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# A scale factor in the long-wave laboratory simulation

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**Abstract.** In the paper the group of variables transformation which is allowed by the shallow-water system of differential equations is obtained. During long wave simulation in water flumes it is necessary to select of the similarity coefficients, that allow to apply the results of modeling in reduced scale to real big-size natural phenomena. Such coefficients are given in this paper.

For a long time laboratory experiments with a water wave were the only instrument for investigation of the wave dynamics near the coast. For this purpose a number of water flumes were built all round the World. Some of them have the size of hundreds meters (Figure 1).

Currently in addition to this kind of research, numerical modeling of various physical and geophysical processes is used. However physical experiments are still widely used for studying hydrodynamic and aerodynamic processes. In particular in the aircraft construction the wind tunnels for

**Figure 1.** Water flume for investigation of beach erosion (Sendai, Japan)

finding parameters (characteristics) of the air mass when flowing round a reduced model of an aircraft are widely used. Usually this is done because of complexity of the air (water) flow structure, which is not quite adequately described by existing mathematical models. In the course of such experiments parameters of a flow are recorded by numerous sensors located both on the surface of a model, and around it. In view of the fact that the model in question is, as a rule, a reduced copy of the created device, there arises a question about the correct interpretation of the results obtained in experiments. In this case it is required to select similarity coefficients correctly. If there is a possibility to carry out an experiment in “full scale”, the question of similarity disappears. For studying aerodynamics there are some huge wind tunnels for purging the full-scale aircrafts.

In geophysics it often happens that the scale of phenomena is such that it is impossible to perform a physical experiment, where the spatial sizes would be, at least, one order less than the original size. Most often it is a question of several orders. Let us consider, for example, the process of tsunami wave propagation in the ocean. In this case the length of real waves reaches hundreds of kilometers, and the ocean depth — ten thousand meters. Therefore a maximum to be achieved is creation of a water flume that is one thousand times smaller than the real scale of this phenomenon. It is necessary to notice that in the pools of hundreds meters in size, the researches into the long wave impact onto the coast is carried out. In particular, one of the tasks is studying the beach erosion process by storm waves with the allowance for the breakwaters consisting of concrete (Figure 2). In these experiments, reduced copies of these breakwaters of 5–10 cm in size are used (see Figure 1). Actually, their size reaches several meters (Figure 2). Thus, in this case a linear scale of the experiment makes up about 1 : 50–1 : 100.

Another example of a laboratory simulation of the 3D dry land inundation by tsunami waves is presented in Figure 3. Here in a small (150×200 cm) water flume the bottom and coastal relief of the southern part of the Okushiri island was reconstructed and the process of shore inundation was studied. Here the horizontal scale factor is about 1 : 1000. For such a kind of research in Saint-Petersburg (Russia) there was built a water flume which is ten times larger (100 × 100 meters).

Rather often the research into the wave motion and transformation is carried out in long (up to several tens meters) pools, wide up to several tens of centimeters. In this case the dynamics of movement of one-dimensional waves in various modes is studied. In all its full length numerous sensors recording parameters of a moving wave are fixed, also, video and photographing are provided. An example of such a “one-dimensional” water flume used in one of laboratories in Japan can be seen in Figure 4. Its length is about 10 meters and the width being 20 cm. Another example of a wave flume for the long wave run-up research is presented in Figure 5. Here


**Figure 2.** Concrete breakwaters which are used for beach protection in Japan

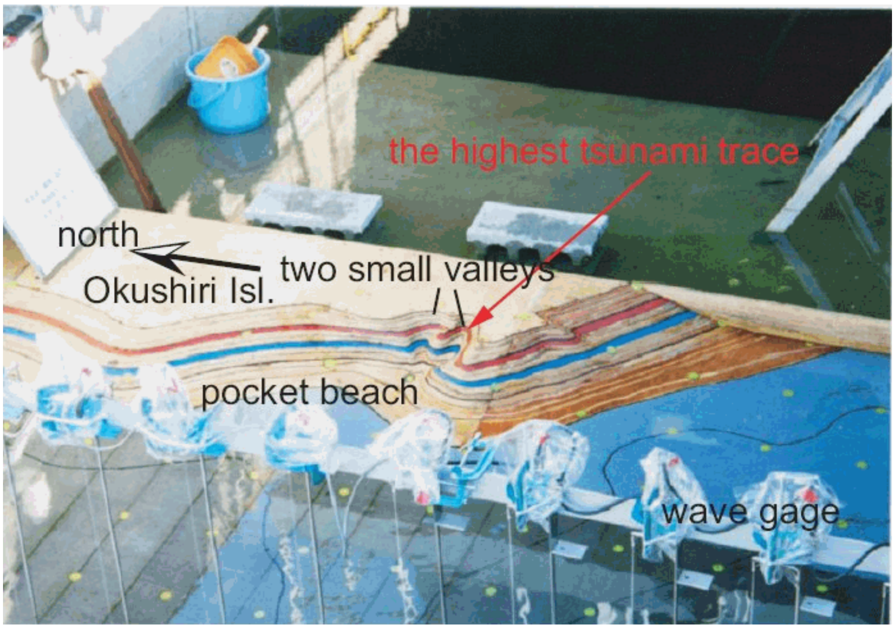


**Figure 3.** The water pool for studying the Okushiri island inundation by tsunami

**Figure 4.** The small one-dimensional water flume in one of laboratories in Japan

**Figure 5.** One-meter wave propagation along the 200-meter long flume

the width as well as the height of 200-meter long flume are about 6 meters. This wave flume permits studying long waves being 1/10 of a real tsunami height.

Now, let us discuss the selection of the similarity coefficients, allowing the application of results of modeling in reduced scale to real big-size natural phenomena. Such an analysis will be carried out on an example of modeling of one-dimensional long-wave propagation in experimental water flumes. This process is well described by system the system of nonlinear shallow-water equations [1]

$$\begin{aligned} &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = 0, \\ &\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\big(u \cdot (\eta + D)\big) = 0. \end{aligned} \tag{1}$$

Here $u$ is the water stream velocity, $\eta$ is the water surface elevation above the still water level, g is the acceleration of gravity, D is depth. It is also necessary to notice that the flow velocity $u$ and elevation $\eta$ in such model are connected by the ratio [1]:

$$u \approx \eta\sqrt{\frac{g}{D}}. \tag{2}$$

Let the behavior of a wave in the flume be investigated, thus all spatial parameters (wave length, its amplitude, depth of the pool and the size of bottom relief roughness) are chosen $n$ times smaller, than in the case of a real tsunami wave propagation. As long waves in the flume also propagate according to the same laws, the parameters of waves there satisfy system of equations (1). Let us clarify, whether parameters of a "magnified" tsunami wave will satisfy the same system of equations whose behavior was required to simulate in this “small” pool. As was already noticed, all linear sizes and parameters will be $n$ times greater, and from ratio (2) it follows that for the flow velocity this coefficient will be equal to $\sqrt{n}$. After substitution of new parameters values in equations (1), we obtain

$$\begin{aligned} &\frac{\partial u}{\partial t} \cdot \sqrt{n} + u\frac{\partial u}{\partial x}\frac{\sqrt{n}\cdot\sqrt{n}}{n} + g\frac{\partial \eta}{\partial x}\frac{n}{n} = 0, \\ &\frac{\partial \eta}{\partial t} \cdot n + \frac{\partial}{\partial x}\big(u \cdot (\eta + D)\big) \cdot \frac{\sqrt{n}\cdot n}{n} = 0. \end{aligned} \tag{3}$$

Obviously, if equations (1) are fulfilled, equations (3), representing the original equations (1) for “a big wave”, can never be fulfilled. This means that wave parameters in the flume should not be connected with parameters of a simulated “big wave” by such simple ratio. Let us consider the situation when in a water flume the vertical size of an initial wave is $n$ times reduced, and the horizontal one — $\sqrt{n}$ times. Really, if in system of differential equations (1) $x$ is increased $\sqrt{n}$ times, $\eta$ and $D$ is $n$ times as much, and the water flow velocity $u$ according to (2) is increased $\sqrt{n}$ times. Then equations (1) can be written down as

$$\begin{aligned} &\frac{\partial u}{\partial t} \cdot \sqrt{n} + u\frac{\partial u}{\partial x}\frac{\sqrt{n}\cdot\sqrt{n}}{\sqrt{n}} + g\frac{\partial \eta}{\partial x}\frac{n}{\sqrt{n}} = 0, \\ &\frac{\partial \eta}{\partial t} \cdot n + \frac{\partial}{\partial x}\bigl(u\cdot(\eta + D)\bigr) \cdot \frac{\sqrt{n}\cdot n}{\sqrt{n}} = 0. \end{aligned} \tag{4}$$

It is clear that the first equation in (4) is the first equation of system of differential equations (1), multiplied by a constant value $\sqrt{n}$, and the second equation of system (4) is nothing but the second equation of initial system (1) multiplied by $n$. Naturally, this means that the shallow-water model is also valid for the description of a "big" wave, which is obtained from a "small" wave by magnification of the horizontal size by $n$ and the vertical size by $\sqrt{n}$. It should be noted that all these conclusions also hold for a two-dimensional case with only difference that all horizontal parameters of the initial process (a real tsunami) should be $\sqrt{n}$ times reduced along both directions.

As a result it is possible to conclude that if we want to study the dynamics of long waves (in particular, tsunami waves) in water flumes of small sizes, the similarity coefficient (a scale coefficient) for the vertical parameters of a problem (depth and wave height) should be equal to a squared coefficient of similarity of horizontal parameters (the sizes of the flume and bottom relief roughness). After carrying out a laboratory experiment and measuring the wave parameters in the water flume it is required to make the inverse transformation: to multiply the wave length by $\sqrt{n}$, and the amplitude by $n$.

## References

[1] Le Mehaute B. An introduction to hydrodynamics and water waves. — Miami, Florida, July 1969. — (ESSA Technical report / Pacific oceanographic laboratories; ERL 118-POL 3-1).