixed-layer parameterization will be functioning. The 1D test experiments with this parameterization is discussed in the fourth section. The fifth section represents the analysis of three thermodynamic regimes following from mathematic formulation of the model.

1. The model of ice dynamics

The basic system of equation is taken as in Hibler' paper [2]:

$$\begin{aligned}
\frac{Du}{Dt} &= (f - mu\cos\theta)v + \frac{\tau_a^\lambda + \tau_w^\lambda}{\rho_i h} - g\,\mathrm{grad}H + F^\lambda, \\
\frac{Dv}{Dt} &= (mu\cos\theta - f)u + \frac{\tau_a^\theta + \tau_w^\theta}{\rho_i h} - g\,\mathrm{grad}H + F^\theta, \\
\frac{\partial h}{\partial t} &+ m\frac{\partial uh}{\partial\lambda} + m\frac{\partial(vhn/m)}{\partial\theta} = \Delta_z h + S_h, \\
\frac{\partial A}{\partial t} &+ m\frac{\partial uA}{\partial\lambda} + m\frac{\partial(vAn/m)}{\partial\theta} = \Delta_z A + S_A.
\end{aligned} \tag{1}$$

The open boundary conditions for any of the previous function ($\psi = u$, v, h, A) are the following:

$$\text{for } (\boldsymbol{u}\cdot\boldsymbol{n}) \geq 0: \quad \frac{\partial\psi}{\partial\boldsymbol{n}} = 0 \qquad \text{(outflow)}$$
$$\text{else} \qquad \psi = \psi_0^\Gamma \qquad \text{(inflow)},$$

where ψ_0^Γ is the value of any function on the boundary at the very beginning ($t = 0$).

The solid boundaries are modelled by the following conditions:

$$\frac{\partial(\boldsymbol{u}\cdot\boldsymbol{l})}{\partial\boldsymbol{n}} = 0, \quad (\boldsymbol{u}\cdot\boldsymbol{n}) = 0, \quad \frac{\partial(h, A)}{\partial\boldsymbol{n}} = 0. \tag{2}$$

In the above equations

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + mu\frac{\partial}{\partial\lambda} + nv\frac{\partial}{\partial\theta}, \quad \Delta_z = m\left(\frac{\partial}{\partial\lambda}\mu m\frac{\partial}{\partial\lambda} + \frac{\partial}{\partial\theta}\mu\frac{n^2}{m}\frac{\partial}{\partial\theta}\right).$$

According to Hibler the right-hand side sources in the equations (1) and (2) are:

$$\begin{aligned} F^\lambda &= m\frac{\partial}{\partial\lambda}\left\{[\eta+\zeta]m\frac{\partial u}{\partial\lambda} + [\zeta-\eta]n\frac{\partial v}{\partial\theta} - \frac{P}{2}\right\} \\ &\quad + n\frac{\partial}{\partial\theta}\left\{\eta\left(n\frac{\partial u}{\partial\theta} + m\frac{\partial v}{\partial\lambda}\right)\right\}, \\ F^\theta &= n\frac{\partial}{\partial\theta}\left\{[\eta+\zeta]n\frac{\partial v}{\partial\theta} + [\zeta-\eta]m\frac{\partial u}{\partial\lambda} - \frac{P}{2}\right\} \\ &\quad + m\frac{\partial}{\partial\lambda}\left\{\eta\left(n\frac{\partial u}{\partial\theta} + m\frac{\partial v}{\partial\lambda}\right)\right\}, \end{aligned} \tag{3}$$

where

$$\begin{gathered} \zeta = \Delta\frac{P}{2}, \qquad \eta = \frac{\zeta}{e^2}, \\ \Delta = \sqrt{(\dot\varepsilon_{11}^2 + \dot\varepsilon_{22}^2)\left(1 + \frac{1}{e^2}\right) + 4\frac{\dot\varepsilon_{12}^2}{e^2} + 2\dot\varepsilon_{11}\dot\varepsilon_{22}\left(1 - \frac{1}{e^2}\right)}, \\ \dot\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \\ P = P^* h \exp[-C(1-A)]. \end{gathered} \tag{4}$$

The thermodynamic ice sources are:

$$\begin{aligned} S_h &= \phi\left(\frac{h}{A}\right) + \phi(0)(1-A), \\ S_A &= \begin{cases} \dfrac{\phi(0)}{h_0}(1-A), & \text{if } \phi(0) > 0 \\ 0, & \text{if } \phi(0) \leq 0 \end{cases} \\ &\quad + \begin{cases} 0, & \text{if } S_h > 0 \\ \dfrac{A}{2h}S_h, & \text{if } S_h \leq 0 \end{cases}, \end{aligned} \tag{5}$$

with $\phi(h)$ - the growth rate of ice thickness h. Really, this function is a parameterization procedure of the ice thermodynamics.

2. Ice thermodynamics

It is assumed that temperature of the ice distributed lineraly from upper surface to bottom. Thus the heat flux through the ice is

$$q_i = -c_{pi}\rho_i k_i \frac{T_{fr} - T_{is}}{h}, \tag{6}$$

where h is the ice thickness.

The thermal balance on the ice-water surface is given by

$$\rho_i \mathcal{L}_f \phi_{iw} = -(q_i - q_w), \tag{7}$$

where q_i is the heat flux through the ice and q_w - heat flux into the water, calculated by

$$q_w = -k(T_w - T_{fr}). \tag{8}$$

For air-ice surface thermal balance is

$$q_a - q_i = 0, \tag{9}$$

where q_a is the total heat flux from atmosphere (sensible, latent, radiative heat fluxes). The unknown parameter of the ice surface temperature can be determined from the relationship between q_i and T_{is}. If the value of T_{is} is greater than the freezing temperature, then ice melting is taking place $q_i = 0$ and

$$\rho_i \mathcal{L}_f \phi_{ai} = -q_a. \tag{10}$$

The total ice growth rate is the sum

$$\phi = \phi_{iw} + \phi_{ai}. \tag{11}$$

3. Mixed-layer model

The mixed-layer model is based on the numerical solution of the primitive equation set. The vertical parts of the equations for temperature and salinity are given by

$$\begin{aligned} \frac{\partial T}{\partial t} &= \frac{\partial}{\partial z}\mu_T \frac{\partial T}{\partial z} - w\frac{\partial T}{\partial z}, \\ \frac{\partial S}{\partial t} &= \frac{\partial}{\partial z}\mu_S \frac{\partial S}{\partial z} - w\frac{\partial S}{\partial z}, \end{aligned} \tag{12}$$

with boundary conditions:

at the bottom

$$\frac{\partial T}{\partial z} = 0, \quad \frac{\partial S}{\partial z} = 0; \tag{13}$$

at the surface

$$\rho c_p \mu_T \frac{\partial T}{\partial z} = q_a^*, \qquad \rho \mu_S \frac{\partial S}{\partial z} = S p_a^*. \tag{14}$$

As it was suggested by Hibler the thermodynamics of open water surface and covered by ice are differing. Hence, the heat and salt flux parameterizations for covered water part (A) is $q_a^* = q_i$ ($p_a^* = p_i$), i.e., the flux from the ice cover, while for open part $(1 - A)$ is the strict atmospheric income. The total flux from the atmosphere is also different, when passing through two types of sea surfaces (covered and open). The flux over open surface is greater by some factor σ (in the following numerical experiments it will be taken equal to 10).

For each time step the new temperature and salinity profiles are built for both cases separately, analysed for convective and dynamic adjustment [1] and then summarized according to the weighting factor A or $1 - A$. This allows to parameterize the polynia circulation cell in some ugly manner.

4. 1D test experiment

Two testing experiments were carried out to illustrate the capability of the thermodynamic model to reproduce adequately the ice growth rate and melting. The upper boundary condition for temperature was built by aerodynamic formulae with sinusoidally varying near-surface atmosphere temperature. The fresh water flux was set to 0. The range of temperature variations was taken according to seasonal one. So the results may be considered as modeling of a seasonal variation of ice-water system. The difference of two experiments is in initial salinity value: it was 0 for the first experiment and equal to 35‰ for the second. Thus the first experiment is dealing with fresh water (lakes, etc.), while the second with oceanic one.

The difference of these two cases is in the fact that according to equation of state the maximum water density for fresh water is at approx. 4^0C, while for sea water the density increases monotonically as temperature decreasing. It means that if there is no salinity counteraction, the ice-covered water temperature is formed by strong convective mixing. It forces water to have temperature of freezing at all horizones. For fresh water the abissal water may be warmer then the sufface one. It gives rise the positive heat flux from below and decreases the ice-growth rate. The maximum ice thickness differes by a factor of approx. 1.5.

5. Thermodynamic regimes

Let us consider the thermodynamics in the absence of the advection process. In accordance with (5) four regimes are formally possible:

1. $\phi(0) > 0, \quad S_h > 0;$
2. $\phi(0) \leq 0, \quad S_h \leq 0;$
3. $\phi(0) > 0, \quad S_h \leq 0;$
4. $\phi(0) \leq 0, \quad S_h > 0.$

First means that ice thickness grows forced by atmospheric cooling, while second represents the case of no atmospheric cooling and, hence, ice thickness decreasing. Third case indicates the situations, when in opposite to the atmospheric cooling, ice thickness may decrease because of the heat fluxes from the underlying water layers. At last, fourth case is practically impossible because ice thickness grows with the heat incoming from the atmosphere, it supposes that ice supplies more heat to the underlying water layers than it receives from atmosphere. It is very courious.

For simplicity of the following considerations, let us suppose that atmospheric cooling is approximatelly constant for a long time period as well as the oceanic warming. The first one may be calculated by

$$q_a = C_D(T_a - T_s), \tag{15}$$

where T_a is the near-surface atmospheric temperature and T_s – sea surface temperature (SST) (both covered or open). Constant atmospheric cooling means the constant atmospheric temperatufe T_a, but variable SST - T_s depending on the ice thickness and ice compactness. The constant oceanic warming means the constant temperature of underlying water layer T_w and hence the constant heat flux from the ocean to the ice (see (8)).

5.1. Ice growth rate over the open surface

The parameterization scheme proposed by Hibler assumes that all the ice formed over the open surface groups into the pieces of thickness h_0, reduces the open area and enlarges ice compactness. At the starting moment the ice compactness is small, so we can consider reduced system (5):

$$\begin{aligned} \frac{\partial h}{\partial t} &= \phi(0)(1 - A), \\ \frac{\partial A}{\partial t} &= \frac{\phi(0)}{h_0}(1 - A). \end{aligned} \tag{16}$$

The latter equation gives

$$A = \Big(1 - \exp\Big(-\frac{t}{t_0}\Big)\Big), \tag{17}$$

where $t_0 = h_0/\phi(0)$ is the time period of formation of the ice cover of thickness h_0, and

$$\phi(0) = \phi_0 = -\frac{C_D}{\rho_i \mathcal{L}_f}(T_a - T_{fr}) = \text{const}$$

in accordance with (15). The substitution of (17) into the first equation of (16) gives

$$h = h_0\Big(1 - \exp\Big(-\frac{t}{t_0}\Big)\Big). \tag{18}$$

As it is seen from (17) and (18), ice compactness and thickness are growing similarly and the time scale of this growth is t_0. The value of h_0 proposed by Hibler is equal to 1 meter. For $T_a = -15°\text{C}$ and $C_D = 100W/(m^2K)$ the ice growth rate

$$\phi_0 = \frac{100 \cdot 15}{917 \cdot 2.8 \cdot 10^6} \simeq 5.82 \cdot 10^{-7} \text{ m/sec}$$

and the time scale

$$t_0 = \frac{1}{5.82 \cdot 10^{-7}} \simeq 1.7 \cdot 10^6 \text{sec} \simeq 19 \text{ days}.$$

This means that the ice of 1m thickness would be formed in 19 days, if atmospheric temperature were 15^0 C, the sea surface were always open. (Really, the value of C_D is larger by a factor of 10, because of the latent heat flux from the open surface. So, the amount of t_0 may be decreased down to the value of 2 days).

5.2. Ice growth forced by atmospheric cooling

The previous ice growing rate is taking place only at the initial phase, when A is too small in comparison with 1. To consider the subsequent dynamics the value $\phi(h)$ should be estimated. If the ice thickness is h, then the heat flux through this ice is given by (6), but this heat flux should be equal to the heat flux into the atmosphere

$$-c_{pi}\rho_i k_i \frac{T_{fr} - T_{is}}{h} = -C_D(T_{is} - T_a).$$

Thus

$$T_{is} = \frac{hC_D T_a + c_{pi}\rho_i k_i T_{fr}}{hC_D + c_{pi}\rho_i k_i}$$

and

$$q_i = -\frac{hC_Dc_{pi}\rho_ik_i}{hC_D + c_{pi}\rho_ik_i}\left(\frac{T_{fr}-T_a}{h}\right) = -C_D(T_{fr}-T_a)\frac{h_*}{h+h_*} = -\rho_i\mathcal{L}_f\phi_0\frac{h_*}{h+h_*}$$

where $h_* = c_{pi}\rho_ik_i/C_D$ is new ice thickness scale (for $C_D = 50W/(\mathrm{m}^2K)$, $c_{pi} = 4.2\cdot 10^3 J/(\mathrm{kg}\cdot K)$, $\rho_i = 917\mathrm{kg/m}^3$, $k_i = 0.6\cdot 10^{-6}\mathrm{m}^2/\mathrm{sec}$, one can calculate $h_* = 0.046\mathrm{m} = 4.6\mathrm{cm}$). For $S_h > 0$ and $\phi_0 > 0$, we have

$$\phi_h = \frac{1}{\rho_i\mathcal{L}_f}(q_w - q_i) = \phi_i - \phi_w \qquad \left(\phi_i = -\frac{q_i}{\rho_i\mathcal{L}_f}, \quad \phi_w = -\frac{q_w}{\rho_i\mathcal{L}_f}\right).$$

The system (5) may be rewritten as

$$\begin{aligned}\frac{\partial h}{\partial t} &= A\phi_0\left(\frac{h_*}{h+h_*}\right) + \phi_0(1-A) - \phi_w,\\ \frac{\partial A}{\partial t} &= \frac{1}{t_0}(1-A) \Longrightarrow A = (1-\exp(-t/t_0)),\end{aligned} \tag{19}$$

and

$$\frac{\partial h}{\partial t} = (\phi_0 - \phi_w) - \phi_0\frac{h}{h+h_*}\left(1-\exp\left(\frac{t}{t_0}\right)\right). \tag{20}$$

Introducing new variables $\chi = h/h_*$ and $\tau = t/t_0$ one can obtain

$$\frac{\partial\chi}{\partial\tau} = r\Big\{(1-f) - \frac{\chi}{1+\chi}\Big(1-\exp(-\tau)\Big)\Big\}, \tag{21}$$

where $r = h_0/h_*$ and $f = \phi_w/\phi_0$ are two parameters depending on the atmosphere and ocean properties. Graphics presented by Figures 1–2 display the ice thickness growing for various regimes defined by the set of parameters r and f. The upper limit of the ice thickness may be obtained from (21), when $\tau \to \infty$, thus

$$(1-f) = \frac{\chi}{1+\chi},$$

and

$$\chi_{\max} = \frac{1-f}{f},$$

or

$$h_{\max} = h_*\frac{\phi_0 - \phi_w}{\phi_w}. \tag{22}$$

As it is seen, $h_{\max}$ depends on the scaling parameter h_* and on the relation between the heat flux from ocean and the heat flux to the atmosphere, if the surface of ocean is open.

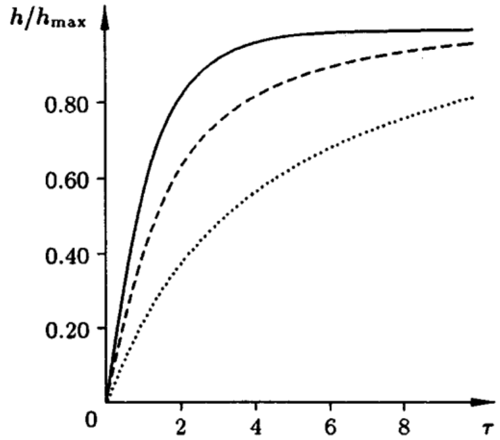

Figure 1. Ice growth for various amount of r parameter: —— $r = 1.5$; - - - $r = 1$; ····· $r = 0.5$

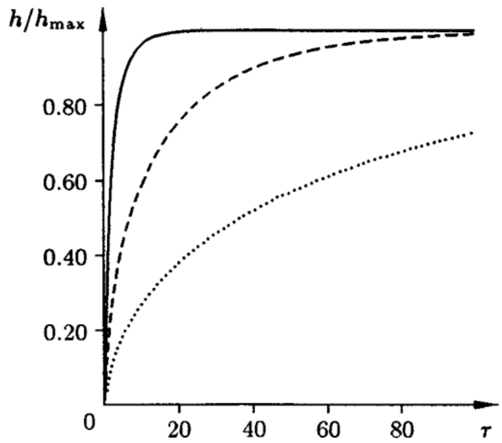

Figure 2. Ice growth for various amount of f parameter: —— $f = 0.5$; - - - $f = 0.2$; ····· $f = 0.08$

5.3. Ice melting caused by the oceanic warming

The first equation of (19) can be written in the following form:

$$\frac{\partial h}{\partial t} = (\phi_0 - \phi_w) - A\phi_0 \frac{h}{h + h_*}.$$

It shows that $\frac{\partial h}{\partial t}$ becomes negative, if values ϕ_w and ϕ_0 satisfy the relation

$$\phi_w > \phi_0\Big(1 - \frac{Ah}{h + h_*}\Big).$$

If the h value corresponds to the stable state with $\tilde{\phi}_w$ and $\tilde{\phi}_0$, then $h = h_*(\tilde{\phi}_0 - \tilde{\phi}_w)/\tilde{\phi}_w$ and $A = 1$. The above inequality in this case has the form

$$\frac{\phi_w}{\phi_0} > \frac{\tilde{\phi}_w}{\tilde{\phi}_0}. \tag{23}$$

This may occure as a sequence of both the reducing of atmospheric cooling or encreasing of oceanic warming. The ice melting will take place untill new limiting ice thickness

$$h' = h_* \frac{\phi_0 - \phi_w}{\phi_w}$$

will be achieved.

5.4. Total ice melting

If both $S_h < 0$ and $\phi_0 < 0$, then the total ice melting is taking place. Hence,

$$\begin{aligned} \frac{\partial h}{\partial t} &= (\phi_0 - \phi_w) - A\phi_0 \frac{h}{h + h_*}, \\ \frac{\partial A}{\partial t} &= \frac{A}{2h}\frac{\partial h}{\partial t}. \end{aligned} \tag{24}$$

The latter equation of (24) yields $A(t)/A(0) = \sqrt{h(t)/h(0)}$ or $A = c\sqrt{h}$ $(c = A(0)/\sqrt{h(0)})$. Then the first equation of (24) becomes

$$\frac{\partial h}{\partial t} = -(|\phi_0| + \phi_w) + c|\phi_0|\frac{h^{3/2}}{h + h_*},$$

or

$$\frac{\partial \chi}{\partial \tau} = -r\Big(1 + f - A_* \frac{\chi^{3/2}}{1 + \chi}\Big), \tag{25}$$

where $\chi = h/h_*$, $\tau = t|\phi_0|/h_0$, $r = h_0/h_*$, $f = \phi_w/|\phi_0|$ and $A_* = c\sqrt{h_*} = c\sqrt{h_0/r} = A(0)\sqrt{h_*/h(0)}$.

Solution of (25) is displayed by Figures 3–5 for various set of parameters r, f and A_*. At the final stage of melting, when $\chi \ll 1$,

$$\frac{\partial \chi}{\partial \tau} = -r(1 + f)$$

yields the rate of the ice melting.

Concluding remarks

We have considered some physical mechanisms of ice growth and melting for better understanding the processes we are going to simulate. The most important fact is that the ice thickness assymtotically equals to some limitted value according to (22). This value is parametrically related to the properties of the atmospheric and oceanic boundary layers. The relaxation time for reaching this value is some number of time scaling parameter t_0.

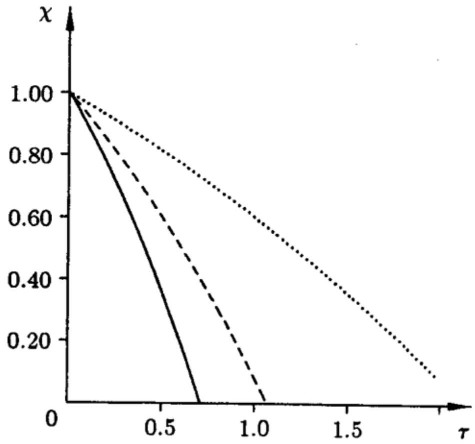

Figure 3. Ice melting for various amount of r parameter: —— $r = 1.5$; - - - $r = 1$; ····· $r = 0.5$

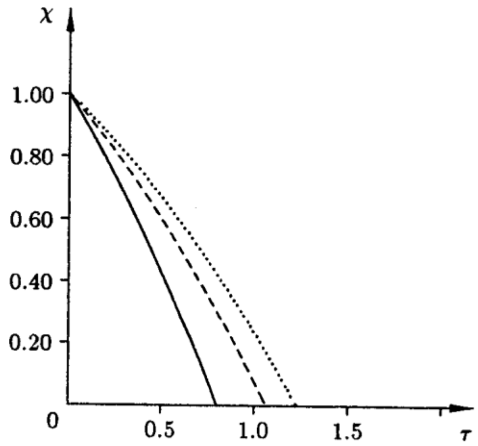

Figure 4. Ice melting for various amount of f parameter: —— $f = 0.5$; - - - $f = 0.2$; ····· $f = 0.08$

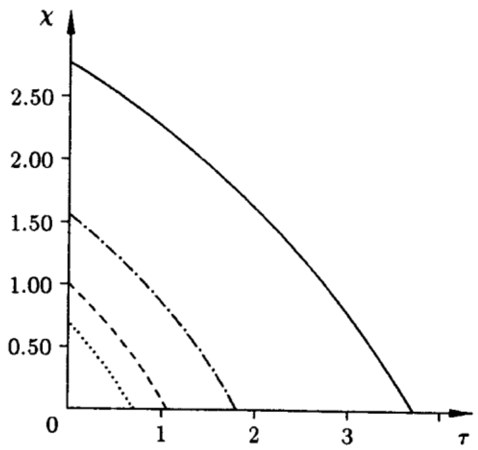

Figure 5. Ice melting for various amount of A_* parameter: —— $A_* = 0.6$; –·– $A_* = 0.8$; - - - $A_* = 1$; ····· $A_* = 1.2$

It should be also mentioned that we have been escaping the problem of ocean mixed-layer, which significantly depends on the ice cover dynamics both through the heat fluxes and through the salinity (or fresh water) fluxes. We have not also considered the variance of atmospheric drag coefficient C_D for the cases of the open surface and covered by ice. The first one should be significantly larger, because it is to parameterize the latent heat flux that is dominant over the open surface and negligible over the ice surface. All these questions are planed to be investigated at the next steps.

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Appendix

Notification list

A	ice compactness (fraction of the area covered by thick ice)
A_*	parameter: $A_* = c\sqrt{h_*}$
a	Earth' radius
C_D	proportionality coefficient of drag heatflux
C	empirical constant
c_{pi}	ice heat capacity at the constant pressure
c_p	water heat capacity at the constant pressure
c	constant
e	ratio of compressive to shear strength
F^λ, F^θ	components of the force due to variation in internal ice strength
f	Coriolis parameter
f	parameter: $f = \phi_w/\phi_0$
g	acceleration due to gravity
H	sea surface dynamic height
h_*	ice thickness scale $h_* = c_{pi}\rho_i k_i/C_D$
h	ice thickness
h_0	demarcation thickness between thin and thick ice (1 m is used here for the standart simulation)
k_i	ice temperature conductivity
k	integral thermal conductivity of ice-water interface
$\mathcal{L}_f$	latent heat of fusion
$\boldsymbol{l}$	unit vector tangent to the boundary
m	factor: $\frac{1}{a\cos\theta}$
n	factor: $\frac{1}{a}$

$\boldsymbol{n}$	unit vector normal to the boundary
P	ice strength
P^*	empirical ice strength parameter
p_a	precipitation rate in atmosphere
q_a	heat flux from the atmosphere
q_i	heat flux through the ice
q_w	heat flux into the water
r	parameter: $r = h_0/h_*$
S_A, S_h	thermodynamic sources in eq uations for ice compactness and thickness
S	water salinity (‰)
T_a	near-surface air temperature
T_{fr}	temperature of freezing (set at the ice bottom)
T_{is}	temperature of the ice upper surface
T_s	sea surface temperature (both open or ice-covered)
T_w	water temperature below the ice bottom
T	water temperature
t_0	time scale $t_0 = h_0/\phi_0$
t	time variable
u_i	two-component ice velocity vector
u	zonal component of the ice velocity
$\boldsymbol{u}$	ice velocity vector
v	meridian component of the ice velocity
w	vertical component of the water velocity
x_i	spheric coordinate vector
z	vertical coordinate (0 at the surface, downwarded)
Δ_z	two-dimensional Laplas operator
Δ	parameter
ε	deformation tensor
λ	longitude angle
μ	diffusion coefficient
ϕ_0	$= \phi(t = 0)$
ϕ	total ice-growth rate
ϕ_{ai}	ice-growth rate caused by snowing
ϕ_{iw}	ice-growth rate at the ice bottom
ϕ_i	$= -q_i/(\rho_i \mathcal{L}_f)$
ϕ_w	$= -q_w/(\rho_i \mathcal{L}_f)$
χ	unscaled ice thickness
ψ_0^Γ	= u,v,h,A determined at the open boundary
ρ_i	sea-ice density
ρ	water density
τ_a	wind stress forcing
τ_w	ice-water friction forcing

τ	unscaled time variable
θ	latitude angle
η	ice shear viscosity
ζ	ice bulk viscosity