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Modeling of the basic melt intrusion into the platform cover*

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Abstract. This paper numerically studies the flow of magmatic melts in a horizontal channel under gravity. A difference approximation of the two-velocity hydrodynamic equations is performed using the control volume method. Non-uniform flows with different model parameter values and their influence on the flow structure are considered.

Introduction

The problem of heterophase magmatic melt emplacement into lithospheric mantle conduits beneath the cratons of the Siberian Platform is studied numerically using a hydrodynamic model of the evolution of magmatic and fluid-magmatic systems. The mathematical model describes the two-velocity dynamics of the redistribution of hot heterophase melts and magmatic fluids in a flow during their movement from generation zones to the platform cover, as well as heat and mass transfer processes between melts and rocks in permeable zones of the lithosphere. The relevance of this problem is determined by the fact that the nature of the flow of liquid fractions of aluminosilicate, sulfide, native, and oxide liquids, in which a subliquidus solid phase appears during movement and decompression boiling occurs, and the characteristics of heat and mass transfer processes determine the type of igneous and magmatic deposits of the Siberian Platform trap formation. This paper examines the flow structure of magmatic melts over a wide range of temperatures, melt phase viscosities, intrusion rates, and the degree of stratification of heterophase magmatic flow. A mathematical model of heat and mass transfer in a heterophase medium can be used to describe the dynamics of various types of magmatic, fluid-magmatic, and hydrothermal systems.

1. Mathematical model

The system of two-velocity hydrodynamic equations, obtained within the framework of the method of conservation laws [1–3], in a gravity field has the form [4]

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$$\frac{\partial \rho_s}{\partial t} + \operatorname{div}(\rho_s \boldsymbol{u}) = 0, \qquad \frac{\partial \rho_l}{\partial t} + \operatorname{div}(\rho_l \boldsymbol{v}) = 0, \tag{1}$$

$$\rho_s \frac{\partial \boldsymbol{u}}{\partial t} + \rho_s(\boldsymbol{u}, \nabla)\boldsymbol{u} + \frac{\rho_s}{\rho}\nabla p + \frac{\rho_s \rho_l}{\rho}\nabla q = -b\rho_l(\boldsymbol{u} - \boldsymbol{v}) + \eta_s \Delta \boldsymbol{u} + \rho_s \mathbf{g}, \tag{2}$$

$$\rho_l \frac{\partial \boldsymbol{v}}{\partial t} + \rho_l(\boldsymbol{v}, \nabla)\boldsymbol{v} + \frac{\rho_l}{\rho}\nabla p - \frac{\rho_s \rho_l}{\rho}\nabla q = b\rho_l(\boldsymbol{u} - \boldsymbol{v}) + \eta_l \Delta \boldsymbol{v} + \rho_l \mathbf{g}, \tag{3}$$

$$\frac{\partial(\rho s)}{\partial t} + \operatorname{div}\Big((\rho_s \boldsymbol{u} + \rho_l \boldsymbol{v})s - \chi \frac{1}{T}\nabla T\Big) = b\frac{1}{T}\rho_l(\boldsymbol{u} - \boldsymbol{v})^2 + \eta_s u_{ik} + \eta_l v_{ik}. \tag{4}$$

Here $\boldsymbol{u}$, $\boldsymbol{v}$, ρ_s, ρ_l are defined as the velocities and partial densities of the phases, respectively; $\rho = \rho_s + \rho_l$; s is the specific entropy of the two-phase medium; T is the temperature; p is the hydrodynamic pressure; q is the parameter of interphase interaction determined by the difference in pressures in the phases $u_{ik} = \partial_i u_k + \partial_k u_i - \frac{2}{3}\delta_{ik} \operatorname{div} \boldsymbol{u}$, $v_{ik} = \partial_i v_k + \partial_k v_i - \frac{2}{3}\delta_{ik} \operatorname{div} \boldsymbol{v}$. Energy dissipation occurs due to viscous friction in the phases, thermal conductivity, and interphase friction: η_s, η_l is the viscosity of the phases, χ is the thermal conductivity coefficient, $b = \eta_l/\rho k$ is the coefficient of interphase friction, k is the permeability coefficient.

The system of equations (1)–(4) is closed by the equations of state

$$\frac{1}{\rho_0}\delta\rho = \alpha_p \delta p - \beta_T \delta T, \quad \delta s = \frac{c_p}{T_0}\delta T - \frac{1}{\rho_0}\beta_T \delta p,$$
$$\frac{1}{\rho_{0s}}\delta\rho_s = \alpha_p \delta p + \rho_{0s}\alpha_q \delta q - \beta_T \delta T,$$

where the heat capacity at constant pressure c_p, the volumetric compression coefficients α_T, α_q and the coefficient of thermal expansion β_T are additive with respect to the thermodynamic parameters of the phases [4]

$$c_p = c_{ps}^{ph}(1 - \phi) + c_{pl}^{ph}\phi, \quad \beta_T = \beta_{Ts}^{ph}(1 - \phi) + \beta_{Tl}^{ph}\phi,$$
$$\alpha_p = \alpha_{ps}^{ph}(1 - \phi) + \alpha_{pl}^{ph}\phi, \quad \alpha_q = \alpha_{qs}^{ph}(1 - \phi).$$

Here ϕ is the volume content of the dispersion phase.

2. Problem statement

This paper examines the intrusion of a hot viscous melt into a horizontal channel containing a heterophase medium under normal conditions. Initially, there is no movement in the channel. The computational domain is shown in Figure 1.

At the lateral boundaries $y = 0$, $y = L_y$ the boundary conditions of no leakage and no adhesion are set, the boundaries are considered adiabatically isolated

$$u_x|_{y=0} = u_y|_{y=0} = 0, \quad v_x|_{y=0} = v_y|_{y=0} = 0, \quad \partial_y T|_{y=0} = 0.$$

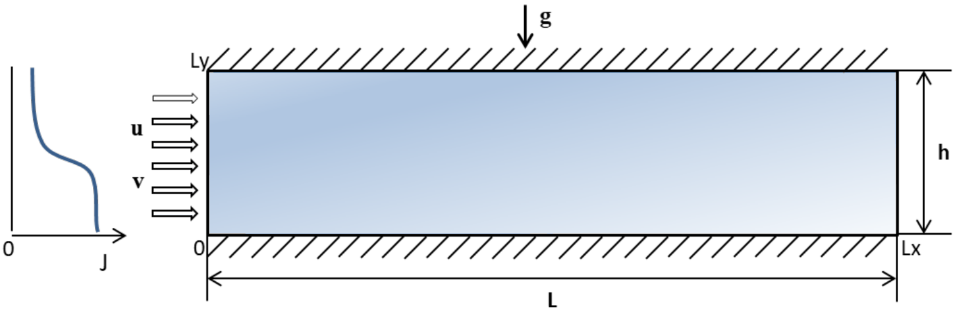

Figure 1. Problem statement

On the left boundary $x = 0$, the components of the velocity vectors of the dispersed and dispersion phases are specified, the temperature of the introduced flow is considered to be given

$$u_x|_{x=0} = u_{x(\text{in})}, \quad u_y|_{x=0} = u_{y(\text{in})}, \quad v_x|_{x=0} = v_{x(\text{in})}, \quad v_y|_{x=0} = v_{y(\text{in})},$$
$$T|_{x=0} = T_{(\text{in})}.$$

In addition, the volume fraction of the dispersed phase is specified on the left boundary.

On the right boundary, the following conditions are set for the components of the velocity vectors of the dispersed and dispersion phases and the temperature:

$$\partial_x u_x|_{x=L_x} = \partial_x u_y|_{x=L_x} = \partial_x v_x|_{x=L_x} = \partial_x v_y|_{x=L_x} = \partial_x T|_{x=L_x} = 0.$$

The difference algorithm is based on the control volume method [5]. Discretization is performed on a rectangular uniform grid with a shift in the computational nodes for the velocity vector components relative to the computational nodes for the remaining model variables. The velocity fields satisfying the continuity equation and the consistent hydrodynamic pressure field are calculated using the SIMPLE iterative algorithm modified for the two-velocity model. The terms of the equations of motion corresponding to the interaction of phases are approximated completely implicitly. Convective terms in calculating flows through the boundaries of control volumes are approximated using a second-order linear-parabolic scheme [6]. Diffusion terms are approximated using a central-difference scheme.

3. Simulation results

Numerical calculations were performed for a model heterophase medium with the following physical parameters: physical densities of the phases $\rho_s^{ph} = 2.6 \cdot 10^3$ kg/m^3, $\rho_l^{ph} = 9.9 \cdot 10^2$ kg/m^3, and phase viscosities varied within the ranges $\eta_l = 10^{-3} \div 10^{-1}$ N·s/m^2, $\eta_s = 1 \div 10^5$ N·s/m^2. The volumetric compression coefficients for the dispersed phase were $\alpha_{ps} = 1.2 \cdot 10^{-10}$ Pa^{-1}, and for the dispersed phase $\alpha_{pl} = 4.7 \cdot 10^{-9}$ Pa^{-1}. The volume fraction of the dispersed phase at the initial moment of time was specified within the range $\phi = 0.2 \div 0.8$. The remaining parameters correspond to normal conditions. The velocities of the dispersed and dispersed phases took on the values $\boldsymbol{u} = 0.02 \div 0.1$ m/s and $\boldsymbol{v} = 0.02 \div 0.1$ m/s.

Below are the results of calculations of the introduction of a hot heterophase flow for different values of temperatures, viscosities and volume contents.

3.1. Variation of the temperature of the introduced flow. The results of calculations for the flow in a two-phase medium channel at different temperature values of introduced flow are presented in Figure 2.

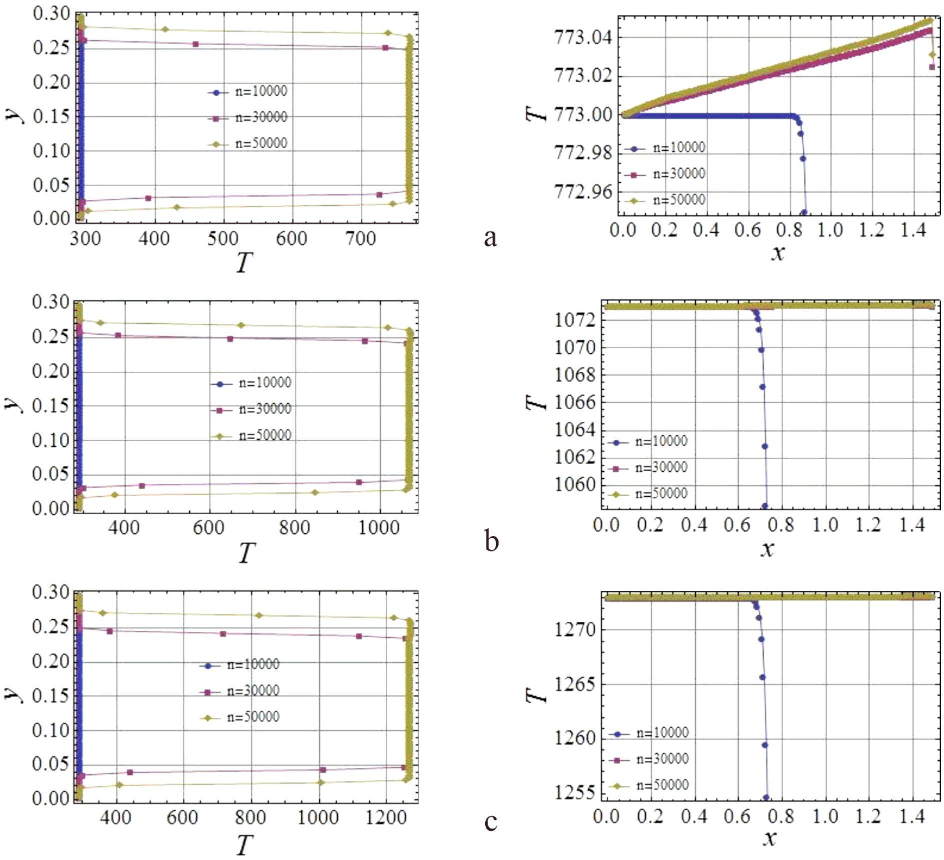

Figure 2. Temperature distribution of a two-phase medium across (left) and along (right) a channel for flows with the initial temperature T (°C): (a) 500, (b) 800, (c) 1000

3.2. Variation in the viscosity of the dispersed phase of the introduced flow. The calculation results for the flow in a two-phase medium channel with the different viscosity values of the dispersed phase η_s are presented in Figure 3. The temperature of the introduced flow was set to $T = 500°C$, and inside the calculation region the temperature at the initial moment of time was $T = 300°C$.

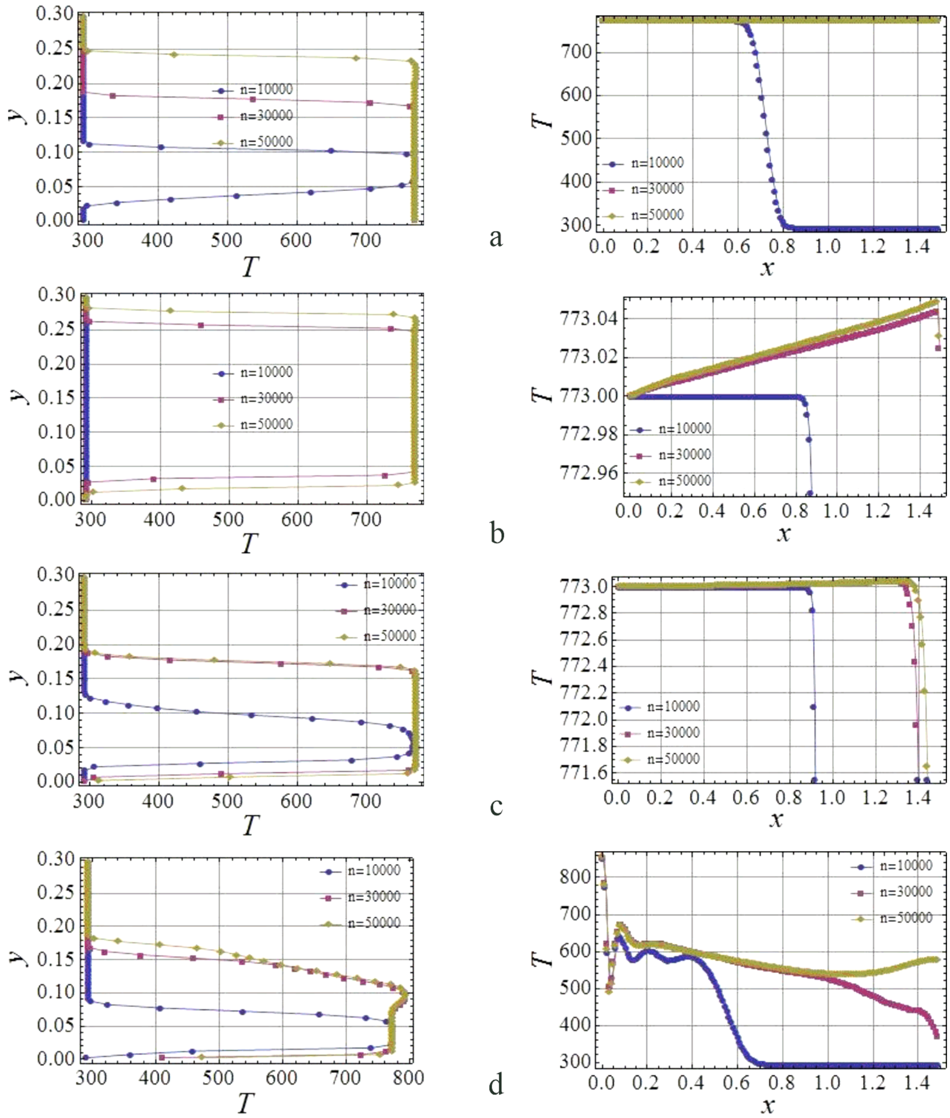

Figure 3. Temperature distribution of a two-phase medium across (left) and along (right) the channel for a flow with the viscosity of the dispersed phase η_s (N·s/m^2): (a) 10^5, (b) 10^3, (c) 10, (d) 1

3.3. Variation of the volumetric content of the dispersed phase of the introduced flow. The calculation results for a two-phase medium flow in a channel with a dispersed phase viscosity $\eta_s = 1$ N·s/m^2 are presented in Figure 4 for two variants of the initial profile of the volumetric content of the dispersed phase: with a higher concentration in the lower part of the flow (see Figure 1) and the opposite profile with a higher concentration in the upper part of the flow. The volumetric content of the dispersed phase in the vertical cross-section varied from $\phi_s = 0.6$ to 0.4. The temperature of the injected flow was set to $T = 500$°C, and within the computational domain, the temperature at the initial time was $T = 300$°C. The phase velocities on the left boundary were $u = v = 0.1$ m/s for the dispersed phase.

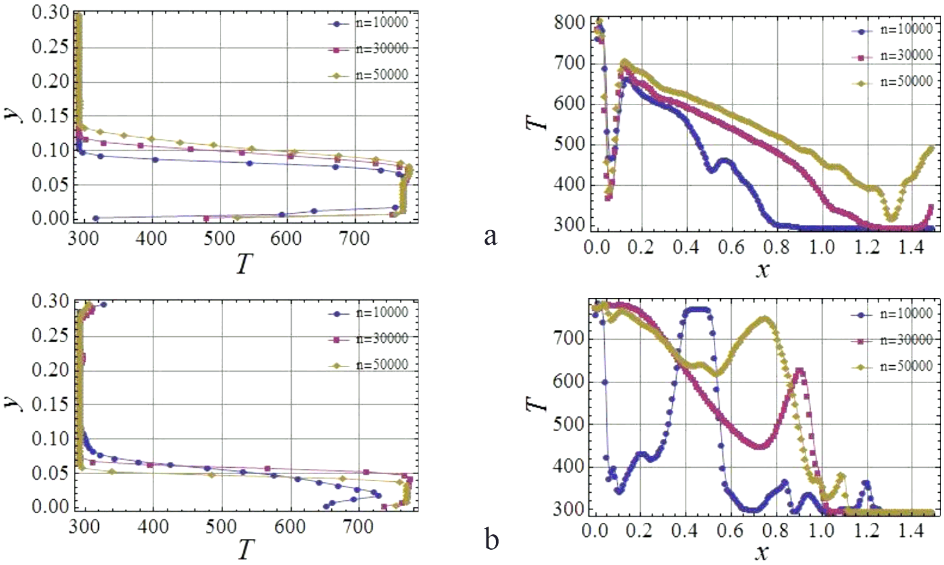

Figure 4. Temperature distribution of a two-phase medium across (left) and along (right) the channel for a flow with a direct (a) and reverse (b) profile of the volume content of the dispersed phase

To compare the character of the flow, Figure 5 shows the temperature distribution for the case of an inverse volumetric content profile (see Figure 4) in the case of a lower viscosity of the dispersed phase of viscosity $\eta_s = 10^{-1}$ N·s/m^2. The volumetric content of the dispersed phase in the vertical section varied from $\phi_s = 0.6$ to 0.4. The temperature of the injected flow inside the region at the initial moment is $T = 300$°C. The phase velocities at the left boundary are $u = v = 0.1$ m/s.

It should be noted that setting different values for the phase velocities at the inlet boundary leads to an increase in the relative phase velocity within the computational domain, but does not significantly affect the flow structure.

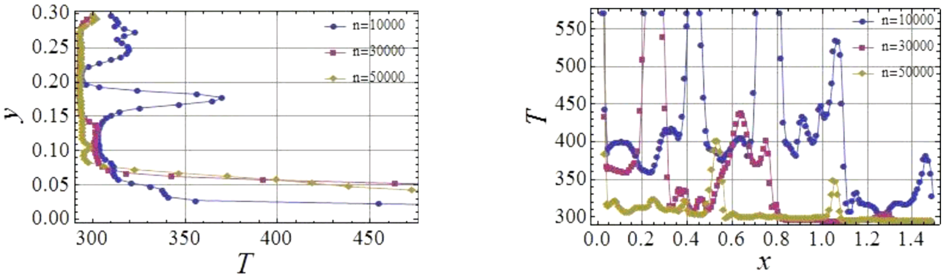

Figure 5. Temperature distribution of a two-phase medium across (left) and along (right) a channel for a flow with the viscosity of the dispersed phase $\eta_s = 10^{-1}$ N·s/m^2

Conclusion

In this paper, a hydrodynamic model of a viscous two-phase medium is applied to simulate the flow structure of heterophase melts in magma conduits. Numerical calculations of unsteady flows were performed over a wide range of temperatures, melt phase viscosities, intrusion velocities, and the degree of stratification of the heterophase magma flow. The mathematical model can be used to describe various types of magmatic, fluid-magmatic, and hydrothermal systems.

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