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Long-period deconvolution of seismometer digital records by recursive filtration*

Yu.V. Mkrtumyan, S.S. Polozov, V.I. Yushin

In this paper, the formulas of digital recursive filters for the long-period deconvolution, which are successfully tested on real records of seismic events, are deduced.

One of "eternal" problems of seismometry is a problem of creating the long-period seismographs. Sensitive mechanics of such instruments in combination with a variety of "know-how" makes the process of their manufacturing a real art, which results in the monopoly of their production and high costs. At the same time, there is another, more cheaper way of getting the long-period records of earthquakes, based on the deconvolution of an record already existing and widespread short-period seismometers. Though this idea is not new, its realization has become practically possible only recently after introducing high-precision (20–24 bit) digital recordings.

The general theory of deconvolution of a short-period record to a long-period one was considered in paper [1]. It was shown that in the general way a deconvolution algorithm must consist of HF-filtration operations with parameters required for a long-period virtual seismometer and subsequent double or triple integration with weights, defined by parameters of the original sensor. In this work the problem of synthesis of the digital recursive filters, ensuring the performance of this algorithm is solved.

A complex frequency response, or a transfer function of sensor-velocimeter with respect to the input displacement has the form [1–3]:

$$V(j\omega) = \frac{(j\omega)^3 V}{(j\omega)^2 + 2\varepsilon(j\omega) + \omega_0^2}, \tag{1}$$

where V is a transformation factor of the end-to-end recording channel (sensor – amplifier – digitizer) of the velocity in the working frequency band (dimensionality is count · s/m), ε is damping of the sensor, ω_0 is a natural frequency of non-damping oscillations. The inverse frequency response will be of the form:

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$$V^{-1}(j\omega) = \frac{1}{V}\left[\frac{1}{j\omega} + \frac{2\varepsilon}{(j\omega)^2} + \frac{\omega_0^2}{(j\omega)^3}\right], \tag{2}$$

In theory, if a signal, received at the output of the sensor (1), goes through the filter (2), there will be received an original signal, in other words, a true "ground" displacement will be restored.

In the time domain this procedure looks like follows:

$$z(t) = \frac{1}{V}\left[\int_0^t y(t)\,dt + 2\varepsilon\int_0^t\int_0^t y(t)\,(dt)^2 + \omega_0^2\int_0^t\int_0^t\int_0^t y(t)\,(dt)^3\right], \tag{3}$$

where $y(t)$ is the original signal from the short-period sensor.

Realization of full deconvolution of such a type is practically impossible. Physical obstacle of this are: apparatus noise and limits of the recording accuracy of a signal (both are of the same nature). A considerable increase in a band of recorded frequencies in the area of long periods is however possible, and this extension is the larger, the higher is the accuracy of the analog-to-digital convertor (ADC).

For this, it is necessary to add an operation of the digital recursive HF-filtrations of the signal $y(t)$, and whereupon it is possible to execute integration over practically unlimited time interval. The HF filter is chosen with parameters, corresponding to a desired long-period sensor, but no less, than one order higher, than the order of the source sensor transfer function numerator. As a result of such processing, the transfer function of a real sensor will be deleted from the signal, and instead of it, an other transfer function more suitable from the researchers viewpoint is introduced.

Algorithm of long-period deconvolution using the recursive digital filters

For realization of a long-period deconvolution it is necessary to digitally simulate the linear filtrations operations, as well as integration and differentiation. In a simple case, integration is executed in the rectangles:

$$\int \quad \longleftrightarrow \quad S_i = x_i\Delta t + S_{i-1}, \tag{4}$$

which corresponds to the conjugate operation of differentiation in finite differences in the form of the so-called inverse difference:

$$\frac{d}{dt} \quad \longleftrightarrow \quad \nabla S_i = \frac{S_i - S_{i-1}}{\Delta t} = (S_i - S_{i-1})f_g = x_i, \tag{5}$$

where Δt and $f_g = (\Delta t)^{-1}$ are the interval and the sampling rate, respectively.

It is necessary to note that if an analog signal before sampling was subject to the anti-aliasing filtration with high Q-factor or sampling was performed on the increased frequency with the subsequent digital filtration and decimation, which is peculiar of modern digital seismographs, the subsequent application to this signal of digital filters containing derivatives of high order (at least of up to fourth order) brings neither to instability nor to any significant mistakes.

Methods of digital modeling of linear dynamic systems [4] allow the simulation of the pass of an analog signal through the analog dynamic system by transformation of a sequence of samples of this signal by a certain recurrent computing procedure named the recursive filter.

An advantage of the recursive filtration consists in the fact that, handling only with a current and limited number of the closest pre-existing samples of input and output signals, it simulates a convolution of an input signal with a weighting function of unlimited extent, which is intrinsic of the seismometer. The recursive filtration allows the online processing of a signal, which is certain to ensure the fulfillment of the principle of causality. The main difficulty is to determine the correlation between parameters of the recursive filter and parameters of the simulated analog system. The problem of the dynamic range of calculations with modern computers has lost its urgency. However, for the long-period deconvolution, it is useful to execute an additional low-pass filtration in order that the visual dynamic range of signals be compressed, which is done for suppressing inessential high-frequency components of a signal spectrum. This can be also done with the help of the appropriate recursive filter.

To find a recursive filter, which would simulate a seismometer, the reasoning is as follows. A seismometer analog transfer function (for simplicity, the gain-factor is equal to 1) can be presented in the operator form in the following versions:

$$W(p) = \frac{T^2p^2}{T^2p^2 + 2dTp + 1} = \frac{p^2}{p^2 + 2d\omega_0 p + \omega_0^2} = \frac{p^2}{(p - p_1)(p - p_2)}, \qquad (6)$$

where ω_0 is the natural angular frequency of non-damping oscillations, T is the time constant equal to ω_0^{-1} (not meaning the eigenperiod), d is dimensionless damping factor ($0 < d \leq 1$), p_1 and p_2 are complex-conjugate poles of the transfer function:

$$p_{1,2} = -d\omega_0 \pm j\omega_0\sqrt{1 - d^2}. \qquad (7)$$

The first expression of formula (6), given in a standard form generally accepted in the automatic control theory [5], shows that the seismometer transfer function with unit gain-factor can be presented in the manner of three consecutively connected units: the oscillatory unit with the transfer function

$$W_1(p) = \frac{1}{T^2p^2 + 2dTp + 1} = \frac{\omega_0^2}{p^2 + 2d\omega_0 p + \omega_0^2}; \tag{8}$$

the double differentiation

$$W_2(p) = p^2, \tag{9}$$

and the constant transfer factor

$$W_3(p) = T^2 = \frac{1}{\omega_0^2} = \frac{1}{(2\pi f_0)^2}. \tag{10}$$

where f_0 is the natural (non-damping) frequency of the simulated seismometer (Hz). Theory of the digital simulation [4] offers the following recurrent algorithm of the oscillatory unit (8):

$$y_i = Ay_{i-1} - By_{i-2} + (1 - A + B)x_i, \tag{11}$$

where x_i and y_i are input and output sequences of recursive filter, respectively; A and B are constants, depending on parameters of the transfer function of the simulated oscillatory unit: the natural frequency f_0 and the damping factor d, as well as on the sampling rate f_g of the input signal:

$$A = 2\exp\left(-\frac{2\pi d f_0}{f_g}\right)\cos\left(\frac{2\pi f_0}{f_g}\sqrt{1-d^2}\right), \tag{12}$$

$$B = \exp\left(-\frac{4\pi d f_0}{f_g}\right). \tag{13}$$

Let us note that a recursive model of the oscillatory unit (11)–(13) in terms of deconvolution problems is of interest not only as part of the seismometer model, but also as a second order low-pass filter (LPF) 2-th order. Using such a filter, as was already mentioned, can be needed when performing the data decimation after deconvolution, as well as when testing the digital models.

For transformation from the oscillatory unit to the seismometer digital model, it is sufficient to twice differentiate an input or an output signal of the oscillatory unit and to take into account the constant factor (10).

The double differentiation in inverse differences is expressed by the formula

$$z_i = (x_i - 2x_{i-1} + x_{i-2})f_g^2. \tag{14}$$

Substituting expression (14) for the second derivative from x_i to formula (11) instead of x_i and multiplying the latter by factor (10), we arrive at the sought for recurrent equation of the virtual seismometer (the HPF of second order)

$$y_i = Ay_{i-1} - By_{i-2} + (1 - A + B)\left(\frac{f_g}{2\pi f_0}\right)^2 (x_i - 2x_{i-1} + x_{i-2}). \quad (15)$$

Thereby, the long-period deconvolution is reduced to the following operations. The original record of the short-period sensor-velocimeter is passed through the recursive filter-pseudo-seismometer (15), whereupon one executes the deconvolution to the velocity consisting of the double integration:

$$a_i = y_i \Delta t + a_{i-1}, \quad (16)$$

$$v_i = a_i \Delta t + v_{i-1}, \quad (17)$$

and the weighted summation of the pseudo-seismometer output signal and its two integrals $\{a_i\}$ and $\{v_i\}$:

$$z_i' = \frac{1}{V}[y_i + 4\pi D F_0 a_i + (2\pi F_0)^2 v_i]. \quad (18)$$

At this stage of processing, the true parameters of a real sensor are introduced into (18) as weighting factors of the summed-up components, as follows: the natural frequency, the damping factor D, and the gain-factor V. As a result, we get a signal proportional to the velocity (in mcm/s) in the operating band of the virtual sensor-velocimeter. For getting a displacement (a signal of the virtual seismometer) it is necessary to integrate the velocity. However, this operation can turn out to be unstable. To avoid this deficiency, one should use the same filter pseudo-seismometer once again (15):

$$u_i = Au_{i-1} - Bu_{i-2} + (1 - A + B)\left(\frac{f_g}{2\pi f_0}\right)^2 (z_i' - 2u_{i-1}' + z_{i-2}') \quad (19)$$

and only after this to fulfil the third integration

$$s_i = u_i \Delta t + s_{i-1}. \quad (20)$$

In this case, no selection of the initial conditions is required.

The most efficient algorithm is in the following operating procedures: centering – the first HP-filtration (the insertion of the pseudo-seismometer) – deconvolution to to the velocity – the second HP-filtration (optional) – integration of the velocity to a displacement. If all these operations have been executed, the deconvolution is ensured without any problems on the interval of any duration. In this case, the selection of the initial conditions is not required – all of them turn to zero. Only the natural frequency of the virtual sensor should be selected. With such a structure of the algorithm one can simultaneously obtain the velocity, brought to the pseudo-seismometer, the velocity after the double pseudo-seismometer and the displacement, brought to the double pseudo-seismometer.

It is necessary to mention a principle difference of the long-period deconvolution from the usual pass-band filtration, which is a standard procedure in packages of the time-series processing. A pass-band filtration enables us to select some frequency range from raw data, but it saves all amplitude-frequency and phase-frequency distortions, due to the sensor, adding to it their own distortions as well. The long-period deconvolution completely deletes the former, but introduces own distortions. But these distortions are strictly known and minimum-phase. Simultaneously, the long-period deconvolution scales a result in parameters of mechanical motion (displacement, velocity or acceleration).

The best evaluation of the long-period deconvolution validity of a short-period record is a comparison of its result with a real long-period seismograph record.

As an example of such a comparison, a fragment of the three-component raw record of 2-second short-period seismometers SM3-KV is presented in Figure 1. This is an earthquake 28 occurred on March, 1999, recorded by the 24-bit seismo-recorder TRAL.

The results of the long-period deconvolution are presented in the upper parts of Figures 2, 3, and 4. The deconvolution was executed to the record of the virtual 20-second seismometer. True records of 20-second seismometers are shown in the lower parts of Figures 2, 3, and 4, respectively. These records were executed by the 20-second galvanometer seismograph SKD at the station "Novosibirsk" and are presented for the comparison. As is seen the virtual and the real records on all the three components coincide sufficiently well.

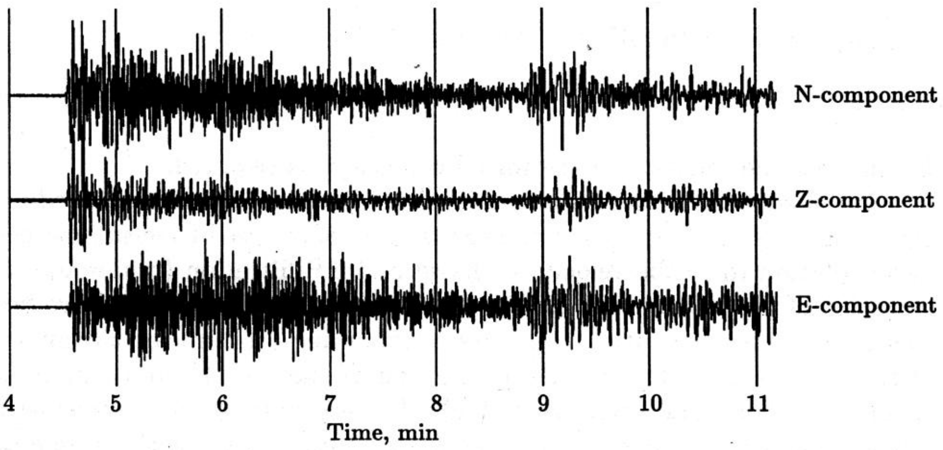

Figure 1. A fragment of the 3-components raw record of the earthquake of March 28, 1999, recorded by seismometers SM3-KV (eigenperiod 2 s) and 24-bit seismo-recorder TRAL on the pedestal of seismological station "Novosibirsk"

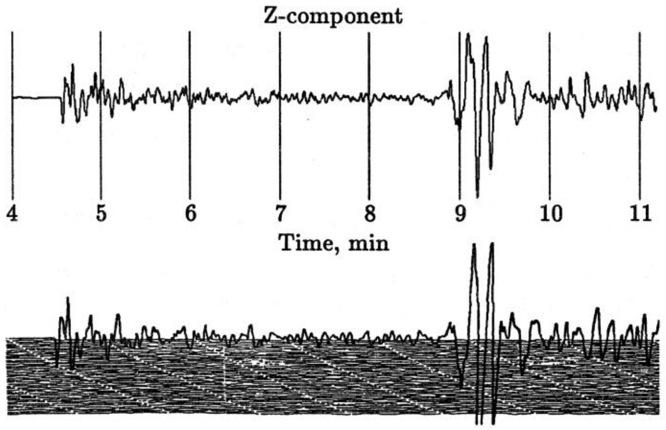

Figure 2

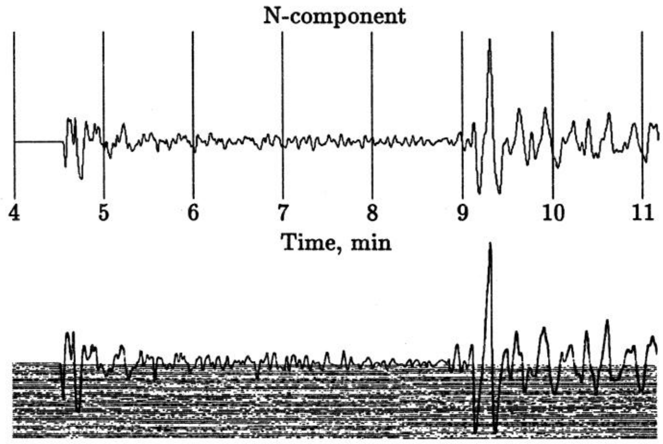

Figure 3

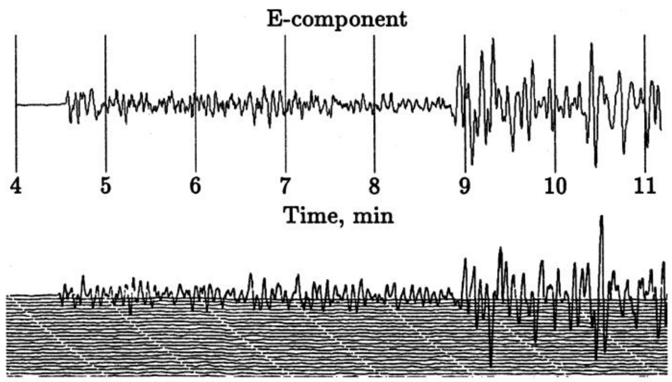

Figure 4

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