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Layered flows of high-temperature suspensions*

Yu.V. Perepechko, K.E. Sorokin, Sh.Kh. Imomnazarov

Abstract. The paper numerically studies the flow of a high-temperature two-phase mixture in a gravity field. Thermodynamically consistent equations of two-speed hydrodynamics of a suspension with a foreign impurity are developed within the framework of the conservation law method. The numerical non-stationary non-isothermal 2D model is implemented based on the modified control volume method. The paper studies the nature of the flow of a high-temperature suspension in a horizontal channel at different values of the viscosity of a two-phase mixture. The effect of non-uniform distribution of solid phase particles on the development of instability of the flow of such a medium is considered.

Keywords: suspension, impurity, two-velocity hydrodynamics, heat and mass transfer, conservation law method, control volume method

Introduction

Modeling of heat and mass transfer in suspensions of solid particles, liquid particles and suspensions of liquid droplets is relevant for solving problems arising in the study of both natural and technological systems. Interest in the problems of heat and mass transfer of suspensions of solid particles is due to the development of modern technologies for the conversion and transmission of energy (for example, the combustion of particles and droplets in fluidized bed reactors, cooling with nanofluids containing nanoparticles) and modeling of natural processes of industrial importance (for example, the formation of ore deposits). The development of consistent models of heat and mass transfer of heterophase media based on multi-velocity hydrodynamics can be the basis for the development of such technologies. One of the most general methods that allows obtaining physically correct models of heat and mass transfer is the conservation method. This phenomenological approach ensures thermodynamic consistency of the equations of thermohydrodynamics of suspensions. In this paper, based on this method, we study the hydrodynamics of high-temperature suspensions in the presence of an impurity taking into account the surface tension of the dispersed phase. The hydrodynamic model also takes into account such dissipative phenomena as thermal conductivity, viscosity of a two-phase suspension, diffusion of impurities, and interphase friction. Modeling of various flows of two-phase suspensions in a channel is carried out for problems with a uniform and

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non-uniform initial distribution of the concentration of solid particles of the suspension.

1. Mathematical model

Simulation of heat and mass transfer of suspensions is based on mathematical models that use various simplifying approximations [1, 2]. In this paper, a complete model of two-velocity thermohydrodynamics of a compressible non-isothermal suspension is investigated. The elementary volume of a two-phase suspension is characterized by partial densities ρ_1, ρ_2 and velocities $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ of the dispersed and dispersion phases, the density of the impurity ρ_a, the number of particles of the dispersed phase n, as well as the concentration of the impurity c and the temperature T. The equations of the dynamics of a heterophase suspension are derived based on the method of conservation laws [3] under the assumption of phase equilibrium in temperature and pressure [4, 5]. Taking into account dissipative processes, conservation laws and balance relations can be represented as follows

$$\frac{\partial\rho_1}{\partial t}+\operatorname{div}(\rho_1\boldsymbol{u}_1)=0,\quad \frac{\partial\rho_2}{\partial t}+\operatorname{div}(\rho_2\boldsymbol{u}_2)=0,\quad \frac{\partial n}{\partial t}+\operatorname{div}(n\boldsymbol{u}_1)=0,$$

$$\frac{\partial\rho_a}{\partial t}+\operatorname{div}\Big(c_1\rho_1\boldsymbol{u}_1+c_2\rho_2\boldsymbol{u}_2-\nu\frac{1}{T}\nabla T-DT\nabla\Big(\frac{\mu_a}{T}\Big)\Big)=0,$$

$$\frac{\partial j_i}{\partial t}+\partial_k\big(\rho_1u_{1i}u_{1k}+\rho_2u_{2i}u_{2k}+p\delta_{ik}-(\eta_1+\eta_{12})u_{1ik}-(\eta_2+\eta_{12})u_{2ik}\big)=\rho \mathrm{g}_i,$$

$$\begin{aligned}\frac{\partial u_{2i}}{\partial t}+(\boldsymbol{u}_2,\nabla)u_{2i}= & -\frac{1}{\rho}\partial_i p+\frac{n}{\rho}\varsigma\partial_i\sigma+\frac{\rho_1}{2\rho}\partial_i\boldsymbol{w}^2+bw_i+\\ & \frac{1}{\rho_2}\partial_k\big(\eta_2u_{2ik}+\eta_{12}u_{2ik}\big)+\mathrm{g}_i,\end{aligned}$$

$$\frac{\partial S}{\partial t}+\operatorname{div}\Big(S_1\boldsymbol{u}_1+S_2\boldsymbol{u}_2-(\kappa+\nu\mu_a)\frac{1}{T^2}\nabla T+(D\mu_a-\nu T)\nabla\Big(\frac{\mu_a}{T}\Big)\Big)=\frac{1}{T}R,$$

where

$$\begin{aligned}R=\rho_2b\boldsymbol{w}^2+\kappa\Big(\frac{\nabla T}{T}\Big)^2+2\nu\nabla\Big(\frac{\mu_a}{T}\Big)\nabla T+DT^2\Big(\nabla\Big(\frac{\mu_a}{T}\Big)\Big)^2+\\ 2\lambda_1\frac{\nabla T}{T}\boldsymbol{w}+2\lambda_2\nabla\Big(\frac{\mu_a}{T}\Big)\boldsymbol{w}+\frac{1}{2}\eta_1u_{1ik}u_{1ik}+\frac{1}{2}\eta_2u_{2ik}u_{2ik}+\eta_{12}u_{1ik}u_{2ik}.\end{aligned}$$

Here $\rho=\rho_1+\rho_2$ and $\boldsymbol{j}=\rho_1\boldsymbol{u}_1+\rho_2\boldsymbol{u}_2$ are the density and momentum of the two-velocity medium; $\boldsymbol{w}=\boldsymbol{u}_1-\boldsymbol{u}_2$ is the relative velocity; p is the pressure; μ_a is the chemical potential of the two-phase medium and the impurity; σ is the surface tension tensor; ς is the specific surface area of the dispersed phase; $\mathbf{g}$ is the acceleration vector of gravity. The notation $\boldsymbol{w}^2$ in formulas above means $(\boldsymbol{w},\boldsymbol{w})$. The notation $\frac{\nabla T}{T}\boldsymbol{w}$ and others vector multiplications mean $\big(\frac{\nabla T}{T},\boldsymbol{w}\big)$.

The impurity concentration in the two-phase medium and in the phases are determined by the relations $c = \rho_a/\rho$, $c_1 = c + 2\lambda_1\rho_2/\rho_1$, $c_2 = c - 2\lambda_1$. S is the entropy of the two-phase medium, $S_2 = S\rho_2/\rho - 2\lambda\rho_2/\rho$ and $S_1 = S\rho_1/\rho + 2\lambda\rho_2/\rho$ are the entropies of the phases, $\lambda = \lambda_2 - \rho\lambda_1\mu_a/T$. The kinetic coefficients of interphase friction b, shear viscosity of the phases η_i, mutual viscosity η_{12}, thermal conductivity of the two-phase medium κ and the coefficients λ_1, λ_2, ν are functions of the thermodynamic parameters. The strain rate tensors are defined by the relations $u_{1ik} = \partial_k u_{1i} + \partial_i u_{1k} - 2/3\delta_{ik}\,\mathrm{div}\,\boldsymbol{u}_1$, $u_{2ik} = \partial_k u_{2i} + \partial_i u_{2ik} - 2/3\delta_{ik}\,\mathrm{div}\,\boldsymbol{u}_2$. The effects of bulk viscosity are not considered in the model.

The equations of state of a two-phase medium closing the dynamic equations above are obtained with the linear approximation:

$$\delta\rho = \rho\alpha\delta p - \rho\beta\delta T, \quad \delta s = \frac{c_p}{T}\delta T - \frac{1}{\rho}\beta\delta p.$$

The coefficients of volumetric compression α, thermal expansion β, and specific heat capacity c_p are additive across phases. The impurity is taken into account in the approximation of an ideal solution $\mu_a = d_1 p + d_2 T + \bar{R}T\ln c$, where $\bar{R}$ is the universal gas constant. The surface tension is determined by the Shishkovsky relation

$$\sigma = \sigma_0\frac{T_c - T}{T_c - T_{\mathrm{ref}}} - \sigma_1\ln(1 + ac).$$

The difference approximation of the equations of two-velocity hydrodynamics is based on the control volume method [6, 7], which ensures accurate integral conservation of mass, momentum and energy in any volume. The differential equations are discretized on a rectangular uniform grid with a shift in the computational nodes for the velocity vector components relative to the computational nodes for the remaining variables. A completely implicit time scheme is used. When approximating convective terms for calculating flows through the edges of control volumes, the second-order HLPA scheme is implemented [7]. When approximating diffusion terms, a central difference scheme is used. To calculate the pressure field consistent with the flow field, an analogue of the IPSA iterative procedure is implemented [8]. The continuity equations are not solved explicitly; their discrete analogue is used to derive the correction equation for pressure and the remaining discrete equations. The difference approximation of the boundary conditions is carried out using a second-order scheme. To calculate the velocity fields satisfying the continuity equation and the pressure field consistent with them, a variant of the SIMPLE iterative procedure [6] is implemented. When switching to a new time step, an initial assumption is made about the approximate value of the pressure field, and the true value is determined through a correction. Corrections for velocities are introduced in a similar manner. The alternating direction method is used to numerically solve systems of linear algebraic equations of discrete analogues of differential equations and a correction equation for pressure.

2. Calculation results

The formulation of the problem of introducing a high-temperature suspension into a channel with an initially non-uniform suspension is shown in Figure 1. The computational domain is taken as a rectangular channel with a size of 0.3×1.5 m. At the initial moment of time, there is no movement in the channel, the thermodynamic parameters of the suspension correspond to normal conditions with a constant temperature

$$u_{1x} = u_{2x} = u_{1y} = u_{2y} = 0, \quad T = T_{\text{in}}.$$

The volume content of solid particles of the suspension is initially non-uniform, shown in Figure 1 in color (blue — low concentration, red — high concentration of the suspension). The distribution of physical and partial densities of the phases at the initial moment of time is specified in agreement with the pressure assignment as a result of the iterative process taking into account the gravity field.

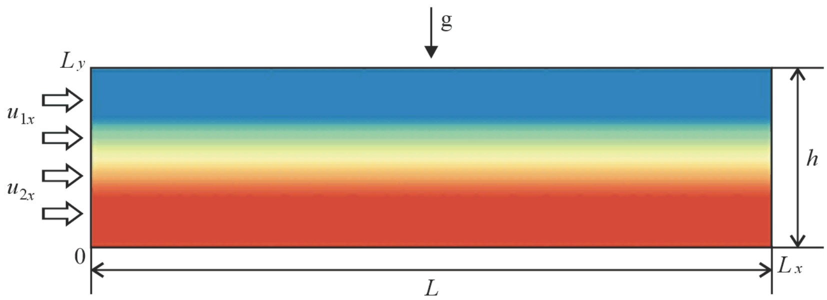

Figure 1. Statement of the problem

On the left boundary, constant velocities of the suspension phases and the flow temperature were set

$$u_{1x}|_{x=0} = u_{1x}^0, \;\; u_{1y}|_{x=0} = u_{1y}^0, \;\; u_{2x}|_{x=0} = u_{2x}^0, \;\; u_{2y}|_{x=0} = u_{2y}^0, \;\; T|_{x=0} = T_0.$$

On the right boundary, the normal derivatives of the phase velocities and temperature were set equal to zero:

$$\frac{\partial u_{1x}}{\partial x}\Big|_{x=L_x} = \frac{\partial u_{1y}}{\partial x}\Big|_{x=L_x} = \frac{\partial u_{2x}}{\partial x}\Big|_{x=L_x} = \frac{\partial u_{2y}}{\partial x}\Big|_{x=L_x} = 0, \quad \frac{\partial T}{\partial x}\Big|_{x=L_x} = 0.$$

The lateral boundaries were assumed to be impermeable and adiabatically isolated. The absence of slippage of the suspension flow and the absence of heat flow through the boundary were specified:

$$u_{1x}|_{y=0} = u_{1y}|_{y=0} = 0, \quad u_{1x}|_{y=L_y} = u_{1y}|_{y=L_y} = 0,$$
$$u_{2x}|_{y=0} = u_{2y}|_{y=0} = 0, \quad u_{2x}|_{y=L_y} = u_{2y}|_{y=L_y} = 0,$$
$$\frac{\partial T}{\partial y}\Big|_{y=0} = \frac{\partial T}{\partial y}\Big|_{y=L_y} = 0.$$

The values of the physical parameters for the dispersed phase are the following: $\rho_1^f = 2.6 \cdot 10^3$ kg/m^3, $\alpha_1 = 1.2 \cdot 10^{-10}$ Pa^{-1}, $\eta_1 = 1 \div 10^5$ N/(ms^2), $\rho_2^f = 9.9 \cdot 10^3$ kg/m^3, $\alpha_2 = 4.7 \cdot 10^{-9}$ Pa^{-1}, $\eta_2 = 10^{-3}$ N/(ms^2). The model parameters are calculated as follows

$$\rho_1 = \rho_1^f(1-\varphi), \quad \rho_2 = \rho_2^f\varphi, \quad \alpha^{-1} = (1-\varphi){\alpha_1}^{-1} + \varphi{\alpha_2}^{-1}.$$

The volume fraction φ of the dispersion phase at the initial moment of time varied within $0.4 \div 0.6$. In addition, the following parameters were set: $d_1 = 0.1$ m^3/kg, $d_2 = 0.001$ m^2/(Ks^2), $T_c = 513$ K, $T_{\rm ref} = 293$ K, $a_1 = 7 \cdot 10^{-2}$ N/m, $\sigma_2 = 0.1 \div 2$ N/m, $\lambda_2 = 10^{-2}$ kg/(ms^2), $\lambda_1 = 10^{-6}$ kg/(ms^2). Diffusion coefficient $D = 2 \cdot 10^{-9}$ m^2/s. The calculations take into account the change in the friction coefficient associated with the change in the density of the dispersed phase during the dynamic process of phase redistribution.

Figures 2–9 show the distributions of the particle number density of the dispersed phase in the suspension n and the temperature distribution T.

Figures 2 and 3 show the results of modeling the introduction of a hot suspension into a horizontal channel in which a suspension of non-uniform concentration is under normal conditions. At the input boundary of the channel, the horizontal components of the velocity vectors of the carrier and dispersed phases are specified $u_{1x} = u_{2x} = 0.1$ m/s. At the initial moment of time, there is no phase movement, from the upper to the lower boundary of the computational domain, the profile of the content of the dispersed phase from 0.4 to 0.6 is specified with a fairly sharp change in the volume content of the phases identified along the central axis of the channel. The flow temperature at the left boundary is $T_0 = 1200°$, the initial temperature in the channel is $T_{\rm in} = 300°$.

An example of the effect of the viscosity of the dispersed phase on the flow structure when introducing a high-temperature suspension is shown in Figures 4 and 5. The flow temperature at the left boundary is $T_0 = 500°$, the initial temperature in the channel is $T_{\rm in} = 300°$. The viscosity of the dispersed phase was taken to be $\eta_s = 10^3$ N/(ms^2) and $\eta_s = 1$ N/(ms^2), respectively. At the input boundary of the channel, the horizontal components of the phase velocity vectors $u_{1x} = u_{2x} = 0.1$ m/s are specified. At the initial moment of time, there is no phase motion, and the dispersion phase content profile from 0.4 to 0.6 is specified from the upper to the lower boundary of the computational domain.

A decrease in the viscosity of the dispersed phase of the suspension leads to the appearance of structural inhomogeneities of the flow associated with

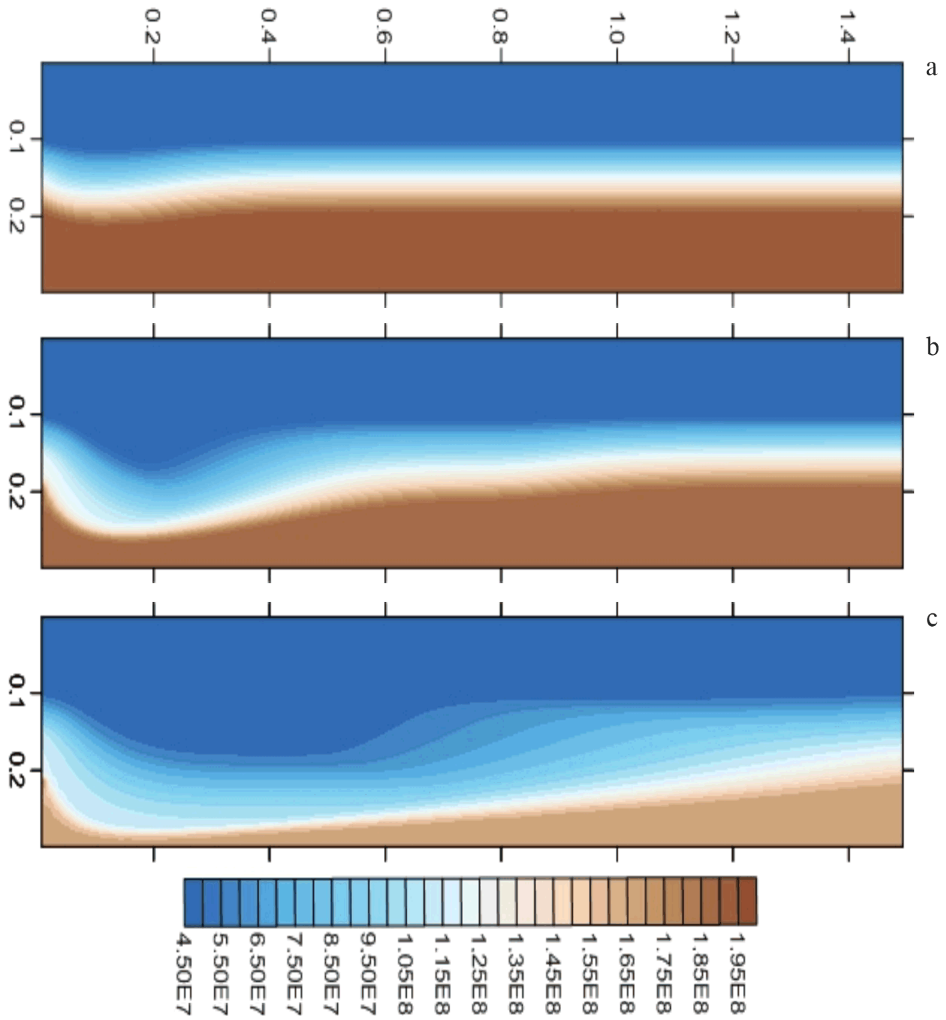

Figure 2. Distribution of the specific content of dispersed phase particles (m^{-3}) in a horizontal channel for counting steps (a) 10^3, (b) $2 \cdot 10^3$, (c) $3 \cdot 10^3$

its stratification. It should be noted that the variation in the viscosity of the dispersed phase has a more noticeable effect on the development of flow inhomogeneity.

The instability of the layered flow of a high-temperature suspension strongly depends on the specified distribution of the solid particle content. Figures 6, 7 and 8, 9 show the results of modeling the introduction of a hot suspension with different distributions of the solid particle content. In Figures 6, 7, the profile of the content of the dispersed (carrier) phase from 0.4 to 0.6 is specified from the upper to the lower boundary of the computa-

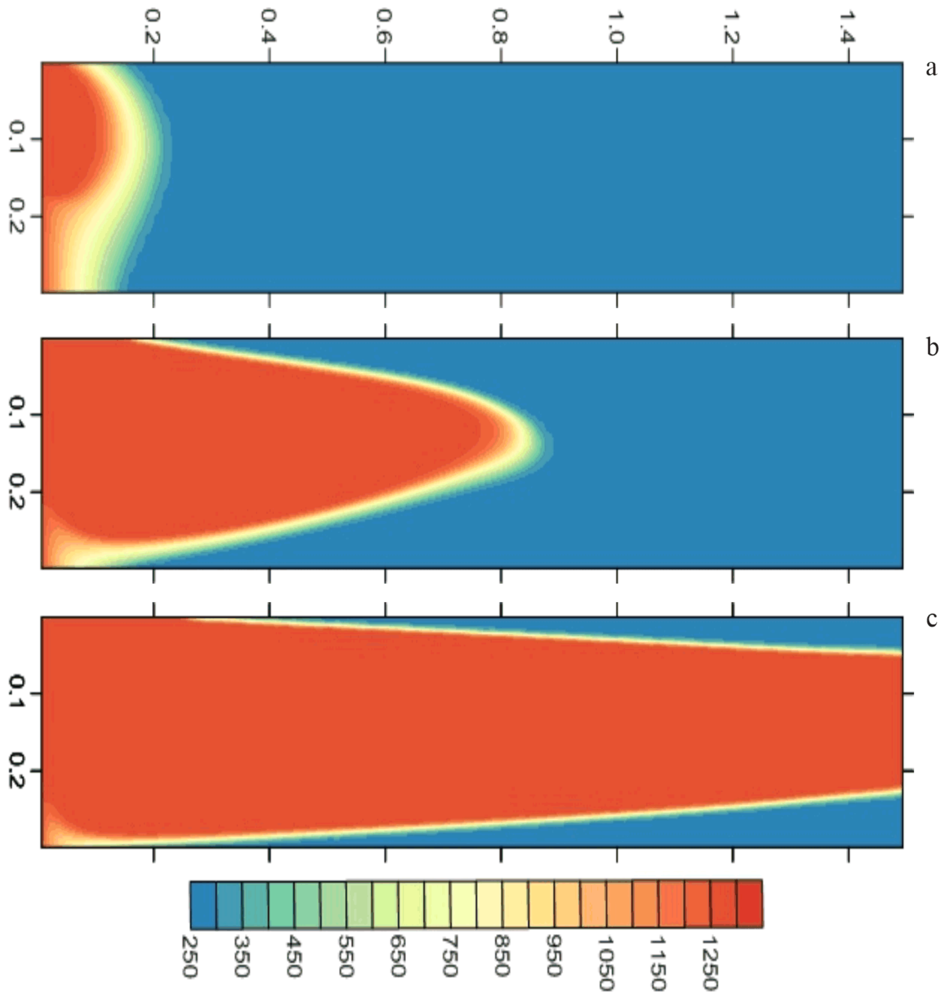

Figure 3. Temperature distribution (°C) in the horizontal channel for counting steps (a) 10^3, (b) $2 \cdot 10^3$, (c) $3 \cdot 10^3$ corresponding to Figure 2

tional domain with a change in the volume content of the phases along the central axis of the channel. In Figures 8, 9, the opposite profile was specified: the profile of the content of the dispersed (carrier) phase from 0.6 to 0.4 is specified from the upper to the lower boundary of the computational domain with a change in the volume content of the phases along the central axis of the channel. The remaining parameters of the problem are the same: the horizontal components of the velocity vectors of the carrier and dispersed phases are $u_{1x} = u_{2x} = 0.1$ m/s specified at the inlet boundary of the channel, the flow temperature at the left boundary is $T_0 = 500°$, and the initial temperature is $T_{\text{in}} = 300°$.

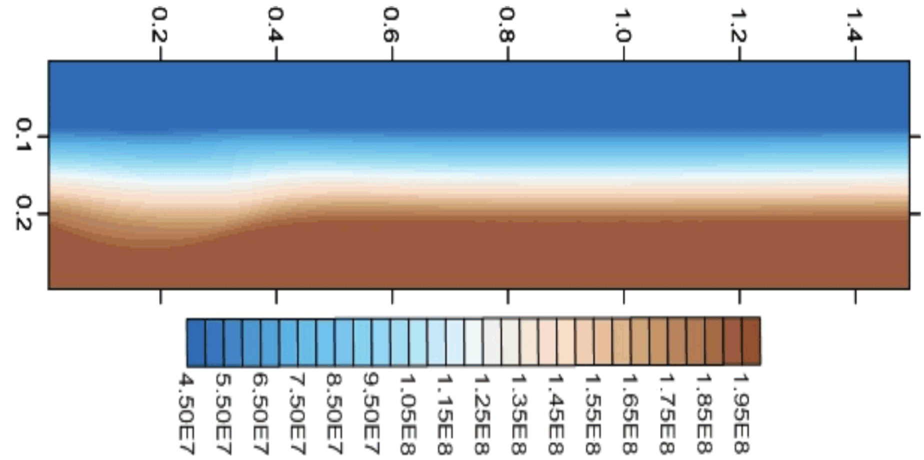

Figure 4. Distribution of the specific content of dispersed phase particles (m^{-3}) with viscosity 10^3 N/(ms^2) for a counting step of $3 \cdot 10^3$

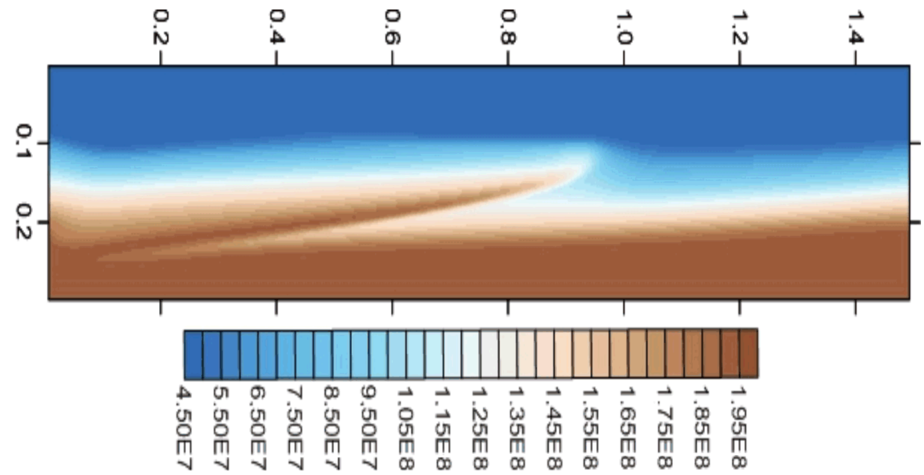

Figure 5. Distribution of the specific content of dispersed phase particles(m^{-3}) with viscosity 1 N/(ms^2) for a counting step of $3 \cdot 10^3$

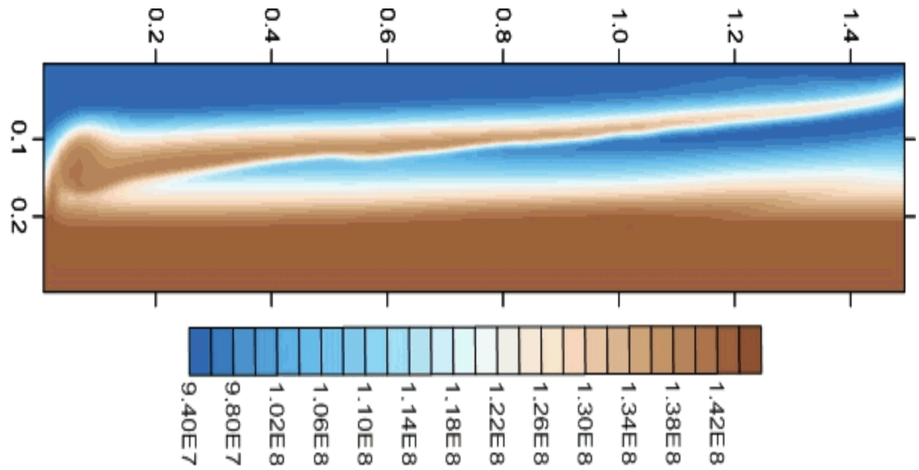

Figure 6. Distribution of the specific content of dispersed phase particles (m^{-3}) in a horizontal channel for step $2 \cdot 10^3$

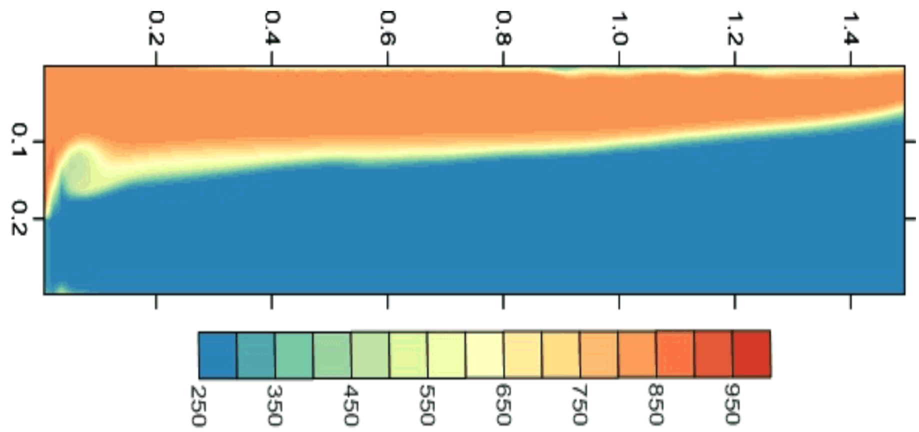

Figure 7. Temperature distribution (°C) in the horizontal channel for step $2 \cdot 10^3$

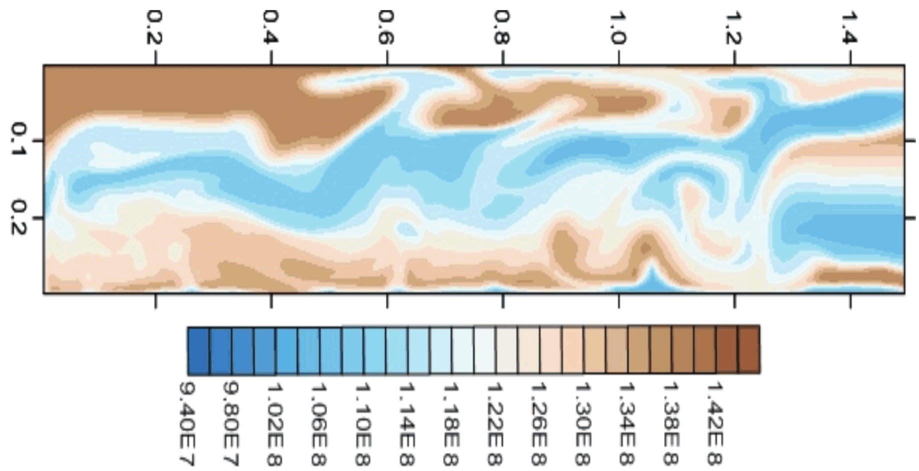

Figure 8. Distribution of the specific content of dispersed phase particles (m^{-3}) in a horizontal channel for step $2 \cdot 10^3$

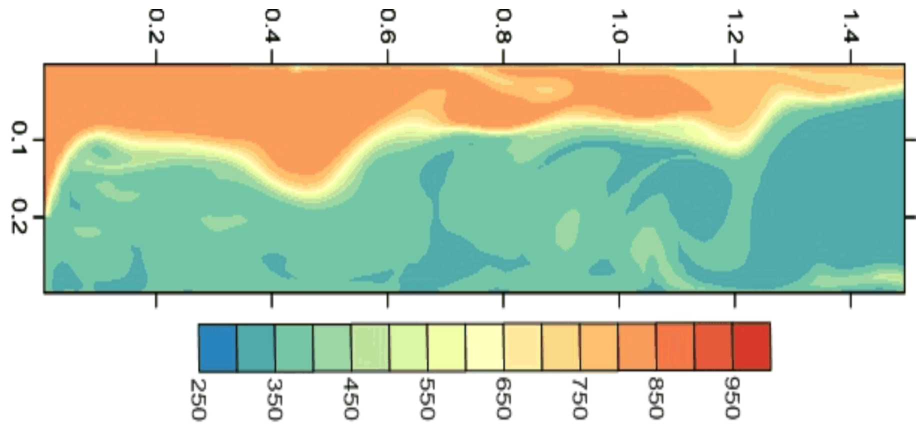

Figure 9. Temperature distribution (°C) in the horizontal channel for step $2 \cdot 10^3$

The assignment of an inverse distribution of the volumetric content of solid particles of a suspension leads to the development of concentration instability of the flow.

Thus, in this work, based on the control volume method, a numerical simulation of the flow of a high-temperature two-phase mixture in a channel in a gravity field was carried out. The numerical model takes into account the compressibility of a heterophase suspension and considers the main dissipative and surface phenomena. The nature of the flow of a high-temperature suspension in a horizontal channel was studied at different values of the viscosity of the two-phase mixture. The effect of the non-uniform distribution of solid particles of the suspension on the development of flow instability was shown.

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