Bull. Nov. Comp. Center, Math. Model. in Geoph., 6 (2000), 115–123

On the influence of the dilatancy, compression of pores, and filtration on stability of shearing sliding in a rock

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Dilatancy, or increase in void volume, is typically associated with the inelastic deformation of a relatively intact rock. However, dilatancy associated with frictional sliding in laboratory samples also has been observed. At low normal stresses, dilatancy may be due to the uplift in sliding over asperity contacts. At higher normal stresses, dilatancy appears to result from the initiation and extension of microcracks adjacent to the sliding surface. If the rock is fluid-saturated and dilatancy occurs more rapidly than a pore fluid can diffuse into the newly created void space, the local pore pressure near the sliding surface decreases. This decrease in the pore pressure increases the effective compressive stress (the total compressive stress minus the pore fluid pressure) and inhibits a further frictional slip.

This paper investigates the stability of the quasi-static shearing sliding along a rock porous layer that can result from the coupling between the change in porosity, pore fluid diffusion and dilatancy accompanying the friction sliding. Analysis was made using a mechanical model of unstable sliding [1] for the evolutional law of friction, when the friction coefficient is a given function of the velocity of sliding and the so-called variable of the state characterizing the evolution of the system [2]. Such effects as shearing dilatancy, the diffusion of fluid and compressibility of the fluid and pores are also taken into account.

A linear analysis of stability has shown that a steady sliding can be both stable and unstable depending on the value of elastic rigidity. An estimate of critical rigidity as function of governing parameters of the problem (the velocity of sliding, the effective normal stress, the coefficients of shearing dilatancy, the diffusion of fluid, and compressibility of pores, as well as parameters of the evolutional law of friction) has been obtained.

A qualitative analysis of the influence of the governing parameters on the stability of sliding is given. In particular, the shearing dilatancy and low values of the effective normal stress ($\bar{\sigma} = \sigma - p$, where σ is the normal stress, p is the pore pressure) favor the stability of steady sliding. Thus, an increase in the pore pressure stabilizes sliding. This is in accord with the experiment.

1. Evolution law of friction

According to the existing concepts, sliding at the surface of contacts occurs when the ratio between the tangential stress τ and the normal stress σ attains the value of the static friction coefficient f_s. The friction drag as well as the friction coefficient decrease in sliding taking the values of the dynamic friction coefficient f_d. This decrease in the drag can be due to rigidity of the system and can result in the instability of sliding. The following facts have been experimentally established in the physics of rocks:

- the static coefficient of the friction f_s depends on the history of sliding of surfaces;
- the dynamic coefficient of the friction f_d for the steady sliding depends on the velocity of sliding. It is also influenced by such factors, as the type of rocks, temperature, and some other parameters.
- if the velocity of sliding varies jumpwise, the evolution of friction to a new steady state takes place at a characteristic distance of sliding d_c.

The time of the contact of surfaces and, hence, the dependence of the friction of sliding on the velocity of the sliding v are important [2]. The effective time of the contact is determined as a ratio between the critical distance of the sliding d_c, at which the contact surface is renewed, and the sliding velocity v. The ratio d_c/v is considered as average time of evolution of the contact θ or as a characteristic time of the contact.

An experimental dependence of the dynamic coefficient of friction on the sliding velocity v and the variable of the state of the system θ is approximated by formula in [2]:

$$f = f_0 + a\ln(v/v_0) + b\ln(\theta/\theta_0), \tag{1}$$

where f_0 is the coefficient of friction at $v = v_0$ and $\theta = \theta_0$, a and b are the empirical constants of the rock, v_0 and θ_0 are the reference constants.

Equation (1) should be considered jointly with the evolution equation for the state variable θ. Various evolutional equations were proposed, for instance

$$\frac{d\theta}{dt} = 1 - \frac{\theta v}{d_c}, \tag{2}$$

$$\frac{d\theta}{dt} = 1 - \frac{\theta v}{d_c}\ln\left(\frac{\theta v}{d_c}\right), \tag{3}$$

$$\frac{d\theta}{dt} = 1 - \left(\frac{\theta v}{2d_c}\right)^2, \tag{4}$$

where t is the time variable.

Experimental data for rocks are well described by using equations (1) and (2) [3], and therefore we use these equations below.

With the use of the expression $\theta_s = d_c/v$ for the case of steady sliding, equation (2) can be written down in the form $d\theta/dt = -\theta v(\theta - \theta_s)/d_c$. It follows that at a constant velocity of the sliding v the value θ exponentially tends to θ_s.

It follows from (1) that there is a continuum of values of the coefficient of friction. If, however, the dynamic coefficient of the friction f_d is determined for a steady velocity of sliding, $df_d/d(\ln v) = a - b$. Similarly, if the static coefficient of the friction f_s is determined for the state after a long period of the contact of surfaces, $df_s/d(\ln v) = b$.

The influence of the jumpwise change of the sliding velocity v and of the state variable θ on the coefficient of friction at $b > a$ is schematically shown in Figure 1. The initial sliding velocity is $v_1 = v$. There is a jump up to $v_2 = 10v$, and then back to $v_3 = v$. The expression for the coefficient of friction at steady sliding is found from equations (1) and (2) and has the following form:

$$f = f_0 + (a - b)\ln\left(\frac{v}{v_0}\right). \tag{5}$$

At a jumpwise increase in the velocity ($\Delta v = v_2 - v_1$), the coefficient of friction increases by a value a. Then it monotonically decreases by a value b due to the evolution effect described by the variable θ. At the time of the jump of the velocity from the high value v_3 to the low value v_2, the coefficient of friction first decreases jumpwise, and then monotonically increases in the velocity interval $v_3 = v$.

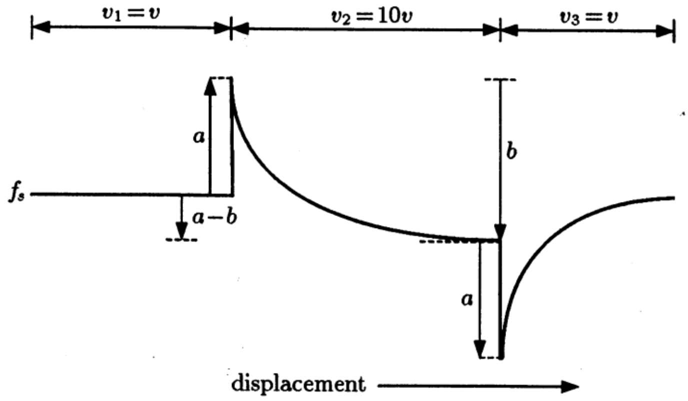

Figure 1. The coefficient of friction for the case of the piecewise constant velocity of sliding, $b > a$

2. A model of the shear sliding with friction

Let us consider a model problem on the stability of quasistatic shear sliding with friction along the porous layer taking into account such effects as shearing dilatancy, diffusion and compressibility of the fluid and pores.

The rock-porous layer horizontal interface is the sliding along a porous layer with the velocity v. The normal stress σ is acting on it vertically (from the surrounding rock) and the porous pressure p (from the porous layer). The tangential stress τ and the elastic force, which has a given velocity v_l, is acting on interface horizontally. The pore pressure p in the porous layer can vary as a result of diffusion of the fluid into the surrounding space with the pore pressure p_o.

Using the law of friction (1), the tangential stress at the rock-porous layer interface can be written in the following form:

$$\tau = \left[f_0 + a \ln\left(\frac{v}{v_0}\right) + b \ln\left(\frac{\theta}{\theta_0}\right)\right]\bar{\sigma}, \tag{6}$$

where τ is the tangential stress, $\bar{\sigma}$ is the effective normal stress, expressed by an applied normal stress σ and the pore pressure p by the relation $\bar{\sigma} = \sigma - p$.

Considering the process of quasistatic sliding of the rock-porous layer interface, we can write the following equation for the variation rate of the tangential stress:

$$\frac{d\tau}{dt} = k(v_l - v), \tag{7}$$

where k is the elastic force rigidity, v_l is the given velocity of motion of this force, and v is the sliding velocity.

Equations for the pore pressure and for the inelastic porosity component are discussed in Sections 3 and 4.

3. Equation for the pore pressure

Let us obtain the equation for the pore pressure using as follows:

1. The equation of discontinuity for the flow of a fluid in a relatively rigid porous skeleton:

$$\frac{dM}{dt} + \operatorname{div}\vec{q} = 0, \tag{8}$$

where $\vec{q}$ is a flow of the fluid mass, M is the fluid mass in unit volume.

2. Darcy's law:

$$\vec{q} = -\rho_0 \frac{K}{\mu} \nabla p, \tag{9}$$

where K is permeability, ρ_0 and μ are, respectively, the actual density and viscosity of the fluid.

3. The equation for the change rate of the fluid mass:

$$\frac{dM}{dt} = \rho\frac{dm}{dt} + m\frac{d\rho}{dt} = \rho\frac{dm}{dt} + m\left(\rho\beta_1\frac{dp}{dt}\right), \tag{10}$$

where m is porosity, and $\beta_1 = \rho^{-1}\partial\rho/\partial p$ is the fluid compressibility coefficient.

4. The equation for the porosity variation as sum of the elastic and the plastic components:

$$\frac{dm}{dt} = m\beta_2\frac{dp}{dt} + \frac{dm_p}{dt}, \tag{11}$$

where $\beta_2 = m^{-1}\partial m/\partial p$ is the coefficient of elastic compressibility of pores, dm_p/dt is the variation rate of the plastic porosity component.

Substituting (11) into (10), we obtain:

$$\frac{dM}{dt} = \rho\left[\beta m\frac{dp}{dt} + \frac{dm_p}{dt}\right], \tag{12}$$

where $\beta = \beta_1 + \beta_2$ is a combination of the compressibility coefficients of the fluid and pores. From (12) we have for $dM/dt = 0$:

$$\frac{dp}{dt} = -\frac{1}{\beta m}\frac{dm_p}{dt}. \tag{13}$$

Hence, it is evident that the pore pressure increases as the rock becomes denser ($dm_p/dt < 0$) and decreases as its density decreases ($dm_p/dt > 0$). Substituting (12) and (9) into (8), we obtain the equation for the pore pressure:

$$\frac{dp}{dt} = D\nabla^2 p - \frac{1}{\beta m}\frac{dm_p}{dt}, \tag{14}$$

where $D = K/(\beta\mu)$ is the diffusion coefficient. For the model being considered with one degree of freedom, we approximate equation (14) for the pore pressure in the following form:

$$\frac{dp}{dt} = D^*(p_o - p) - \frac{1}{\beta m}\frac{dm_p}{dt}, \tag{15}$$

where p is the pore pressure in the sliding layer, p_o is the pore pressure outside this layer, $D^* = K/(\beta\mu L^2)$ is the coefficient of fluid diffusion, and L is the characteristic scale of diffusion.

4. Equation for the inelastic porosity component

The inelastic change in porosity depends on the sliding velocity and is taken, by analogy with relation (7), in the following form:

$$\frac{dm_p}{dt} = -\frac{v}{d_c}(m_p - m_{ps}), \tag{16}$$

where m_{ps} is the value of porosity for the steady sliding. Its value depends on the sliding velocity:

$$m_{ps} = m_0 + c_d \ln \frac{v}{v_0}, \tag{17}$$

where c_d is the coefficient of dilatancy. It can be seen from (17) that the steady porosity m_{ps} increases with the increase in the velocity.

5. Analysis of linear stability of sliding

In the case of the steady sliding of the rock-porous layer interface, friction and elastic forces are in equilibrium. Hence,

$$(\sigma - p)f(v, \theta) = k(v_l t - u), \tag{18}$$

where $f(v, \theta)$ is the function for the friction coefficient given by expression (1), u is the displacement, t is time. The variables for the steady sliding take the following values: $v_s = v_l$, $\theta_s = d_c/v_l$, $p_s = p_0$, $\tau_s = (\sigma - p_0)f_s$, $f_s = f_0 + (a - b)\ln(v_s/v_0)$, $m_{ps} = m_0 + c_d \ln(v_l/v_0)$.

Linearizing the system of governing equations (2), (7), (16), (15), and (18) in the vicinity of the equilibrium state, we obtain the following linear system of equations:

$$\frac{d\delta\theta}{dt} = -\frac{v_l}{d_c}\delta\theta - \frac{1}{v_l}\delta v, \qquad \frac{d\delta m_p}{dt} = -\frac{v_l}{d_c}\delta m_p + \frac{c_d}{d_c}\delta m_p, \tag{19a}$$

$$\frac{d\delta p}{dt} = -D^*\delta p - \frac{1}{\beta}\frac{d\delta m_p}{dt}, \qquad \frac{d\delta u}{dt} = \delta v, \tag{19b}$$

$$\frac{a}{v_l}(\sigma - p)\frac{d\delta v}{dt} = -\frac{b v_l}{d_c}\frac{d\delta\theta}{dt} + f_s\frac{d\delta p}{dt}, \tag{19c}$$

where δ denotes small deviations of the variables from their values at steady sliding.

Equation (19c) was obtained under the assumption that the normal stress σ is kept constant during the process of sliding.

The characteristic equation for the system of equations (19) has the following form:

$$[a(\sigma - p)\lambda + k d_c z](D^* + \lambda)(z + \lambda) - b(\sigma - p)z\lambda(D^* + \lambda) + \frac{f_s c_d}{\beta} z\lambda^2 = 0, \tag{20}$$

where $z = v_l/d_c$.

Equation (20) is a polynomial of the third degree with respect to λ. As is known, if real parts of all the roots $\mathrm{Re}(\lambda_i)$ less than zero, $i = 1, 2, 3$, system of equations (19) is linearly stable. If $\mathrm{Re}(\lambda_i) > 0$ for some λ_i, the system is unstable.

The critical rigidity is determined by the largest value of kd_c, for which $\mathrm{Re}(\lambda_i) > 0$ for a certain i. The solution of equation (20) gives the following estimate of the critical rigidity:

$$k_c = (\sigma - p)\frac{b-a}{d_c} - \frac{c_d f_s}{\beta d_c}\Psi(D^*), \tag{21}$$

where the function $\Psi(D^*)$ is given by the expression

$$\Psi(D^*) = \left[1 + e_1 + e_2 - \sqrt{(1 + e_1 + e_2)^2 - 4e_2}\right]/2, \tag{22a}$$

$$e_1 = \frac{\beta(\sigma - p)a}{c_d f_s}\frac{q^2}{q+1}, \qquad e_2 = \frac{\beta(\sigma - p)(b-a)}{c_d f_s}\frac{1}{q+1}, \tag{22b}$$

$$q = D^* d_c / v_l. \tag{22c}$$

Estimates of critical rigidity are obtained from (21) and (22) for the following two limiting cases:

1) an infinitely large diffusion of the fluid ($D^* \to \infty$, $e_2 \to 0$, and $\Psi(D^*) \to 0$):

$$k_{c\infty} = (\sigma - p)(b-a)/d_c. \tag{23}$$

and 2) an infinitely small diffusion ($D^* \to 0$, $e_1 \to 1$, and $\Psi(D^*) \to 1$):

$$k_{c0} = \left[(\sigma - p)(b-a) - \frac{c_d f_s}{\beta}\right]/d_c. \tag{24}$$

The limiting case of the infinitely small diffusion of fluid implies the absence of flow of the pore fluid.

Estimate (21) means the following: for small perturbations of the sliding velocity and an inconsiderable deviation of the elastic rigidity coefficient k from a critical value k_c given by the estimate (22), the amplitudes of oscillations of all variables (sliding velocity, tangential stress, pore pressure, and porosity) will either attenuate under the condition $k/k_c > 1$ (stable sliding) or increase under the condition $k/k_c < 1$ (unstable sliding) with time.

Using estimates (21)–(24), we can conclude that:

1. The critical rigidity $k_c \to 0$ as the effective normal stress tends to zero ($\bar{\sigma} = \sigma - p \to 0$). Hence, an increase in the pore pressure contributes to the stability of sliding.
2. The stability of sliding is influenced by the ratio between the coefficients of the dilatancy c_d and the compressibility of the fluid and pores β, i.e., the parameter c_d/β.

3. The instability of sliding under the condition of the absence of the flow of pore fluid will be suppressed, when the coefficient of dilatancy c_d will be higher than the critical value determined by (24).
4. For the instability of sliding, under the condition of the absence of flow of the porous fluid, the effective normal stress $\bar{\sigma}$ must exceed the value $c_d f_s/\beta(b-a)$ (since $k_{c0} \geq 0$) even for sufficiently small values of rigidity. Hence, the requirement of the instability of sliding in these conditions leads to the severe limitation on the ratio between the pore pressure and the normal stress:

$$\frac{p}{\sigma} < 1 - \frac{c_d f_s}{\beta\sigma(b-a)}. \tag{25}$$

Figure 2 shows the limiting values of the ratio between the pore pressure and the normal stress versus the coefficient of dilatancy for unstable sliding in the absence of flow of the pore fluid. The area of instability is under the curve, and the area of stability is over the curve. It is seen from the figure that for each value of the parameter $b-a$ there exist critical values of the

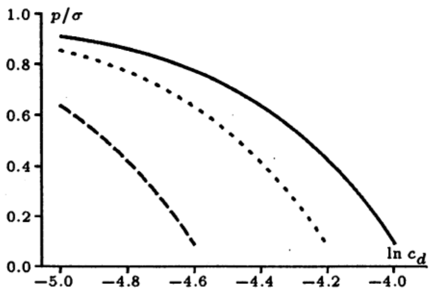

Figure 2. Limiting values curves: $b - a = 0.001$ (solid curve), $b - a = 0.0025$ (dotted curve), $b - a = 0.004$ (dashed curve)

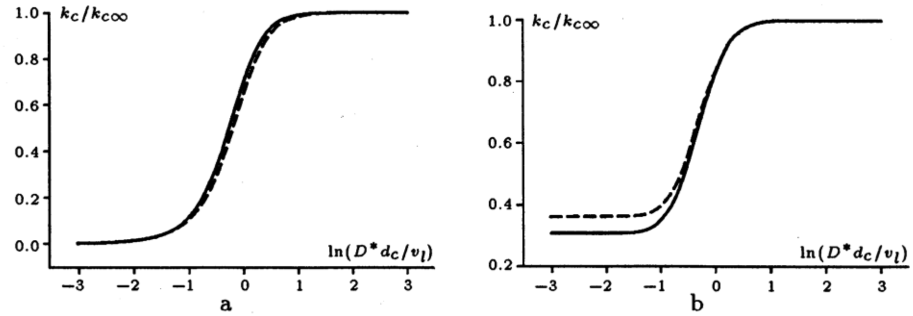

Figure 3. The critical rigidity $k_c/k_{c\infty}$ versus the diffusion $D^* d_c/v_l$:
a) $h = 1300$–2000 m ($\bar{\sigma} = 32$ MPa), $\beta = 0.00045$ MPa^{-1} (solid curve), $\beta = 0.0007$ MPa^{-1} (dashed curve) and
b) $h = 2500$–4000 m ($\bar{\sigma} = 64$ MPa), $\beta = 0.00035$ MPa^{-1} (solid curve), $\beta = 0.00032$ MPa^{-1} (dashed curve)

coefficient of dilatancy d_c^* such that at $d_c > d_c^*$ and any $p/\sigma > 0$ only a stable sliding takes place.

The dependence of the critical rigidity $(k_c/k_{c\infty})$ on the diffusion $(D^* d_c/v_l)$ given by equation (21) is shown in Figure 3. Figures 3a and 3b present the results for two rocks: 1) sandstone with cemented clay cement and 2) sandstone and aleurite tightly cemented with clay-carbonate cement, with depths h = 1300–2000 m and h = 2500–4000 m. The difference between the rocks influences the values of the combined coefficient of compressibility β, and the difference between the depths influences the values of the effective normal stress $\bar{\sigma} = \sigma - p$. For other parameters we took the following values: $f_s = 0.6$, $b - a = 0.005$, $c_d = 0.001$, $d_c = 0.01$ m.

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