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Problem of determining the source from a Hopf-type system

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Abstract. A one-dimensional inverse problem for a quasilinear hyperbolic system with an unknown excitation source is considered. The Cauchy problem for a nonlinear Hopf-type system is studied. The Fourier transform is used to reduce the inverse problem to a direct problem, and the existence and uniqueness theorem is proved. The approach used can become the basis for constructing an effective numerical algorithm for the inverse problem.

Keywords: Two-velocity hydrodynamics, viscous fluid, relative velocity, direct problem, inverse problem, Darcy coefficient

Introduction

The theory of two-phase filtration finds important application in solving problems of petroleum engineering, soil science, biomechanics and others practical areas. Increasing attention is being paid to modeling of multiphase flows in connection with burial radioactive waste. Simulation and numerical analysis of two-phase filtration in elastically deformable porous media are important element in the development of cost-effective and safe cleaning devices, reducing the number of laboratory and field experiments, identify the main mechanisms, optimize existing strategies and evaluate possible risks. In recent years, interest in processes has significantly increased of multiphase filtration in low-permeability fractured porous collectors. One of the important reasons for this is the fact that fractured hydrocarbon deposits contain more than 20 percent of world oil reserves [1].

In this paper, we study the inverse problem for a system of the Hopf-type equations with an unknown source, under the condition of overdetermination of solutions, specified on a fixed line. The original problem is reduced to the study of the Cauchy problem for a system of ordinary nonlinear integrodifferential equations containing a convolution, for which unique solvability has been proven. Unique solvability of the inverse problem is proven, and a representation of its solution is obtained through the solution of the abovementioned Cauchy problem [2]. Similar problems for linear and semilinear equations are considered in [3–5]. Inverse problems with final overdetermination are studied for parabolic equations and equations of a viscous incompressible fluid in [6–8]. For studies of direct problems for Burgers-type equations and systems, see, for example, [9–11]. The issues of correctness of the linear inverse problem for a three-dimensional equation of mixed type of the second kind of the second order in an unbounded parallelepiped are considered in [12]. Similar problems for a system of the Hopf-type equations in the class of analytic functions are considered in [13–16].

1. The Hopf type system of equations

The Cauchy problem in a strip $\Pi_{[0,T]} = \{(t,x):\ 0 \le t \le T,\ x \in R\}$ for a system of the Hopf-type equations is considered [17–19]:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -b(u-v) + f(x)g_1(t), \tag{1}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \varepsilon b(u-v) + f(x)g_2(t), \tag{2}$$

$$u|_{t=0} = u^0(x), \quad v|_{t=0} = v^0(x). \tag{3}$$

where the function $f(x)$ is given, $\varepsilon = \rho_1/\rho_2$ is a dimensionless positive constant, b is a positive constant. The unknown functions $g_k = g_k(t)$ $(k = 1, 2)$, $t \in [0, T]$, and solutions u, v of the system of equations (1), (2) must be determined. The system (1), (2) differs from the system of two-velocity hydrodynamics in the dissipative case due to the coefficient of friction, the absence of pressure and the condition of incompressibility. For this reason, problems arise associated with the Hopf-type system, which gives the simplest quasi-linear system of equations [20].

2. Inverse source problem for a Hopf-type system

Let us assume we have additional override conditions

$$u|_{x=0} = \varphi(t), \quad v|_{x=0} = \psi(t), \quad t \in [0, T], \tag{4}$$

and the functions $\varphi(t)$, $\psi(t)$ satisfy the matching conditions

$$\varphi(0) = u^0(0), \quad \psi(0) = v^0(0),$$

The functions $u^0(x)$, $v^0(x)$, $f(x)$ and $\varphi(t)$, $\psi(t)$ are assumed to be real. Next, we study the real solution to the classical inverse problem.

Suppose that there exist the Fourier transforms $U(t,y)$, $V(t,y)$ (with respect to x) of the solution $u(t,x)$, $v(t,x)$ for (1)–(3)

$$(U(t,y), V(t,y)) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(u(t,x), v(t,x))e^{ixy}dx = F(u,v)(t,y), \tag{5}$$

$$(u(t,x), v(t,x)) = \int_{-\infty}^{\infty}(U(t,y), V(t,y))e^{-ixy}dx = F^{-1}(U,V)(t,x).$$

Applying the Fourier transform in variables x to (1), (2) we have

$$\frac{\partial U(t,y)}{\partial t} + \frac{1}{2}F\Big(\frac{\partial u^2}{\partial x}\Big)(t,y) = -b(U(t,y) - V(t,y)) + \tilde{F}(y)g_1(t), \tag{6}$$

$$\frac{\partial V(t,y)}{\partial t} + \frac{1}{2}F\Big(\frac{\partial v^2}{\partial x}\Big)(t,y) = \varepsilon b(U(t,y) - V(t,y)) + \tilde{F}(y)g_2(t), \tag{7}$$

where $\tilde{F}(y) = F(f)(y)$.

Let in (1) and (2) $x = 0$. Using (3) and (5), we obtain

$$\varphi_t(t) + i\varphi(t)\int_{-\infty}^{\infty} yU(t,y)\,dy = -b(\varphi(t) - \psi(t)) + f(0)g_1(t),$$

$$\psi_t(t) + i\psi(t)\int_{-\infty}^{\infty} yV(t,y)\,dy = \varepsilon b(\varphi(t) - \psi(t)) + f(0)g_2(t),$$

what does it mean

$$g_1(t) = \frac{1}{f(0)}\Big\{\tilde{\varphi}(t) + i\varphi(t)\int_{-\infty}^{\infty} yU(t,y)\,dy\Big\}, \tag{8}$$

$$g_2(t) = \frac{1}{f(0)}\Big\{\tilde{\psi}(t) + i\psi(t)\int_{-\infty}^{\infty} yV(t,y)\,dy\Big\}. \tag{9}$$

In formulas (8) and (9)

$$\tilde{\varphi}(t) = \varphi_t(t) + b(\varphi(t) - \psi(t)), \quad \tilde{\psi}(t) = \psi_t(t) - \varepsilon b(\varphi(t) - \psi(t)).$$

Further, without loss of generality, we can assume that $f(0) = 1$.

Since we are looking for a real solution $u(t,x)$, $v(t,x)$, $g_1(t)$, $g_2(t)$, it is worth considering the real parts of the functions $g_1(t)$, $g_2(t)$ in (8) and (9) (see [21, Remark 3.1])

$$\mathrm{Re}\Big\{\tilde{\varphi}(t) + i\varphi(t)\int_{-\infty}^{\infty} yU(t,y)\,dy\Big\},$$

$$\mathrm{Re}\Big\{\tilde{\psi}(t) + i\psi(t)\int_{-\infty}^{\infty} yV(t,y)\,dy\Big\}.$$

Suppose that the functions $U^0(y) = F(u^0)(y)$, $V^0(y) = F(v^0)(y)$ are continuously differentiable on $(-\infty,\infty)$, $\tilde{F}(y)$ and $\tilde{F}_y(y)$ continuous on $(-\infty,\infty)$, the functions $\varphi(t)$, $\psi(t)$ are continuously differentiable in $[0,T]$ and

$$\big(1 + |y|^{k+\lambda}\big)|U^0(y)| + \big(1 + |y|^{k+\lambda}\big)|\tilde{F}(y)| + \Big|\frac{\partial}{\partial y}U^0(y)\Big| + \Big|\frac{\partial}{\partial y}\tilde{F}(y)\Big| \le d_1(k), \quad y \in (-\infty,\infty), \tag{10}$$

$$\big(1 + |y|^{k+\lambda}\big)|V^0(y)| + \big(1 + |y|^{k+\lambda}\big)|\tilde{F}(y)| + \Big|\frac{\partial}{\partial y}V^0(y)\Big| + \Big|\frac{\partial}{\partial y}\tilde{F}(y)\Big| \le d_2(k), \quad y \in (-\infty,\infty), \tag{11}$$

where $\lambda = \mathrm{const} > 0$ and $k > 0$ is an integer.

Since

$$F(u^2)(t,y) = \int_{-\infty}^{\infty} U(t,z)U(t,y-z)dz$$

we represent

$$F\Big(\frac{\partial u^2}{\partial x}\Big)(t,y) = iyF(u^2)(t,y) = iy\int_{-\infty}^{\infty} U(t,z)U(t,y-z)\,dz$$

and substitute the real parts of $g_1(t)$, $g_2(t)$ from (8), (9) into (6), (7) to obtain the integro-differential equation

$$\frac{\partial U(t,y)}{\partial t} + iy\int_{-\infty}^{\infty} U(t,z)U(t,y-z)dz = -b(U(t,y)-V(t,y)) + \operatorname{Re}\Big\{\tilde{\varphi}(t) + i\varphi(t)\int_{-\infty}^{\infty} yU(t,y)\,dy\Big\}\tilde{F}(y), \quad (12)$$

$$\frac{\partial V(t,y)}{\partial t} + iy\int_{-\infty}^{\infty} V(t,z)V(t,y-z)\,dz = \varepsilon b(U(t,y)-V(t,y)) + \operatorname{Re}\Big\{\tilde{\psi}(t) + i\psi(t)\int_{-\infty}^{\infty} yV(t,y)\,dy\Big\}\tilde{F}(y), \quad (13)$$

with parameter and initial Cauchy data

$$U(0,y) = U^0(y), \quad V(0,y) = V^0(y). \quad (14)$$

Note that system (12) and (13) are not the result of applying the Fourier transform to system (1) and (2), since, instead of g_1 and g_2 in (8), (9), we take only their real parts.

We will prove the existence and uniqueness of solution to (12)–(14) using the method of cutting functions [21]. We introduce a sequence of cutting functions in the class such that

$$S_N(y) = \begin{cases} 1, & |y| \le N-2, \\ 0, & |y| > N \end{cases} \quad (15)$$

and approximate (12)–(14) by the problem

$$U_t^N(t,y) + iy\int_{-\infty}^{\infty} S_N(z)U^N(t,z)S_N(y-z)U^N(t,y-z)\,dz$$

$$= -b(U^N(t,y) - V^N(t,y)) + \operatorname{Re}\Big\{\tilde{\varphi}(t) + i\varphi(t)\int_{-\infty}^{\infty} yS_N(y)U^N(t,y)\,dy\Big\}\tilde{F}(y), \quad (16)$$

$$V_t^N(t,y) + iy\int_{-\infty}^{\infty} S_N(z)V^N(t,z)S_N(y-z)V^N(t,y-z)\,dz$$

$$= \varepsilon b(U^N(t,y) - V^N(t,y)) + \mathrm{Re}\Big\{\tilde{\psi}(t) + i\psi(t)\int_{-\infty}^{\infty} yS_N(y)V^N(t,y)\,dy\Big\}\tilde{F}(y), \tag{17}$$

$$U^N(0,y) = S_N(y)U^0(y), \quad V^N(0,y) = S_N(y)V^0(y), \quad N \geq 3. \tag{18}$$

By virtue of (15), we can replace the integrals in (16), (17) with integrals over a segment $[-N,N]$ and obtain

$$U_t^N(t,y) + iy\int_{-N}^{N} S_N(z)U^N(t,z)S_N(y-z)U^N(t,y-z)\,dz$$

$$= -b(U^N(t,y) - V^N(t,y)) + \mathrm{Re}\Big\{\tilde{\varphi}(t) + i\varphi(t)\int_{-N}^{N} yS_N(y)U^N(t,y)\,dy\Big\}\tilde{F}(y), \tag{19}$$

$$V_t^N(t,y) + iy\int_{-N}^{N} S_N(z)V^N(t,z)S_N(y-z)V^N(t,y-z)\,dz$$

$$= \varepsilon b(U^N(t,y) - V^N(t,y)) + \mathrm{Re}\Big\{\tilde{\psi}(t) + i\psi(t)\int_{-N}^{N} yS_N(y)V^N(t,y)\,dy\Big\}\tilde{F}(y), \tag{20}$$

Solving the Cauchy problem for system (18)-(20) we obtain a system of nonlinear integral Volterra equations of the second kind

$$U^N(t,y) = \frac{\varepsilon + e^{-b(1+\varepsilon)t}}{1+\varepsilon}S_N(y)U^0(y) + \frac{1 - e^{-b(1+\varepsilon)t}}{1+\varepsilon}S_N(y)V^0(y) +$$

$$\frac{\varepsilon + e^{-b(1+\varepsilon)t}}{(1+\varepsilon)^2}\int_0^t\Bigg[\big(\varepsilon + e^{b(1+\varepsilon)\tau}\big)\Bigg[\mathrm{Re}\Big\{\tilde{\varphi}(\tau) + i\varphi(\tau)\int_{-N}^{N} yS_N(y)U^N(\tau,y)dy\Big\}\tilde{F}(y) -$$

$$iy\int_{-N}^{N} S_N(z)U^N(\tau,z)S_N(y-z)U^N(\tau,y-z)\,dz\Bigg] +$$

$$\big(1 - e^{b(1+\varepsilon)\tau}\big)\Bigg[\mathrm{Re}\Big\{\tilde{\psi}(\tau) + i\psi(\tau)\int_{-N}^{N} yS_N(y)V^N(\tau,y)dy\Big\}\tilde{F}(y) -$$

$$-iy\int_{-N}^{N} S_N(z)V^N(\tau,z)S_N(y-z)V^N(\tau,y-z)\,dz\Bigg]\Bigg]d\tau +$$

$$\frac{1 - e^{-b(1+\varepsilon)t}}{(1+\varepsilon t)^2}\int_0^t\Bigg[\varepsilon\big(1 - e^{b(1+\varepsilon)\tau}\big)\Bigg[\mathrm{Re}\Big\{\tilde{\varphi}(\tau) + i\varphi(\tau)\int_{-N}^{N} yS_N(y)U^N(\tau,y)dy\Big\}\tilde{F}(y) -$$

$$iy\int_{-N}^{N} S_N(z)U^N(\tau,z)S_N(y-z)U^N(\tau,y-z)\,dz\Bigg] +$$

$$\big(1+\varepsilon e^{b(1+\varepsilon)\tau}\big)\Bigg[\mathrm{Re}\Big\{\tilde{\psi}(\tau)+i\psi(\tau)\int_{-N}^{N} yS_N(y)V^N(\tau,y)dy\Big\}\tilde{F}(y)-$$
$$iy\int_{-N}^{N} S_N(z)V^N(\tau,z)S_N(y-z)V^N(\tau,y-z)\,dz\Bigg]\Bigg]d\tau, \tag{21}$$

$$V^N(t,y)=\frac{\varepsilon(1-e^{-b(1+\varepsilon)t})}{1+\varepsilon}S_N(y)U^0(y)+\frac{1+\varepsilon e^{-b(1+\varepsilon)t}}{1+\varepsilon}S_N(y)V^0(y)+$$
$$\frac{\varepsilon(1-e^{-b(1+\varepsilon)t})}{(1+\varepsilon)^2}\int_0^t\Bigg[\big(\varepsilon+e^{b(1+\varepsilon)\tau}\big)\Bigg[\mathrm{Re}\Big\{\tilde{\varphi}(\tau)+i\varphi(\tau)\int_{-N}^{N} yS_N(y)U^N(\tau,y)dy\Big\}\tilde{F}(y)-$$
$$iy\int_{-N}^{N} S_N(z)U^N(\tau,z)S_N(y-z)U^N(\tau,y-z)\,dz\Bigg]+$$
$$\big(1-e^{b(1+\varepsilon)\tau}bigr)\Bigg[\mathrm{Re}\Big\{\tilde{\psi}(\tau)+i\psi(\tau)\int_{-N}^{N} yS_N(y)V^N(\tau,y)dy\Big\}\tilde{F}(y)-$$
$$-iy\int_{-N}^{N} S_N(z)V^N(\tau,z)S_N(y-z)V^N(\tau,y-z)dz\Bigg]\Bigg]d\tau+$$
$$\frac{1+\varepsilon e^{-b(1+\varepsilon)t}}{(1+\varepsilon)^2}\int_0^t\Bigg[\varepsilon\big(1-e^{b(1+\varepsilon)\tau}\big)\Bigg[\mathrm{Re}\Big\{\tilde{\varphi}(\tau)+i\varphi(\tau)\int_{-N}^{N} yS_N(y)U^N(\tau,y)dy\Big\}\tilde{F}(y)-$$
$$iy\int_{-N}^{N} S_N(z)U^N(\tau,z)S_N(y-z)U^N(\tau,y-z)\,dz\Bigg]+$$
$$\big(1+\varepsilon e^{b(1+\varepsilon)\tau}\big)\Bigg[\mathrm{Re}\Big\{\tilde{\psi}(\tau)+i\psi(\tau)\int_{-N}^{N} yS_N(y)V^N(\tau,y)dy\Big\}\tilde{F}(y)-$$
$$iy\int_{-N}^{N} S_N(z)V^N(\tau,z)S_N(y-z)V^N(\tau,y-z)dz\Bigg]\Bigg]d\tau. \tag{22}$$

Using the method of contraction mappings, it can be shown that for fixed $N\geq 3$, there exist classical solutions $U^N(t,y), V^N(t,y)\in C^{1,0}_{t,y}\big(\Pi_{[0,t_N]}\big)$ of problem (18)–(20) in $\Pi_{[0,t_N]}$. Here the constant t_N is positive and, generally speaking, depends on N.

Following [5], taking into account [21, Lemma 3.1], a priori estimates of solutions $U^N(t,y)$, $V^N(t,y)$ are established:

$$|y|^{3+\lambda}|U^N(t,y)|\leq c_1,\quad |y|^{3+\lambda}|V^N(t,y)|\leq c_2,\quad (t,y)\in\Pi_{[0,t_*]}. \tag{23}$$

Here and below, the constants c_1, c_2, ... do not depend on N, while t_* depends on $d_1(4)$, $d_2(4)$, $\|\varphi\|_{C^1[0,T]}$, $\|\psi\|_{C^1[0,T]}$ and does not depend on N, for all $N\geq 3$. From equations (19), (20) we obtain

$$|U_t^N(t,y)|\leq c_3,\quad |V_t^N(t,y)|\leq c_4,\quad (t,y)\in\Pi_{[0,t_*t]}. \tag{24}$$

Differentiating both parts of system (21), (22) with respect to y, we can show that the following estimates are valid

$$|U_y^N(t,y)| \le c_5, \quad |V_y^N(t,y)| \le c_6, \quad (t,y) \in \Pi_{[0,t_*]}. \tag{25}$$

Using (23)–(25) and Arzela's compactness theorem in C, we can choose subsequences $\{U^{N_k}\}$, $\{V^{N_k}\}$ such that

$$U^{N_k} \to U, \quad V^{N_k} \to V, \quad N_k \to \infty, \tag{26}$$

uniformly on each compact K in $\Pi_{[0,t_*]}$.

The uniqueness of the solution is proved in the usual way. Thus, we arrive at the following

Theorem. *Let conditions* (10), (11), $f(0) = 1$ *be satisfied and* $\varphi, \psi \in C^1[0,T]$. *Then there exists a unique solution* $U(t,y)$, $V(t,y)$ *to system* (12)–(14) *in the strip* $\Pi_{[0,t_*]}$. *The value* $0 < t_* \le T$ *depends only on the constants* $d_1(4)$, $d_2(4)$ *and* $\|\varphi\|_{C^1[0,T]}$, $\|\psi\|_{C^1[0,T]}$.

Let us now prove that the solution $u(t,x)$, $v(t,x)$, $g_1(t)$, $g_2(t)$ to the original problem (1)–(4) is

$$(u(t,x), v(t,x)) = \int_{-\infty}^{\infty} (U(t,y), V(t,y)) e^{-ixy}\, dx, \tag{27}$$

$$g_1(t) = \mathrm{Re}\Big\{\tilde{\varphi}(t) + i\varphi(t)\int_{-\infty}^{\infty} yU(t,y)\, dy\Big\}, \tag{28}$$

$$g_2(t) = \mathrm{Re}\Big\{\tilde{\psi}(t) + i\psi(t)\int_{-\infty}^{\infty} yV(t,y)\, dy\Big\}. \tag{29}$$

It is easy to see that $g_1(t)$ and $g_2(t)$ are real functions. We will show that $u(t,x)$, $v(t,x)$ are also real functions and satisfy (1)–(4) (where $g_1(t)$ and $g_2(t)$ are defined in (28) and (29), respectively). We apply the inverse Fourier transform to (12)–(14) by y and see that $u(t,x)$, $v(t,x)$ are a solution to the problem

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -b(u-v) + f(x)g_1(t), \quad (t,x) \in \Pi_{[0,t_*]}, \tag{30}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \varepsilon b(u-v) + f(x)g_2(t), \quad (t,x) \in \Pi_{t[0,t_*]}, \tag{31}$$

$$u|_{t=0} = u^0(x), \quad v|_{t=0} = v^0(x). \tag{32}$$

or

$$\frac{\partial u_1}{\partial t} + u_1\frac{\partial u_1}{\partial x} - u_2\frac{\partial u_2}{\partial x} = -b(u_1 - v_1) + f(x)g_1(t), \quad (t,x) \in \Pi_{[0,t_*]}, \tag{33}$$

$$\frac{\partial v_1}{\partial t} + v_1\frac{\partial v_1}{\partial x} - v_2\frac{\partial v_2}{\partial x} = \varepsilon b(u_1 - v_1) + f(x)g_2(t), \quad (t,x) \in \Pi_{[0,t_*]}, \tag{34}$$

$$\frac{\partial u_2}{\partial t} + u_1\frac{\partial u_2}{\partial x} + u_2\frac{\partial u_1}{\partial x} = -b(u_2 - v_2), \quad (t, x) \in \Pi_{[0,t_*]}, \tag{35}$$

$$\frac{\partial v_2}{\partial t} + v_1\frac{\partial v_2}{\partial x} + v_2\frac{\partial v_1}{\partial x} = \varepsilon b(u_2 - v_2), \quad (t, x) \in \Pi_{[0,t_*]}, \tag{36}$$

$$u_1|_{t=0} = u^0(x), \quad v_1|_{t=0} = v^0(x), \quad u_2|_{t=0} = 0, \quad v_2|_{t=0} = 0. \tag{37}$$

where u_1, v_1 and u_2, v_2 are the real and imaginary parts of the functions u, v ($u = u_1 + iu_2$, $v = v_1 + iv_2$), and $g_1(t)$, $g_2(t)$ are the functions in (28), (29). Since u_1, v_1 and u_2, v_2 is a classical bounded solution of (33)–(37) (see (26)), we can consider system (35), (36) as a linear system with respect to u_2, v_2 and apply the method of characteristics (see, for example, [10, 22]) to obtain $u_2 = 0$, $v_2 = 0$. Consequently, $u = u_1$, $v = v_1$ is a real solution of (30)–(32) or (which is the same) (1)–(3), and $g_1(t)$, $g_2(t)$ are given by equations (28), (29).

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