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Two and three-dimensional modeling of tzunami generation due to a submarine mudslide

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Introduction

The mathematical models of the tsunami wave generation by an underwater landslide should include a set of the fluid flow equations and the landslide motion equations along the sloping surface taking into account the energy of the wave generation and the necessary initial and boundary conditions. Examples of the two-dimensional (2D) and three-dimensional (3D) numerical modeling of the tsunami generation are represented.

In Section 1, the numerical modeling of tsunami generation due to the submarine slides for simple models of a landslide has been developed. In Section 1.1, the linear and the nonlinear 2D incompressible shallow water and the long wave equations, including the friction effects and the Coriolis parameter are solved by the SWAN code [1]. In Section 1.2, the solution of the 2D time dependent Navier–Stokes equations for an incompressible flow solved by the ZUNI code is presented [1]. In Section 1.3, the simplified slide models are used to consider the slide as a solid nontransformed body [2–4] moving due to the gravity forces, fluid resistance and the Coulomb friction along the bed. In Section 1.4, examples of the numerical modeling of some hypothetical and observed slide tsunamis in the past are discussed, for instance: the hypothetical 105 Ka Lanai tzunami [5], the observed underwater landslides of Storegga [6–10], the landslide lapse occurred in the Tafjord fjord in western the Norway in 1934 [11] and the underwater landslides of Floras Island in Indonesia [12].

In Section 2 the 2D, unsteady, single-layer, depth-averaged turbidity currents driven by nonuniform, noncohesive sediments have been mathematically modelled [13]. The model comprises the fluid, momentum and sediment conservation laws for hydrodynamics, and a bed sediment conservation equation for the bed dynamics. The finite volume method was selected, because it is useful for solving hyperbolic, time dependent equations such as conservation laws.

In Section 3, unstable sedimentary bodies of the slide and their hydraulic effects are numerically examined [14]. The 2D fluid mechanics model based on the Navier–Stokes equations has been developed assuming sediments and water as mixture [15]. The sediments are treated as the Bingham fluid that can diffuse into water. The model includes the viscous–plastic relations and the Fick law of diffusion for the sediments [14].

In Section 4, the impact of a debris avalanche with a volume of 40×10^6 m^3 into the sea and tsunamis it generates are considered [16]. It has been numerically simulated by a mixture model [17] solving the 3D Euler equations. The mixture composed of sediments and water is treated as a homogeneous fluid. Numerical tests show that the generated waves are sensitive to both the initial impact velocities and the avalanche fronts of the landslide. The water surface and velocities calculated by the 3D mixture model are used as input data in the nonlinear shallow water model [18] to calculate tsunami propagation along the coasts of Montserrat.

In Section 5, the numerical 3D model [19] is developed to simulate tsunami generation due to a viscous mudslide on a gentle uniform slope. The problem of the mudslide dynamics is formulated, where the mudslide is treated as an incompressible 3D viscous flow. The seawater is treated as an nonviscous fluid, and the water motion is assumed irrotational. The long wave approximation is adopted for both the water waves and the mudslide. The resulting differential equations are solved by the finite difference method. Numerical results are presented for successive profiles of the mud surface, the horizontal velocities of the mudslide, the evolution of the surface elevations, and the water motion velocities. Comparisons of the present 3D calculations with the previously published 2D results have been done.

In Section 6, there is a brief review of several papers on the 2D numerical simulation of the submarine landslides.

1. Numerical modeling of tsunami generation due to submarine slides for simple models of a landslide

The wave formation and propagation due to landslides and avalanches are complex phenomena that may be divided into three parts: energy transfer from the slide motion to the water motion, the wave propagation in open water, and the wave run-up on the shores. From the viewpoint of modeling, the second part may be the simplest one, since well-established hydrodynamic equations may be directly applied. The physical processes involved in the first part are much more complex, and no common model equations are available that would describe the motion of different slide materials as rock, clay, mud, ice and snow and the energy transfer mechanism between the slide and the fluid.

The waves generated by slides can often be classified as long waves. Most energy transferred from the slide to the water motion is distributed among the waves with a typical wavelength λ, which is much larger than the characteristic water depth D_0. From the assumption $D_0/\lambda \ll 1$ it follows that the pressure is approximately hydrostatic, and the vertical variations of the horizontal velocity are small. It is also assumed that the characteristic amplitude of the waves a is much less than D_0. Nonlinear effects may be important in the wave generation area, but only in a restricted region and for a short period of time. The accumulative effect of the nonlinearity will not probably exceed errors that originate from uncertainties in the slide shape and motion. On the basis of these assumptions, the linearized shallow water equations are used.

The estimations of sliding tsunamis are often obtained by the numerical solution of nonlinear or linear shallow water equations and considering landslide lapses approximately. In other cases, they are taken into account through the bed topography change due to the landslide motion. The bottom topography is set either as a function of coordinates and time, or it is determined by the velocity change law of the solid landslide body motion of a given shape, or it results from the solution of the solid landslide dynamics equation taking into consideration the gravity and the resistance forces of the bottom and liquid. In these cases, the shallow water equations and the boundary conditions comprise the terms with the function $h_s(x, y, t)$ reflecting the bottom topography change.

The formation of tsunami waves because of an initial sea surface displacement similar to the tsunami waves due to the landslide has been examined. Given an initial sea surface displacement the characteristics of the tsunami wave formed by the landslide using the nonlinear shallow water model and the incompressible Navier–Stokes model are compared.

1.1. The shallow water numerical model

The incompressible shallow water and the long wave equations including the friction effects and the Coriolis parameter solved by the SWAN code are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial \zeta}{\partial x} = fv + F^{(x)} - g\frac{uV}{C^2H}, \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial \zeta}{\partial y} = -fu + F^{(y)} - g\frac{vV}{C^2H}, \tag{2}$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial Hu}{\partial x} + \frac{\partial Hv}{\partial y} = \frac{\partial h_s}{\partial t}, \tag{3}$$

where t is time, u is the velocity in x direction, v is the velocity in y direction, g is the gravitational acceleration, $-h(x,y) + h_s(x,y,t) < z < \zeta(x,y,t)$ is the liquid flow in the area, h is the unperturbed depth of water close to $z = 0$, ζ is the water surface displacement, $h_s = h_s(x, y, t)$ is the bottom topography change due to the landslide motion, $H = h + \zeta - h_s$ is the bed depth of water, h_s is the bottom motion, f is the Coriolis parameter, C is the DeChezy coefficient for the bottom friction, $F^{(x)}$, $F^{(y)}$ are forcing functions of the wind stress in x and y directions, $V = (u^2 + v^2)^{1/2}$ is the water module velocity.

The details of the computational procedure SWAN are described in the reference [1].

The linear shallow water and the long wave equations including the friction effect and the Coriolis parameter are the following:

$$\frac{\partial u}{\partial t} + g\frac{\partial \zeta}{\partial x} = fv + F^{(x)} - g\frac{uV}{C^2H},$$

$$\frac{\partial v}{\partial t} + g\frac{\partial \zeta}{\partial y} = -fu + F^{(y)} - g\frac{vV}{C^2H},$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial Hu}{\partial x} + \frac{\partial Hv}{\partial y} = \frac{\partial h_s}{\partial t},$$

where $H = h(x, y) - h_s(x, y, t)$ is the water layer thickness. The symbols of the other variables are the same as for the system of equations (1)–(3).

1.2. The Navier–Stokes numerical modeling

The two-dimensional time dependent Navier–Stokes equations for incompressible flow solved by the ZUNI code are the following:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\mu}{\rho_0}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right),$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{\mu}{\rho_0}\left(\frac{\partial^2 v}{\partial y \partial x} + \frac{\partial^2 v}{\partial x^2}\right),$$

where p is pressure and μ is the viscosity coefficient. A partial cell treatment that allows a rigid free slip obstacle to be placed through cell diagonals is included. The desired boundary slope is obtained by choosing the appropriate aspect ratio for mesh cells. Thus, the numerical technique can be used to calculate the wave run-up on the exposed coasts in addition to the submerged coasts. The details of the calculational procedure ZUNI are described in reference [1].

The detailed Navier–Stokes numerical simulation of the gravity waves that resemble the profile of actual tsunami waves was first done in reference [1]. The interaction of tsunami waves with slopes that resemble the continental slope and shelf was simulated. Wave heights were calculated to increase by a factor of four as they shoaled up 6.66 percent continental slope.

1.3. Slide modeling

Obviously, the energy transfer from the slide mass to water is a very complicated process that is impossible to simulate in detail. The composition of the slide may vary over a wide range from large blocks to fine particles that will experience resistance from the viscous drag, form a drag and added mass. The slide will also lose energy due to collisions and friction among slide particles and because of the bottom friction. The slide characteristics can considerably change during the slide process; blocks can be crushed, mass may be released or deposited along the sea bed, and water can entrain into the total slide mass, thereby generating the turbidity currents.

The complexity of the real slide process accounts for developing simplified slide models. Such models do not take into consideration many of the enumerated factors and reflect only certain physical features of the slide material. Given below are examples of slide models according to their complexity.

Modeling of the surface waves generated by the motion of a submerged body is a specific application to underwater landslides. In such problems, an underwater "bump" initially at rest on the sloping bottom is gradually moving under the action of gravity.

Consider a slide as a solid nontransformed body [2–4] moving due to the gravity forces, the fluid resistance and the Coulomb friction along the bed. The friction between the fluid and the slide body can be neglected if it is infinitesimal in comparison with inertia, i.e., the dimensionless parameter $\epsilon = c_D U l / (2h\sqrt{gD_0}) \ll 1$, where c_D is the coefficient of the slide resistance, l is its length, h is the maximum height, U is the slide velocity, D_0 is the basin depth. This model is useful for describing the motion of slides of consolidated material which preserve their form during the motion. It is applied in studying tsunami waves due to the fall of rocks, ice blocks and so on into the sea.

1.4. Examples of the numerical modeling of underwater landslides

Let us consider the examples of the numerical modeling of hypothetical and observed slide tsunamis occurred in the past.

1.4.1. Modeling the 105 Ka Lanai tzunami. The first example refers to the large tsunami wave run-up with the height of $\zeta \sim 325$ m which happened 105 thousand years ago on Lanai island in the Hawaiian chain. One of the reasons for this historical event could be the underwater landslide. In future, there will be a great possibility for very large shore landslide lapses which might generate gigantic tsunamis and cause the greater part of the Hawaiian islands to sink. Therefore, the analysis of such events is of primary concern.

The numerical simulation of the hypothetical event on Lanai island was made in [5] based on the system of equations (1)–(3) solved by the obvious finite difference method on the code SWAN [1]. The landslide lapse was approximately described by the instantaneous topography change of the sea bottom (descent $20 \times 20 \times 1.5$ km of a shore slope part of the island and ascent $28 \times 28 \times 0.75$ km of a flat ocean bottom) in such a way that the landslide volume of 600 km^3 was preserved. The calculations have shown that the earlier estimated volumes of local underwater landslides appear to be insufficient to generate a gigantic tsunami wave.

1.4.2. The underwater landslides of Storegga. Let us consider another aspect of the problem, namely, the aspect connected with the description of the landslide motion. The lapse of an underwater landslide actually means the collapse of nonconsolidated sediments moving along the sea bottom slope until they reach its flat part. While the landslide moves, the topography of the sea bottom changes, and its duration is not negligibly small as compared to the time of the sea surface displacement. Thus, the actual landslide lapse cannot be adequately simulated by the instantaneous change of the sea bottom topography.

The so-called underwater landslides of Storegga which occurred at least three times in the Storegga area of a continental shelf of the western Norway are well studied. They formed a sediment layer of 450 m wide and 800 km long on a flat part of the sea bottom at the depth of 3.6 km. The greatest landslide happened 30–50 thousand years ago. Its volume was large enough to cover the entire surface of Alaska with a thick sediment layer.

However, the greatest attention was given to the analysis of tsunami induced by the second landslide which occurred 6–8 thousand years ago. Two slide bodies of the horizontal size of 10×30 km moved downwards along the continental slope for 200 km and generated tsunami with the wave run-up found along the shore line of the eastern coast of Northern Atlantic [6, 7]. The deposition of sediments was retrieved in the areas located 4 m higher than marks of the highest water level. In the numerical modeling of the water waves generated by the second landslide of Storegga, the landslide motion was described by the solid body dynamics equation, and the water motion – by simple linear shallow water equations (1)–(3) solved with the help of obvious finite difference method [8, 9]. It was found that the values of tsunami wave run-up considerably depended on the average velocity of the landslide motion and the shear stresses at the water-slide body interface. The landslide moving at the average velocity of 35 m/sec should make a wave run-up from +3 to +5 m high along the eastern coast of Greenland, Iceland, Scotland and the western coast of Norway. There is a marked initial lowering of a water level (below − 10 m) along the western coast of Norway. The second landslide probably caused two main tsunami waves as well as some minor fluctuations of the water level.

In another numerical model of the second landslide of Storegga [10], the landslide lapse was not considered. The estimation of the initial wave amplitude in the center (∼ 10 m) was obtained by the simple formula:

$$\ln \frac{\zeta}{H} = k_1 + k_2 \ln \frac{V}{H^3},$$

where V is the landslide volume, H is the average water depth. Then using the linear shallow water equation, the numerical calculation of tsunami wave in the direction of the eastern coast was done. The calculated values of the tsunami wave runup appeared to be close to the measured ones [8].

The numerical model similar to the model [8] was applied to analyze the drastic event connected with the landslide lapse in Tafjord fjord in the western Norway in 1934 [11]. The predicted heights of the wave run-up according to the numerical simulation well agree with the measured ones, the calculated oscillations of quiescent waves coinciding with the observed ones.

1.4.3. The underwater landslides of Floras Island in Indonesia. Underwater earthquakes together with tectonic tsunamis can be called slide tsunamis, their amplitude can be higher than the amplitude of the former. Probably, the following event testifies to this. On December 12, 1992, the earthquake with a magnitude $M = 7.5$ on the Richter scale struck the center of Floras Island in Indonesia. Nearly 2,000 people were killed by the earthquake and the tsunami.

The measured values of a tsunami wave runup in some places of the northern shore of Hading Bay appeared to be much higher than the values predicted by numerical simulation results of tectonic tsunamis. Great heights of the wave run-up were bounded by the area around the underwater ground sliding and could be caused by the waves generated by a underwater landslide rather than by tectonic tsunamis.

A tsunami model including the motion of a landslide using a circular arc slip model and a subsidence model is proposed and applied to the wave generation for the case of significant phenomena on the southern shore of Hading Bay, Floras Island, Indonesia [12]. The one-dimensional propagation and simple topography are introduced to simplify the numerical conditions due to the lack of field data. The effects of the soil diffusion, bottom friction and drag on the wave generation are also discussed. The results of this research suggests that two circular arc slip models properly describe the significant phenomenon in Hading Bay.

A few models were developed for the case of the Floras Earthquake Tsunami with allowance for the kinematic and the dynamic processes of the landslide near the coastal line. The subsidence model is simple. The slope just drops vertically downward. The force causing the vertical drop is obtained as follows:

$$m\frac{d^2\zeta}{dt^2} = g[1 - (C + \tan\theta)],$$

where m is mass of the soil, ζ is displacement of the sea bottom/slope, C is the soil cohesiveness, and $\tan\theta$ is the soil friction.

In the second model, the circular arc slip model, two types of the slip are considered: toe slip and base slip. The failure surface for both models is assumed to be a circular arc profile.

To confirm this, a numerical model of tsunamis including three models of a landslide slip which differed from those mentioned above was proposed in [12]. Two of them considered the material slip of the shore slope along an arc circle, and the third describes vertical subsidence of this material. The numerical simulation of the one-dimensional wave in Hading Bay was performed on the basis of the shallow water theory supposing a simple bottom topography, taking into account the resistance forces and the effects of mixing on the landslide edge.

The results presented in [12], allowed one to explain the cause of a considerable difference among heights of the wave run-up for the northern and the southern shores of Hading Bay. The paper summarizes the geologic data for these tsunamis. These data are compared to the data of the present mathematical modelling of landslides and tsunamis. It is remarkable that there is rather a good consent between estimations of the tsunami wave run-up obtained on the base of the sediment data and those obtained in model experiments.

2. Mathematical model of the two-dimensional, time dependent turbidity current driven by nonuniform sediments

A mathematical model has been developed for the 2D, unsteady, single layer, depth averaged turbidity currents driven by a nonuniform, noncohesive sediments [13]. The sediment entrainment and deposition are explicitly accounted for, which encourages the simulation of the turbidity growth and evolution. The model consists of the fluid, momentum and sediment conservation laws for hydrodynamics, and a bed sediment conservation equation for the bed dynamics. The finite volume method was selected because it is an ideal technique for solving the hyperbolic, time dependent equations such as conservation laws.

2.1. The equations for the two-dimensional turbidity current driven by nonuniform sediments

The equations that form the basis of the model are: vertically integrated fluid, momentum, and sediment conservation equations. They constitute a coupled system of nonlinear, hyperbolic, spatial differential equations. Turbidity currents occur as underflows in the deep sea, when the flow thickness does not exceed approximately 7.5 % of the overall ambient fluid depth, hydrodynamics of the turbidity current can be accurately described by a "single layer" formulation. The equations are valid for the 2D turbidity current driven by nonuniform, noncohesive sediments flowing beneath an infinitely deep layer of the quiescent fluid with the constant density, they are written in the integral form [13]:

$$\frac{\partial}{\partial t}\int_{\Omega} \boldsymbol{U}\, d\Omega + \oint_{\partial\Omega} (\boldsymbol{F}\, dy - \boldsymbol{G}\, dx) = \int_{\Omega} \boldsymbol{Q}\, d\Omega,$$

where $\boldsymbol{U}^T = (h, hU, hV, hC_1, \ldots, hC_{n_s})$ is the vector of conservative variables, and

$$\begin{aligned}
\boldsymbol{F}^T &= (hU, hU^2 + \frac{1}{2}gh^2RC_T, hUV, hUC_1, \ldots, hUC_{n_s}),\\
\boldsymbol{G}^T &= (hV, hUV, hV^2 + \frac{1}{2}gh^2RC_T, hVC_1, \ldots, hVC_{n_s}),\\
\boldsymbol{Q}^T &= (E_w\sqrt{U^2+V^2}, -ghRC_Ts_x - u_*^2, -ghRC_Ts_y - v_*^2,\\
&\quad v_{s_1}(P_1E_{s_1} - c_{b_1}), \ldots, v_{s_{n_s}}(P_{n_s}E_{s_{n_s}} - c_{b_{n_s}})),
\end{aligned}$$

where $RC_T = \sum_{i=1}^{n_s} R_iC_i$.

The term h represents the flow thickness; U and V are vertically averaged velocities in the x and y directions, respectively, and C_i is the vertically averaged volume concentration of the i-th sediment. The total number of sediments contained in the current is denoted by n_s. The parameter $R_i = (\rho_{s_i} - \rho)/\rho$, where ρ_{s_i} is the density of the i-th sediment, and ρ is the density of the ambient water, while g denotes the acceleration due to the gravity.

The term E_w is the fluid entrainment coefficient. The following expression is used in the present model:

$$E_w = \frac{0.075}{\sqrt{1 + 718\mathrm{Ri}^{2.4}}},$$

where Ri is the Richardson number: $\mathrm{Ri} = gRC_T/(U^2 + V^2)$.

The parameters s_x and s_y are bed slopes in the x and y directions, respectively, while u_* and v_* represent the shear velocities in the x and y directions, respectively. The shear velocities are defined as:

$$u_*^2 = c_D U\sqrt{U^2 + V^2}, \qquad v_*^2 = c_D V\sqrt{U^2 + V^2},$$

where c_D is the bed drag coefficient. A typical range of values presented is 0.002–0.06.

The volume fraction of the i-th sediment in the bed is rerepresented by p_i, while E_{s_i} is the i-th sediment entrainment coefficient. The expression E_{s_i} is used for the closure, i.e.,

$$E_{s_i} = \frac{1.3 \times 10^{-7} Z_{m_i}^5}{1 + 4.3 \times 10^{-7} Z_{m_i}^5},$$

where $Z_{m_i} = k\sqrt{u_*^2 + u_*^2}/v_{n_i} f(R_{p_i})$, and k is the strain parameter defined as:

$$k = 1 - 0.288\sigma_*,$$

where σ_* is the standard deviation of the grain-size distribution based on the phi-scale, $\phi = \log_2 D_s$. The function f is dependent on the particle Reynolds number, $R_{p_i} = \sqrt{gR_iD_{s_i}}D_{s_i}/\nu$, i.e.,

$$R_{p_i} = R_{p_i}^{0.6}, \quad R_{p_i} \geq 3.5, \qquad R_{p_i} = 0.586 R_{p_i}^{1.23}, \quad 1 < R_{p_i} < 3.5,$$

where c_{b_i} is the near-bed concentration of the i-th sediment. This expression is

$$\frac{c_{b_i}}{C_i} = 0.40\left(\frac{D_i}{D_{sg}}\right)^{1.64} + 1.64,$$

where D_{sg} denotes the geometric mean size of the suspended sediment mixture. Finally, v_{s_i} is the fall velocity of the i-th sediment in the quiescent water.

2.2. The bed-sediment conservation equation

The equations governing the hydrodymamics have been coupled to the evolution of the bed through a bed-sediment conservation equation. The bed continuity equation is needed to keep track of the amount of loose sediments at the bed. This is critical in order to limit sediment erosion in areas where loose sediments are not covering the bed. In addition, tracking the bed elevation allows one to calculate bed slopes, which are needed to solve the momentum equations.

The bed-sediment conservation equation has the form

$$(1-\gamma)\frac{\partial z}{\partial t} = \sum_{i=1}^{n_s} v_{s_i}(c_{b_i}\cos\theta - p_i E_{s_i}), \tag{4}$$

where x and γ are the elevation and porosity of the bed, respectively, and it has been assumed that $\frac{\partial z}{\partial t} \approx 0$. Note that p_i may also change with time. In order to compute its variation, each grain size may be considered individually, i.e.,

$$(1-\gamma)\left[p_i\frac{\partial z}{\partial t} + z\frac{\partial p_i}{\partial t}\right] = \sum_{i=1}^{n_s} v_{s_i}(c_{b_i}\cos\theta - p_i E_{s_i}), \tag{5}$$

assuming θ not changing rapidly with time. Substituting the expression for $\partial z/\partial t$ from (4) into (5) yields:

$$(1-\gamma)z\frac{\partial p_i}{\partial t} = f_{s_i} - p_i f_i,$$

$$f_s = \sum_{i=1}^{n_s} f_{s_i} = \sum_{i=1}^{n_s} v_{s_i}(c_{b_i}\cos\theta - p_i E_{s_i}).$$

2.3. The finite volume numerical model

The numerical solution is obtained by a high resolution total variation diminishing, finite volume numerical model, which is known to accurately capture sharp fronts. The monotone upstream scheme for conservation laws is used in conjunction with predictor–corrector time stepping to provide a second order accurate solution. Flux limiting is implemented to prevent the development of spurious oscillations near discontinuities.

3. Diffusive viscous–plastic model of a two-dimensional submarine slide

The sliding unstable sedimentary bodies and their hydraulic effects are studied numerically. A 2D fluid mechanics model based on the Navier–Stokes equations has been developed assuming the sediments and water as a mixture. The sediments are treated as the Bingham fluid that may diffuse into water. The viscous–plastic relations and the Fick law of diffusion for sediments have been introduced into the model [14].

3.1. Rheological model of the dense part of the slide

Bingham plastic model is employed to describe the behaviour law of these fluids by combining the yield stress τ_0, called also the Bingham yield, and the plastic dynamic viscosity μ_B. These non-Newtonian fluids are also called the "generalized Newtonian fluids" with a viscosity $\mu(\overline{D})$. The nonlinear stress-strain relation between the shear stress and the rate of shear is expressed by:

$$\bar{\tau} = 2\mu(\overline{D})\overline{D}, \tag{6}$$

where

$$\mu(\overline{D}) = \mu_B + \frac{\tau_0}{\sqrt{\overline{D}^2_{xx} + \overline{D}^2_{yy}}}, \qquad \text{if} \quad \bar{\tau}^2_{xx} + \bar{\tau}^2_{yy} \geq \bar{\tau}^2_0,$$

$$\mu(\overline{D}) = \infty, \qquad \overline{D} = 0, \qquad \text{if} \quad \bar{\tau}^2_{xx} + \bar{\tau}^2_{yy} \leq \bar{\tau}^2_0,$$

$\bar{\tau}$ is the non-diagonal part of the stress tensor, $\overline{D}$ is the shear rate or non-diagonal part of the strain rate tensor.

The nonlinear constitutive relation (6) leads to two distinct zones in the flow, a shear zone and a plug zone, or only one of them according to the value of the yield stress.

3.2. Nasa–VOF2D numerical model and its extension

The Eulerian code Nasa–VOF2D solves the complete incompressible and nonlinear Navier–Stokes two-dimensional equations for one single fluid with a free surface [15] and its extension [14].

Modeling of the water waves generated by a submarine slide on the long wave approximation gives wrong results for steep slopes of the sea bottom. In this case, we have to discard the long-wave approximation and consider models on the basis of the complete Navier-Stokes equations. Such an approach was used, for example, in [14] with the following assumptions:

- The mechanism which may initiate the landslide is not examined
- The submarine sediments are treated as a monophasic isotropic continuity; the motion of interstratal fluid is then neglected
- Both water and sediments are assumed to be incompressible
- An impermeable rigid slope is considered, and the erosion of the sea bottom caused by the slide flow is neglected.

The mixture equations of conservation of mass and momentum, the water surface equation and one additional diffusion equation, which takes into account the concentration changes are expressed as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0, \qquad \rho = \rho_1 + c\partial\rho, \quad \partial\rho = \rho_2 - \rho_1 \tag{7}$$

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v}) = \rho \boldsymbol{g} - \nabla p + \nabla \cdot \bar{\tau}, \qquad \bar{\tau} = 2\mu(\bar{\boldsymbol{D}})\bar{\boldsymbol{D}}, \tag{8}$$

$$\frac{\partial F}{\partial t} + \nabla \cdot (F\boldsymbol{v}) = 0. \tag{9}$$

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\boldsymbol{v}) = \frac{1}{\rho_2}\nabla \cdot (\chi \nabla c), \tag{10}$$

where $\nabla = \frac{\partial}{\partial x} \cdot \vec{i} + \frac{\partial}{\partial y} \cdot \vec{j}$, $\vec{i}$ and $\vec{j}$ are the horizontal x and the vertical y axes orts in the Cartesian system of coordinates, $\boldsymbol{v}$ is the fluid velocity vector of the mixture, $\bar{\tau}$ is the shear stress tensor, $\boldsymbol{g}$ is the gravity acceleration, $c(x, y, t)$ is the volume fraction of the solid phase, $\rho(x, y, t)$ is the mixture flow density, ρ_1 and ρ_2 are the water and sediment densities, $F(x, y, t)$ is the fractional volume of the cell filled in with the mixture and c is used for the calculation, $p(x, y, t)$ is the pressure, $\boldsymbol{j} = \chi \nabla c$ is the diffusion flux.

So far, the sediment diffusivity χ has been assumed constant in time and space for numerical reasons. The value of the diffusion coefficient is then an artificial value.

The Navier-Stokes equations are solved by an Eulerian finite difference technique. The resolution of equation (7) leads to the problem of the numerical diffusion at the water-sediments interface. The method has been developed based on a donor-acceptor technique. This technique is adapted for the convection of F (see equation (9)). When no physical diffusion is calculated ($\chi = 0$), the convection of c from a donor cell to an empty downstream cell ($c = 0$) is carried out provided that the upstream donor cell is full ($c = 0$). This method allows one to follow the interface water-sediments without the numerical diffusion. When physical diffusion is introduced into the model, the method still calculates an interface for each cell. Each cell is conventionally divided into two parts, one part with the concentration (c_u) of the upstream cell and the other part with the concentration (c_d) of the downstream cell.

The initiation mechanism of a slide runup in [14] as well as in the works mentioned above is not considered. The slide is thought to start its motion and move along the inclined slope. The friction force of the slide against the slope surface is taken into account, the friction force of the slide against the fluid being neglected.

3.3. The flow of an underwater landslide in different approximations within the system of equations (7)–(10)

The considered model of waves generated by an underwater landslide includes four parameters: the diffusion coefficient χ, the Bingham yield stress τ_0, the viscosity coefficient μ_B, and the friction coefficient along the slope surface k_f. Assuming some values of these parameters equal to zero, it is possible to simulate the flow of an underwater landslide in different approximations within the system of equations (7)–(10), for example:

- nonviscous fluid ($\mu_B = 0$, $\tau_0 = 0$, $\chi = 0$, $k_f = 0$);
- viscous fluid ($\mu_B \neq 0$, $\tau_0 = 0$, $\chi = 0$, $k_f \neq 0$);
- plastic fluid ($\mu_B = 0$, $\tau_0 \neq 0$, $\chi = 0$, $k_f \neq 0$);
- the Bingham viscous-plastic fluid ($\mu_B \neq 0$, $\tau_0 \neq 0$, $\chi = 0$, $k_f \neq 0$);
- nonviscous fluid solid phase diffusion ($\mu_B = 0$, $\tau_0 = 0$, $\chi \neq 0$, $k_f = 0$);
- viscous fluid with solid phase diffusion ($\mu_B \neq 0$, $\tau_0 = 0$, $\chi \neq 0$, $k_f \neq 0$);
- the Bingham viscous-plastic fluid with solid phase diffusion ($\mu_B \neq 0$, $\tau_0 \neq 0$, $\chi \neq 0$, $k_f \neq 0$).

The diffusive model from equations (7)–(10) allows us to describe the water penetration into the granulated mass. The rheological law ($\mu_B \neq 0$, $\tau_0 \neq 0$) is used only for the dense part of the slide body, i.e., in the cells of the calculated area with a high concentration of the solid phase ($c(x, y, t) = 1$). The diffusion mechanism of the granulated mass into the surrounding liquid is triggered only in those cells, where shear stress exceeds yield stress.

3.4. Sand flows at small-scale and water waves they generates

The experiments are to generate water waves by allowing a mass of sand to slide freely down a frictionless inclined plane with the angle of 45°. The channel is 4 m long, 0.03 m wide and 2 m high. For the water depth of 1.60 m, the wave celerity is about 4 m/s. The sand mass is as wide as the channel so that experiments are two-dimensional in a vertical plane. The initial vertical profile of the coarse gravel mass is triangular. This mass is of 0.65×0.65 m in its cross section.

The computational domain is 4 m by 2 m in x and y directions. The mesh consists of 300 columns with variable spacings and 200 rows. The computed density maps and the computed wave profiles are presented at $t = 0.4$ and 0.8 s.

In the first simulation, the sediments are modelled by an ideal fluid without rheological law and without further diffusion. The slide is governed by inertia forces, but not by viscosity forces. The computed mud mass is mainly concentrated in the mud front, whereas in the experiments, the most important part of the mud remains close to the initial position.

The following simulation has been carried out using the Bingham law without diffusion. The plastic viscosity μ_B is set at 0 ($\mu_B = 0$ Pa-s) assuming the yield stress to be exceeded. According to numerical results for the yield stress value of $\tau_0 = 1000$ Pa at $t = 0.8$ s a part of the mass is located on top of the slope, but the most important mass is still concentrated in the mud front. The same distribution of the density ρ is observed for the case with a lower yield stress value.

The concluding simulation has been carried out with an artificial diffusion and the Bingham model $\mu_B = 0$ Pa-s, where the value of $\tau_0 = 1000$ Pa is high. The diffusion model based on the Fick law $\chi = 0.004$ Pa-s allows one to reproduce the penetration of water into the gravel mass, which induces the motion of the mass. The computed density maps can be considered close to the experimental results. The yield stress has a sufficient value, an important part of the mass is close to the top of the slope and the sand volumes are well distributed along the slope, particularly, in the slide front where the mass of sand is decreased.

The comparison of numerical simulation results and small-scale laboratory experiments concerning the subside of some sand mass into water along the slope surface under the angle of 45° has shown [14] that the Bingham model together with the diffusion mechanism reproduce experimental observations rather accurately.

4. Three-dimensional numerical modeling of the debris avalanche impact upon the sea

An impact of a debris avalanche of 40×10^6 m^3 volume upon the sea and the generated tsunami have been numerically simulated by a mixture model solving 3D Euler equations [16]. The mixture composed of sediments and water is considered as a homogeneous fluid. Numerical tests show that generated waves are sensitive to both the initial impact velocities and avalanche fronts of the landslide. The water surface and velocities calculated by the 3D mixture model are used as input data in a nonlinear shallow water model to calculate tsunami propagation along the coast of Montserrat [17].

The 3D hydrodynamics model [16] is developed for a single fluid. The Euler equations are solved with a free surface for a mixture of two incompressible fluids using the Eulerian finite difference method. The debris avalanche is assumed to be a non-viscous fluid flowing down a frictionless slope and non-porous while sliding into water. Mixture equations of conservation of mass and momentum, water surface equation and one additional diffusion equation with allowance of concentration changes are expressed as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0, \qquad \rho = \rho_1 + c\partial\rho, \quad \partial\rho = \rho_2 - \rho_1,$$

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v}) = \rho \boldsymbol{g} - \nabla p,$$

$$\frac{\partial F}{\partial t} + \nabla \cdot (F\boldsymbol{v}) = 0, \qquad \nabla \cdot \boldsymbol{v} = \frac{\rho_2 - \rho_1}{\rho_1 \rho_2} \nabla \cdot \boldsymbol{j},$$

where $\nabla = \frac{\partial}{\partial x} \cdot \vec{i} + \frac{\partial}{\partial y} \cdot \vec{j} + \frac{\partial}{\partial z} \cdot \vec{k}$, $\vec{i}$, $\vec{j}$, and $\vec{k}$ are the axes orts in the Cartesian coordinate system (x, y, z), $\boldsymbol{v}$ is the fluid velocity vector of the mixture, $\boldsymbol{g}$ is the gravity acceleration, $\rho(x, y, t)$ is the mixture flow density, ρ_1 and ρ_2 are the water and sediment densities, $F(x, y, t)$ is the fractional volume of the cell filled in with the mixture, and c is used for the calculation, $p(x, y, t)$ is the pressure, j is the diffusion flux at 0, since no dilution of debris material in water is calculated.

The continuity equation leads to the classical problem of numerical diffusion at the interface between water and debris flow. The method has been developed based on a donor–acceptor technique used for the convection of F. The convection of c from the donor cell to the entry downstream cell $c = 0$ is carried out provided that the upstream donor cell is full $c = 1$. This method allows one to follow the water–sediments interface without the numerical diffusion.

The 3D computational domain extends over 8 km in the x direction, over 4.5 km in the y direction and from -750 m to 400 m in the vertical direction. The mesh consists of 100×80 cells in the horizontal direction and 60 cells in the vertical direction.

The relative importance of the initial avalanche fronts and the initial velocities is investigated by varying these parameters in 12 cases (4 velocities and 3 front heights). The numerical experiments are performed for a mass entering the sea at 40 m/s with a 25 m avalanche front. The waves propagate in a semi-circular fashion outside the slide area, with the maximum wave heights occurring in the slide direction.

A series of numerical results also shows that the mean velocity of the mass penetrating into the sea decreases for the initial velocities of 55 m/s and 40 m/s, whereas it increases for velocities lower than 25 m/s.

The propagation is simulated by a standard shallow water model initialized by the results of the 3D model. The selected case is the numerical experiment with the impact velocity of 40 m/s and the avalanche front 25 m. The considered nonlinear shallow water equations are referred to as $2 + 1$ model in literature [18] indicating to the fact that there are two spatial horizontal propagation directions (x and y) and one temporal (t). The equations are solved by a finite difference method using the upwind scheme which is iterative in time to limit numerical oscillations due to nonlinearities.

The water wave heights and velocities used as initial conditions in $2 + 1$ model have to be calculated by the 3D model at times when most energy was transferred from the landslide to the water wave.

The 3D mixture model is used to calculate water waves generated by a debris avalanche entering the sea. Water waves are propagating at further distances by a shallow water model. The calculated water heights along the Montserrat coast are in the range of those estimated for the event occurred in Old Town on December 26, 1997.

5. The three-dimensional modeling of tzunami generated by a submarine mudslide

The general case of the three-dimensional viscous slide flow is considered in [19], where they deal with the effect of slides spreading along the slope both in the longitudinal and the transverse directions on the characteristics of water waves generated by them. The landslide material is treated as incompressible and nonviscous fluid. The long wave approximation is used for both water waves and the slide. The characteristic length scale of the wave motion is more than the local water depth and the slide thickness is much smaller than the characteristic length scale of the slide along the slope. The long wave approximation is valid only for waves on small slopes (1–10°). There are governing equations for the three-dimensional viscous slide and the surface waves generated by the slide. The surface waves and the water circulation produced by three initial slides of various shapes with different width/length ratios were calculated using finite difference method.

5.1. Governing equations for a viscous slide

The problem of the viscous fluid flowing down a rigid impermeable slope inclined at a small angle θ with respect to the horizon is represented. Let x, y, z be the Cartesian coordinates with their origin at the upper margin of the slope, with x measured seaward at still water level, y measured along the shore, z normal to the $x-y$ plane. The free surface is designated by $z=\eta(x,y,t)$, and the sloping bottom by $z=-\eta_s(x,y,t)$. By the long-wave approximation slide velocities are essentially horizontal, and the vertical momentum equation reduces hydrostatic pressure distribution, that is,

$$p(x,y,z,t)=\rho_1 g[(\eta(x,y,t)+h(x,y,t)]-\rho_2 g[z+h(x,y,t)], \qquad (11)$$
$$-h_s(x,y)\le z\le -h(x,y,t),$$

where $p(x,y,z,t)$ is the pressure in the mud layer, t is time, ρ_1 and ρ_2 are densities of water and mud, respectively; $h(x,y,t)$ is the undisturbed water depth.

The viscous slide is assumed laminar and the nonlinear Navier–Stokes equations will be employed. The x- and y-direction momentum equations can be written down as:

$$\rho_2\left(\frac{\partial U_m}{\partial t}+U_m\frac{\partial U_m}{\partial x}+V_m\frac{\partial U_m}{\partial y}+W_m\frac{\partial U_m}{\partial z}\right)$$
$$=(\rho_2-\rho_1)g\sin\theta-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 U_m}{\partial z^2}, \qquad (12)$$

$$\rho_2\left(\frac{\partial V_m}{\partial t}+U_m\frac{\partial V_m}{\partial x}+V_m\frac{\partial V_m}{\partial y}+W_m\frac{\partial V_m}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\frac{\partial^2 V_m}{\partial z^2}, \qquad (13)$$

where $U_m(x,y,z,t)$, $V_m(x,y,z,t)$, and $W_m(x,y,z,t)$ are the slide velocities in the x, y, and z directions, respectively, μ is the dynamic coefficient of the slide viscosity. The viscous forces involving x and y derivatives are neglected because of the relative thinness of the slide.

The corresponding boundary conditions are 1) the zero-shear condition on the water-slide interface,

$$\frac{\partial U_m}{\partial z} = \frac{\partial V_m}{\partial z} = 0, \quad z = -h(x,y,t), \tag{14}$$

and 2) the no-slip condition on the seafloor,

$$U_m = V_m = W_m = 0, \quad z = -h_s(x,y,t). \tag{15}$$

The mudslide rapidly reaches its equilibrium velocity so that the vertical parabolic approximation may be used for horizontal velocities $U_m(x,y,z,t)$ and $V_m(x,y,z,t)$; namely,

$$U_m(x,y,z,t) = U(x,y,t)\left[2\left(\frac{z+h}{D}\right) - \left(\frac{z+h_s}{D}\right)^2\right], \tag{16}$$

$$V_m(x,y,z,t) = V(x,y,t)\left[2\left(\frac{z+h_s}{D}\right) - \left(\frac{z+h_s}{D}\right)^2\right], \tag{17}$$

which satisfy the boundary conditions on the water-slide interface and on the slide bottom (14), (15); $U(x,y,t)$ and $V(x,y,t)$ are horizontal velocities of the slide at the water-slide interface, $z = -h(x,y,t)$ and $D(x,y,t)$ is the thickness of the slide.

The vertical component of the velocity $W_m(x,y,z,t)$ can be derived from continuity equation:

$$\frac{\partial U_m}{\partial x} + \frac{\partial V_m}{\partial y} + \frac{\partial W_m}{\partial z} = 0, \tag{18}$$

integrating (18) with respect to z from $z = -h_s(x,y)$ to $z = -h(x,y)$ and substituting (16) and (17) yield

$$W_m(x,y,z,t) = -D(x,y,t)\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)\left[2\left(\frac{z+h_s}{D}\right) - \left(\frac{z+h_s}{D}\right)^2\right]. \tag{19}$$

Integrate momentum equations (12) and (13) with respect to z from $z = h_s(x,y)$ to $z = h(x,y,t)$ and substitute (11), (14), (16), (17), and (19) to obtain x and y direction depth-averaged momentum equations for the slide in terms of the unknown $D(x,y,t), U(x,y,t)$, and $V(x,y,t)$:

$$\rho_2\left[\frac{2}{3}D\frac{\partial U}{\partial t} - \frac{1}{3}U\frac{\partial D}{\partial t} + \frac{2}{15}D\left(5U\frac{\partial U}{\partial x} + 4V\frac{\partial U}{\partial y} - U\frac{\partial V}{\partial y}\right)\right]$$
$$= D\left[(\rho_2 - \rho_1)g\left(\alpha - \frac{\partial D}{\partial x}\right) - \rho_1 g\frac{\partial \eta}{\partial x}\right] - 2\mu\frac{U}{D}, \qquad (20)$$

$$\rho_2\left[\frac{2}{3}D\frac{\partial V}{\partial t} - \frac{1}{3}V\frac{\partial D}{\partial t} + \frac{2}{15}D\left(5V\frac{\partial V}{\partial y} + 4U\frac{\partial V}{\partial x} - V\frac{\partial U}{\partial x}\right)\right]$$
$$= D\left[-(\rho_2 - \rho_1)g\frac{\partial D}{\partial y}\right) - \rho_1 g\frac{\partial \eta}{\partial y}\right] - 2\mu\frac{V}{D}, \qquad (21)$$

where $\alpha = \sin\theta$.

The mass conservation equation for the entire slide can be written down as:

$$\frac{\partial D}{\partial t} + \frac{\partial}{\partial x}\left(\int_{h_s}^{-h_s+D} U_m\,dz\right) + \frac{\partial}{\partial y}\left(\int_{h_s}^{-h_s+D} V_m\,dz\right) = 0,$$

or

$$\frac{\partial D}{\partial t} + \frac{2}{3}\left[\frac{\partial}{\partial x}(UD) + \frac{\partial}{\partial y}(VD)\right] = 0. \qquad (22)$$

There are the following dimensionless variables:

$$(x^*, y^*, z^*, t^*) = ([L]^{-1}(z, y), [H]^{-1}z, (g'/[L])^{1/2}t), \qquad (23a)$$
$$(\eta^*, D^*, h^*, H_s^*) = [H]^{-1}(\eta, D, h, h_s), \qquad (23b)$$
$$(U_m^*, V_m^*, W_m^*, U^*, V^*) = [U]^{-1}(U_m, V_m, W_m, U_m, V_m). \qquad (23c)$$

g' is the reduced gravity defined as $g' = g(\rho_2 - \rho_1)/\rho_2$.

With (23), the governing equations of the slide (20)–(22) take on the form (with asterisks omitted):

$$\frac{2}{3}D\frac{\partial U}{\partial t} - \frac{1}{3}U\frac{\partial D}{\partial t} + \frac{2}{15}D\left(5U\frac{\partial U}{\partial x} + 4V\frac{\partial U}{\partial y} - U\frac{\partial V}{\partial y}\right)$$
$$= D\left(\alpha - \epsilon\frac{\partial D}{\partial x} - \frac{\epsilon}{r-1}\frac{\partial \eta}{\partial x}\right) - \mu\frac{2}{\epsilon R}\frac{U}{D},$$

$$\frac{2}{3}D\frac{\partial V}{\partial t} - \frac{1}{3}V\frac{\partial D}{\partial t} + \frac{2}{15}D\left(5V\frac{\partial V}{\partial y} + 4U\frac{\partial V}{\partial x} - V\frac{\partial U}{\partial x}\right)$$
$$= -D\left(\epsilon\frac{\partial D}{\partial y} + \frac{\epsilon}{r-1}\frac{\partial \eta}{\partial y}\right) - \mu\frac{2}{\epsilon R}\frac{V}{D},$$

$$\frac{\partial D}{\partial t} + \frac{2}{3}\left[\frac{\partial}{\partial x}(UD) + \frac{\partial}{\partial y}(VD)\right] = 0.$$

where $\epsilon = [H]/[L]$, $r = \rho_2/\rho_1$, $R = \rho_2[H](g'[L])^{1/2}/\mu$, R may be called the slide intensity parameter which combines parameters that affect the slide flow mode. The Reynold number is defined as $\mathrm{Re} = \rho_2 DU/\mu$ to characterize the mud flow.

The laminar flow mode is expected for the Reynold numbers $\mathrm{Re} = \rho_2 U h_0/\mu < \mathrm{Re}_c \approx 1000$, and the contact surface stability in the two-layer shear flow (landslide body – water) demands that the Keulegan number $\mathrm{Ke} = \rho_2^2(\Delta U)^3/((\rho_2 - \rho_1)g\mu) < \mathrm{Ke}_c \approx 180$, where h_0 is the initial maximum height of the landslide, $\Delta U = U - u$, U is the typical velocity of the landslide, u is the typical velocity of water, ρ_2 is the density of the landslide, ρ_1 is the density of water. It follows from the expression for the Keulegan number that the contact boundary in the two-layer shear flow is steadier for large values of viscosity and density differences.

5.2. Governing equations for waves

For waves on a gentle slope in shallow water the long-wave approximation is adopted, i.e., the vertical length scale is much smaller than the horizontal one. Thus, the water motion is essentially horizontal and the pressure distribution in water can be assumed hydrostatic. Neglecting the wave dispersion, depth averaged dynamic equations for nonlinear shallow-water waves due to the impermeable seabed motion are the following:

$$\frac{\partial(\eta + h)}{\partial t} + \frac{\partial[u(\eta + h)]}{\partial x} + \frac{\partial[v(\eta + h)]}{\partial y} = 0, \tag{24}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial \eta}{\partial x} = 0, \tag{25}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial \eta}{\partial y} = 0, \tag{26}$$

where $u(x, y, t)$ and $v(x, y, t)$ are horizontal particle velocities of the water motion. Nonlinear terms are retained in (24)–(26) but the wave dispersion is neglected.

Let us adopt the same length, velocity and time scales as those used for the mudslide and the relation $h(x, y, t) = h_s(x, y) - D(x, y, t)$, then equations (24)–(26) in dimensionless variables (with asterisks omitted) are the following:

$$\begin{aligned}
\frac{\partial \eta}{\partial t} &= \frac{\partial D}{\partial t} - \frac{\partial}{\partial x}[u(h_s + \eta - D)] - \frac{\partial}{\partial x}[v(h_s + \eta - D)], \\
\frac{\partial u}{\partial t} &= -\frac{\epsilon r}{r-1}\frac{\partial \eta}{\partial x} - v\frac{\partial u}{\partial y} - u\frac{\partial u}{\partial x}, \\
\frac{\partial v}{\partial t} &= -\frac{\epsilon r}{r-1}\frac{\partial \eta}{\partial y} - v\frac{\partial v}{\partial y} - u\frac{\partial v}{\partial x}.
\end{aligned}$$

5.3. Numerical results

The slide is initially at rest. Consider a parabolic initial slide surface with a rectangular bottom periphery. The initial slide surface can be expressed as:

$$h(x,y,0) = h_0[1-[2(x-x_0)/L_0]^2][1-[2(y-y_0)/(nL_0)]^2], \qquad (27)$$

where x_0 and y_0 are the initial x- and y- coordinates of the center of the slide bottom, L_0 is the initial length of the slide bottom, h_0 is the initial maximum slide thickness, and n is the ratio between the breath (in y direction) and the width (in x direction) of the slide. To examine the three-dimensional effects of the slide on the surface wave generation and water circulation, one has to examine the cases for two different width/length ratios of the slides. Equation (27) in dimensionless variables reads:

$$h(x,y,0) = h_0[1-4(x-x_0)^2]\left[1-\frac{4}{n^2}(y-y_0)^2\right].$$

Given the slope angle, the typical parameters and the initial positions of the slide, one can calculate $\eta(x,y,t)$, $u(x,y,t)$, and $v(x,y,t)$ for waves and water current, and $D(x,y,t)$, $U(x,y,t)$, and $V(x,y,t)$ for the slide. The governing equations of the slide and the waves are solved with a finite difference method. The numerical scheme employed is an explicit finite difference scheme in space and time.

Numerical results are presented for successive profiles of the mud surface, horizontal velocities of the mudslide, the evolution of surface elevations, and velocities of the water motion. Comparisons of the presented 3D calculations with the previously published 2D results [20] indicate to small differences for large length/width ratios after the initiation of the slide. Generally, however, water surface profiles deviate significantly from the 2D results. Adequate simulations thus require an accurate representation of the aspect ratio of the sliding mass.

6. Some numerical results unavailable in literature

There are several papers on the 2D numerical simulation of the submarine landslides. A two-phase description of the sediment motion was introduced to simulate an underwater landslide in [21]. The computation of waves generated by submarine landslides was performed in [22]. Fluid mechanics models are used to simulate subaerial or submarine landslides due to their observed fluidlike behaviors [23, 24]. The coupling of two absorbing boundary conditions for the 2D time-domain was shown, and simulations of free surface gravity waves were examined in [25].

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