Bull. Nov. Comp. Center, Math. Model. in Geoph., 2 (1996), 63–70

Method for geophysical exploration based on the interaction of waves with the Earth's magnetic field

B.G. Mikhailenko and O.N. Soboleva

The induction seismomagnetic effects arising in the seismic wave motion in the constant Earth's magnetic field are numerically studied in this artical. The phenomenon is described as a simultaneous solution of the system of elastic equations and quasistationary Maxwell's equations with displacement velocity components. For solving the problem we use numerical-analitical algorithm based on the finite Fourie transform. The obtained system of ordinary differential equations is solved by the factorization method.

1. Introduction

It is known that when a conductive wire frame is moving in a constant magnetic field, an electric current and a variable magnetic field are generated in the frame.

Similar phenomena can be observed when a seismic wave propagates in the Earth's constant magnetic field. The seismic wave, with its forward and back wave fronts is analogous to the wire frame. A conductive medium between the forward and the back wave vibrates in the Earth's constant magnetic field, which brings about local geomagnetic variations. Local geomagnetic variations, propagating simultaneously with the seismic wave diffusing into the medium are termed seismomagnetic waves. These waves contain information about both electromagnetic and elastic parameters of a medium.

The electromagnetic wave rides the "back" of the seismic wave, that is, the induced electromagnetic wave is "frozen" into the seismic wave and propagates either with *P*- or with *S*-seismic wave velocity, depending on the type of waves. The dominant frequency and the velocity of the induced seismomagnetic wave is equal to the frequency and velocity of the seismic wave.

Lately, some attempts have been made to measure and implement the electrical currents generated by seismic waves and to develop methods of electroseismic prospecting [1]. The electroseismic approach is different from the seismomagnetic method. Whereas the electroseismic effect results from a local effect of the seismic wave interactions with the interface between elastic media and shallow layers of subsurface fluids, the seismomagnetic effect is based on the interaction of seismic waves with the Earth's magnetic field. The result of such an interaction is the induced electromagnetic wave which propagates with the seismic wave, but not with the light speed, as in the case of the electroseismic effect.

The results (see [2, 5]) indicate to the simultaneous propagation of the seismic wave together with the induced geomagnetic variation, and to the fact that it is possible to record the geomagnetic variation.

2. Mathematical model for seismomagnetic effect

The phenomenon of the seismomagnetic effect caused by an explosion in an elastic medium is described as a simultaneous solution of the self-consistent system of elastic equations with the Lorenz force and quasi-stationary Maxwell's equations with displacement velocity components. The first paper in this area was by Knopoff [6], who concluded that the Earth's magnetic field weakly affects the wave propagation velocity; however, Knopoff did not study seismomagnetic waves. The theory of the interaction of elastic and magnetic fields proposed by Knopoff was further developed in the solid state physics. At present a new 1 area of research of the magneto-elasticity theory is gaining more importance Novacki [7]. We use the system of equations derived by Novacki

$$V_p^2 \operatorname{grad}\operatorname{div} \boldsymbol{U} - V_s^2 \operatorname{rot}\operatorname{rot} \boldsymbol{U} = \frac{\partial^2 \boldsymbol{U}}{\partial t^2} + \boldsymbol{F}, \tag{1}$$

$$\frac{\partial \boldsymbol{h}}{\partial t} = \operatorname{rot}\Big(\frac{\partial \boldsymbol{U}}{\partial t} \times \boldsymbol{H}_0\Big) + \beta \Delta \boldsymbol{h}, \tag{2}$$

$$\operatorname{div} \boldsymbol{h} = 0, \tag{3}$$

For the vacuum $z < 0$:

$$\Delta \boldsymbol{h} = 0, \tag{4}$$

Components of the seismoelectric field can be found from the following equations:

$$\boldsymbol{E} = \sigma^{-1} \operatorname{rot} \boldsymbol{h} - \mu_0 \frac{\partial \boldsymbol{U}}{\partial t} \times \boldsymbol{H}_0, \tag{5}$$

for the vacuum $z < 0$:

$$\operatorname{rot} \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{h}}{\partial t}. \tag{6}$$

Here $\boldsymbol{U}$ is the displacement vector of the elastic medium, and $\boldsymbol{H}_0$ is the strength vector of the Earth's magnetic field.

Local geomagnetic variations of the magnetic field induced into the medium due to the seismic wave propagation can be represented as $\boldsymbol{H}_0 \pm \boldsymbol{h}$, where $\boldsymbol{h}$ is assumed to be small as compared to $\boldsymbol{H}_0$. Velocities of longitudinal V_p- and transverse V_s-waves, density ρ, and conductivity σ are assumed to be constant within each layer. Magnetic permeability μ_0 is assumed constant for all the layers.

The continuity conditions should be satisfied for the displacement components, strengths τ_{xz}, τ_{yz}, τ_{zz} of the elastic medium and magnetic field components, tangent components of the electrical field for the contact boundary for any two layers.

On the boundary with vacuum for $z = 0$, the strength $\tau_{xz} = 0$, $\tau_{yz} = 0$, $\tau_{zz} = 0$, magnetic field components and the normal component of the electrical field are continuous. At the infinity, movement is absent. The source of the seismic wave:

$$\boldsymbol{F} = f(t)\Big[M_{ij}\frac{\partial\delta(\boldsymbol{x}-\boldsymbol{x}_0)}{\partial x_j} + \boldsymbol{F}_0\delta(\boldsymbol{x}-\boldsymbol{x}_0)\Big], \tag{7}$$

where $\boldsymbol{x} = (x, y, z)$, $M_{i,j}$ are the components of seismic moment, $f(t)$ is Puzyrerv's impulse:

$$f(t) = \exp\Big[-\frac{2\pi\omega_0(t-t_0)^2}{\gamma^2}\Big]\sin(2\pi\omega_0(t-t_0)),$$

$\boldsymbol{F}_0$ is vector of the force.

3. Numerical method

To solve the system of equations (1)–(4) we make use of the earlier developed numerical-analytical algorithms [9] employing the finite Fourier transform along the coordinates x, y, t. The obtained system of ordinary equations is the following:

$$\frac{d^2u_x}{dz^2} - \Big(\frac{V_p^2}{V_s^2}k_x^2 + k_y^2 - \frac{\omega^2}{V_s^2}\Big)u_x - k_xk_y\frac{(V_p^2-V_s^2)}{V_s^2}u_y + ik_x\frac{(V_p^2-V_s^2)}{V_s^2}\frac{du_z}{dz} = 0,$$
$$\frac{d^2u_y}{dz^2} - k_xk_y\frac{(V_p^2-V_s^2)}{V_s^2}u_x - \Big(k_x^2 + \frac{V_p^2}{V_s^2}k_y^2 - \frac{\omega^2}{V_s^2}\Big)u_y + ik_y\frac{(V_p^2-V_s^2)}{V_s^2}\frac{du_z}{dz} = 0,$$
$$\frac{d^2u_z}{dz^2} + \frac{(V_p^2-V_s^2)}{V_p^2}\Big(ik_x\frac{du_x}{dz} + ik_y\frac{du_y}{dz}\Big) - \Big(\frac{V_s^2}{V_p^2}k_x^2 + \frac{V_s^2}{V_p^2}k_y^2 - \frac{\omega_2}{V_p^2}\Big)u_z = 0. \tag{8}$$

$$\frac{d^2h_x}{dz^2} - \tau^2 h_x = \frac{i\omega}{\beta}\Big[H_{0x}\Big(\frac{du_z}{dz} + ik_y u_y\Big) - H_{0y} ik_y u_x - H_{0z}\frac{du_x}{dz}\Big],$$
$$\frac{d^2h_y}{dz^2} - \tau^2 h_y = \frac{i\omega}{\beta}\Big[-H_{0x} ik_x u_y + H_{0y}\Big(\frac{du_z}{dz} + ik_x u_x\Big) - H_{0z}\frac{du_y}{dz}\Big],$$
$$\frac{d^2h_z}{dz^2} - \tau^2 h_z = \frac{i\omega}{\beta}[-H_{0x} ik_x u_z - H_{0y} ik_y u_z + H_{0z}(ik_x u_x + ik_y u_y)], \quad (9)$$

where $\tau^2 = k_x^2 + k_y^2 + \frac{i\omega}{\beta}$. The source $\vec{F}$ from the right-hand side of equations (8) is carried over to the point $z = z_0$ by the standard technique.

The solution to equations (9) we find in the form

$$h_x = C_{1l}^j e^{\tau_j z} + C_{2l}^j e^{-\tau_j z} + \varphi_l^j, \quad l = x, y, z, \quad (10)$$

where j is a number of the layer. If the partial solutions φ_x, φ_y, φ_z of equations (9) are known, we can find constants C^j using the well-known (for such problems) recurrent formulas taking into account the boundary conditions and conjugation conditions for layers. It is more difficult to define the partial solution of equations (9) as the difficulties of construction of the analytical solution in every layer of the elastic theory equations by matrix methods are well-known. For solving equations (8) we used the factorization method [9]. Here we introduce the potentials:

$$u_x = ik_x w_1 + \frac{dw_2}{dz}, \quad u_y = ik_y w_1 + \frac{dw_3}{dz}, \quad u_z = -ik_x w_2 - ik_y w_3 + \frac{dw_1}{dz}.$$

Then from (8):

$$\frac{d^2w_1}{dz} = R^2 w_1, \qquad \frac{d^2w_2}{dz} = S^2 w_2, \qquad \frac{d^2w_3}{dz} = S^2 w_3, \quad (11)$$

where $R^2 = k_x^2 + k_y^2 - \frac{w^2}{V_p^2}$, $S^2 = k_x^2 + k_y^2 - \frac{w^2}{V_s^2}$.

Following [9] we introduce a new unknown matrix of the functions A:

$$\frac{dw}{dz} = Aw. \quad (12)$$

Provided that (12) identically satisfies the equations in potentials (11) we obtain Riccati's matrix equations for determining a_{ij} in each layer. Riccati's equation admits an analytical solution in the closed form. In its final form, the algorithm is as follows. First, we perform the same procedure of calculating a_{ij} from above up to the point $z = z_0$, where the source is located, using the conjugation conditions between layers and Riccati's equations. Then this procedure is repeated from the bottom upwards. Taking into account the conditions in the source, we obtain values of potentials in each layer. If we present the prototypes of displacements in the form

$$u_l = C^j_{3l} e^{R_j z} + C^j_{4l} e^{-R_j z} + C^j_{5l} e^{S_j z} + C^j_{6l} e^{-S_j z}, \quad l = x, y, z, \tag{13}$$

we will be able to find the constant C^j_{il} in each layer and to construct the partial solution for equations (9). Note, this approach has no computational restrictions when doing calculations with high frequencies.

4. Some results of numerical modeling of seismomagnetic wave

Consider some dynamic features of seismomagnetic waves. Each kind of the seismic wave generates an electromagnetic wave associated with it and propagating with the same velocity. The electromagnetic wave generated by a seismic wave of a given kind we call the seismomagnetic wave of the same kind (e.g., Rayleigh seismomagnetic wave, longitudinal seismomagnetic wave, transverse seismomagnetic wave, etc.). As compared to the longitudinal wave the seismomagnetic wave is transverse, as any other electromagnetic wave. However, the longitudinal seismomagnetic wave propagates with longitudinal seismic wave velocity. Basic dynamic features of seismomagnetic waves for homogeneous elastic media were considered in [10]. Further, we convert components of both seismic and seismomagnetic fields from x, y, z to the spherical coordinate system for stratified elastic media. We transformed all the components into dimensionless units. We divided the components of the seismomagnetic field by the maximum amplitude of all its components and the components of seismic field by the maximum amplitude of all its components.

Figure 1 shows radial, tangential components of the displacement of the elastic wave at the point $r_0 = 3\lambda$, where λ is the dominant P – wavelength in the elastic medium and the radial, tangential components of seismomagnetic field at the same point for different angles $\hat{\theta}$; $\hat{\theta}$ is the angle between the strength vector of the Earth's magneticfield $\boldsymbol{H}_0$ and the vertical axis z. The elastic model is used with an explosive point source near $z = 0$, for this case transverse components of all waves are equal to zero. Parameters of the model are the following: $V_{p1} = 1000$ m/s, $V_{p2} = 2000$ m/s, $V_{si} = V_{pi}/1.73$, $i = 1, 2$, the depth of layer $h = \lambda$, strength of geomagnetic field $H_0 = 40$ A/m, conductivity $\sigma = 0.01$ cm/m. The figure shows, that the phase and first arrivals of geomagnetic variations coincide with the analogous characteristics of the seismic waves. The first wave is the longitudinal wave P, the second wave is the Rayleigh wave, the third wave is a wave reflected from the boundary of the layer. The radial and tangential components of P and the Rayleigh seismomagnetic waves have the same circular polarization. The amplitude of P-seismomagnetic wave decreases with the increase of the angle $\hat{\theta}$, while the amplitude of the Rayleigh wave increases.

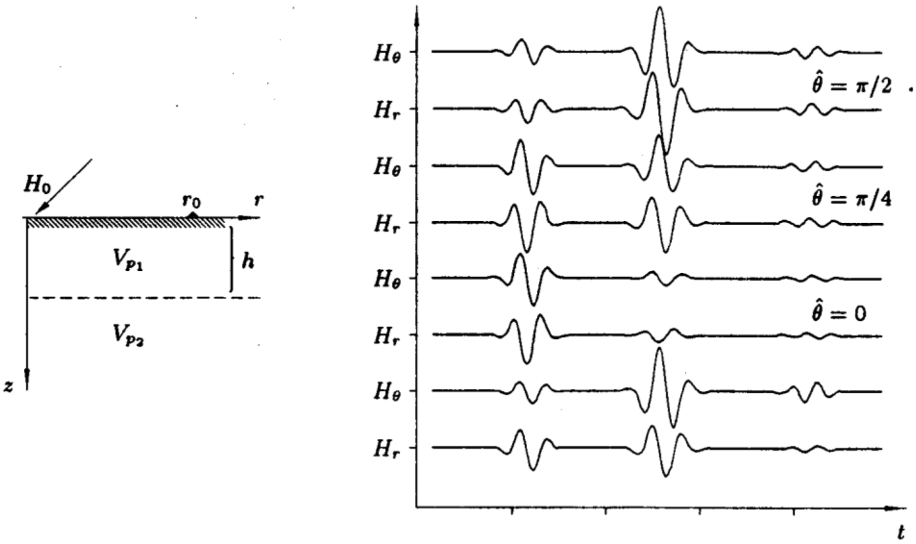

Figure 1

Figure 2 shows the radial, tangential, transverse components of elastic displacements and seismomagnetic waves. The elastic model and its parameters are the same as in Figure 1. The point source located near $z = 0$ is the source of the horizontal force type. In this case we have radial, tangential and transverse components of all waves. The amplitude of P-seismomagnetic wave decreases with the increase of the angle $\hat{\theta}$, while, the amplitude of the Rayleigh wave increases. Results of numerical modeling of seismomagnetic wave propagation in an elastic medium indicate that a few interpretation parameters may be used for geophysical exploration parameters:

1. The ratio between the amplitudes, respectively, of the seismic wave and induced seismomagnetic wave. This ratio is a function of conductivity at the location of the receiver, inclination of the Earth's magnetic field, angle of incidence of the seismic wave with the surface at the receiver location from any subsurface reflector. The amplitude of the seismomagnetic wave is equal to zero, when displacement in the seismic wave is directed parallel to the Earth's magnetic force lines and, on the contrary, the amplitude of the seismomagnetic wave is maximum when displacement in a seismic wave is directed perpendicular to the Earth's magnetic force lines.
2. The ratio between the electrical and magnetic horizontal components of the seismomagnetic wave. Theoretical analysis indicates that this ratio gives us the possibility to direct instantaneous velocity measurement in the vicinity of a few seismic wavelengths.
3. The circular polarization of seismomagnetic waves near the boundary of two layers with different conductivities.

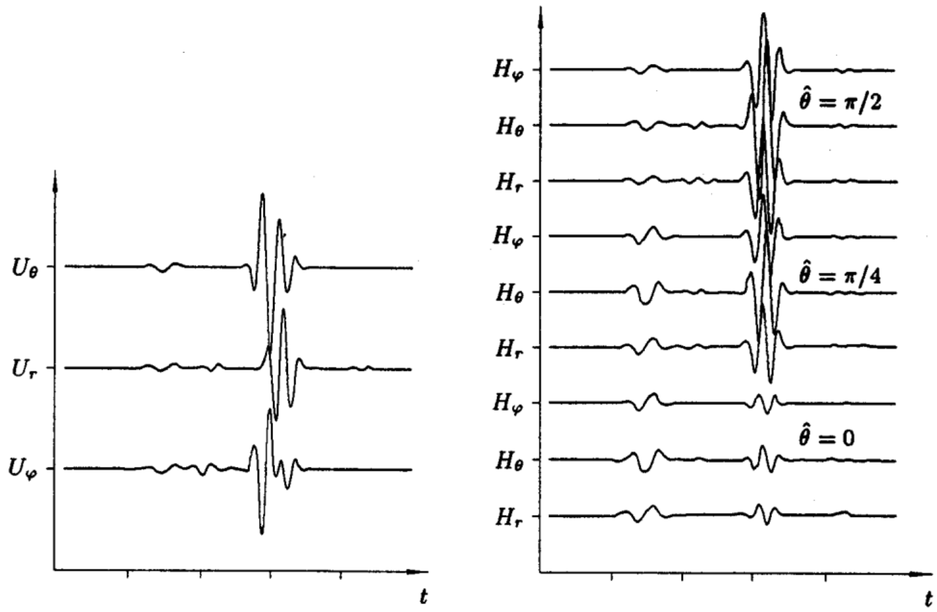

Figure 2

This seismomagnetic approach to geophysical investigation will allow one to complement the conventional procedure of processing and interpreting seismic data by synchronously recording seismic and electromagnetic data. The combined interpretation of the seismic field and the induced electromagnetic field makes it possible to determine not only elastic, but also electromagnetic parameters of the subsurface. The seismomagnetic approach also opens the way for the creation of multicomponent electromagnetic methods. Seismic body and surface waves generate seismomagnetic waves of different types and different propagation velocity and polarization. The new seismomagnetic technology will allow the use of dynamic features of electromagnetic waves – such features as amplitude, polarization, shape and recording duration, frequency spectrum, etc. – which, in turn, bring about better determination of rock properties and complex subsurface structures.

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