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Modeling of Stoneley wave generated by seismic vibrators

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The paper deals with mathematical problems of physical phenomena, connected with generation, long-distance propagation, and interaction of the acoustic and the seismic waves from powerful vibrational sources operating on the interface of the elastic Earth–atmosphere. Three problems of mathematical modeling are considered: modeling of acoustic waves radiation of a vibrational source, modeling of the long-distance propagation of acoustic waves in a low velocity near- surface air channel, and modeling of an acoustoseismic induction process. The conditions of the resonance increase on the amplitude of the induced surface seismic wave are determined.

1. Introduction

Experimental survey with the use of powerful seismic vibrators has allowed us to learn the processes of radiation and interaction of the acoustic and seismic fields generated by powerful vibroseismic low-frequency sources. The excitation effect of acoustic waves and subsequent induction of surface waves from powerful vibrational sources at large distances between a radiator and a receiver was experimentally detected. When operating powerful seismic vibrators, an infrasonic acoustic wave is radiated simultaneously with seismic waves. In the presence of the near-surface sound channel, this wave can propagate to a distance of tens kilometers and induce surface seismic waves. These waves are recorded by seismic receivers together with radiated seismic waves from vibrators. The propagation of acoustic waves of infrasonic frequencies (6–7 Hz) up to distances of 20–50 km is possible due to the phenomenon of refraction of sonic waves in the atmosphere and occurrence of a near-surface wave channel.

When studying acoustoseismic effects, caused by operation of powerful seismic vibrators, one can distinguish the following three interrelated processes: 1) radiation of acoustic waves by a vibrational source on a free surface, 2) propagation of acoustic waves from a vibrational source to long distances over the Earth's surface, and 3) excitation of surface seismic waves by a harmonic acoustic wave coming to the recording point [1, 2].

2. Modeling of the acoustic waves radiation by a vibrator

A variety of papers are devoted to the modeling of the infrasonic waves radiation into the atmosphere while the vibrational sources are running [3, 4]. Two homogeneous (elastic and gas) media, contacting along a plane boundary, are taken as a model. The dynamic elasticity equations with constant characteristics (density and velocities of longitudinal and transverse waves) are solved for an elastic half-space, and the wave equation with a constant density and the sound speed are solved for a gas medium. Boundary conditions are the equality of the normal components of stresses and velocities at the interface between the two media. A point harmonic force that acts normally to the interface is considered as a source. The asymptotics of the acoustic and the seismic body waves in the far zone and the corresponding radiation powers were found in [4]. The solution is given in the form of integral representations, and for some relations between parameters of the media, it admits the following analytical representation:

$$W_a = 3.16\frac{\rho}{\rho_1}\frac{P^2\omega^2}{\pi\rho_1 V_p^3}; \quad W_s = 0.085\frac{P^2\omega^2}{\pi\rho_1 V_p^3}; \tag{1}$$

where P is the amplitude of the force, ρ is the frequency, c and c are the density and the speed of sound in the gas, and ρ_1, V_p, and V_s are the density and the velocities of longitudinal and transverse waves in the elastic half-space. The formulas are valid for the velocity relations $V_p = \sqrt{3}V_s = \sqrt{3}c$.

The relation between the powers of seismic and acoustic waves does not depend on frequency and $W_a/W_s = 0.0186$, that is, approximately only 2% of the total radiation power is transferred by acoustic waves. Numerical values for a vibrational source with the amplitude of force $P = 100$ tons, the density and the sound speed in the gas $\rho = 1.2$ kg/m^3 and $c = 340$ m/s, and the density and the velocity of longitudinal waves in the ground $\rho_1 =$ 2 000 kg/m^3 and $V_p = 588$ m/s are $W_s = 94$ W and $W_a = 2.1$ W with a frequency of 6 Hz, and $W_s = 261$ W and $W_a = 5.8$ W with a frequency of 10 Hz, respectively.

A beam pattern of the acoustic radiation of vibrational source considerably changes if a low-velocity air layer occurs near the surface, which takes place in the atmosphere in the case of temperature inversion and occurrence near the surface of a cool air layer with a velocity lower the sound speed. Let us consider the formation of a beam pattern of a point source near the surface in the ray approximation.

Let a gas medium with a low-velocity layer $0 < z < h$ of thickness h and the velocity of sonic waves c1 be above an elastic half-space $z < 0$ of a cylindrical system of coordinates r, φ, z. Above the gas medium, let there

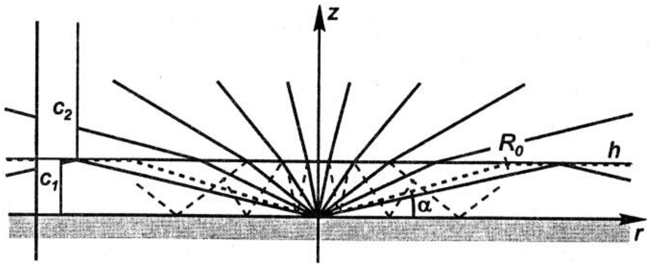

Figure 1. Acoustic rays propagation with a near-surface low-velocity air level

be a half-space with the velocity of sonic waves c_2. The gas density in the entire gas medium, ρ, is the same (Figure 1). The point source at the point $z = r = 0$.

In the ray approximation, a wave field is formed of the rays originating from the source and the rays refracted and reflected from the media interfaces. It is known that in the presence of a low-velocity layer, there is an angle of limiting reflection, and all the rays below this angle have a complete internal reflection in the layer. The value of this angle is determined from the Snell's law, and depends on the relation between the velocities c_1 and c_2. The acoustic field at large distances from the source is formed of waves with a slip angle less than critical. They have the complete internal reflection in the layer. The passed waves are inhomogeneous with a real wave vector in the radial direction and an exponential decrease in the amplitude along z-axis. The part of acoustic energy getting into the wave channel is determined in the ray approximation by the ratio between the solid angle in which waves have a supercritical reflection and the solid angle of the hemisphere:

$$\frac{W_k}{W_a} = \frac{h}{R_0} = \sin\alpha \approx \sqrt{\frac{2(c_2 - c_1)}{c_2}}, \tag{2}$$

where W_a and W_k are the total power of the acoustic radiation and the radiation power kept in the wave channel, and R_0 is a radius from the origin of coordinates to the point of contact of the wave of the critical reflection angle of the layer upper boundary. It is seen from (2) that the value of wave energy in the layer does not depend on its thickness, but is determined only by the relation between the sound speeds.

In spite of the fact that with an insignificant difference in the sound speeds the coefficient in (2) is small, the presence of a wave channel has an effect on amplitudes of the acoustic wave at large distances from the source. If we compare the amplitude of the acoustic wave at the distance R in the half-space without channel with its amplitude in the channel, then, with allowance for the cylindrical symmetry in the channel, we will obtain:

$$W_a = A_0^2(R) \cdot 2\pi R^2; \quad W_k = A_k^2(R) \cdot 2\pi Rh;$$

$$\frac{A_k^2(R)}{A_0^2(R)} = \sqrt{\frac{2(c_2 - c_1)}{c_2} \frac{R}{h}}. \tag{3}$$

Here A_0 is the acoustic wave amplitude at the distance R in the half-space, A_k is the amplitude of the wave in the channel at the distance R, and R_0 is a radius to the channel formation point.

Table 1

ΔT, °C	c_2, m/s	h, m	α, °	W_k/W_a	R_0, m	A_k/A_0
5	334	10	7.8	0.13	75	19.9
5	334	25	7.8	0.13	188	12.6
5	334	50	7.8	0.13	376	8.9
5	334	100	7.8	0.13	752	6.3
10	337	10	11	0.19	53	23.6
10	337	25	11	0.19	133	14.9
10	337	50	11	0.19	267	10.5
10	337	100	11	0.19	534	7.4

The numerical values of velocities, the relations between the powers and amplitudes at a distance of 30 km from the source for the temperature differences in the layer of 5 and 10°C, and three values of layer thickness of 25, 50, and 100 m are presented in Table 1. The temperature coefficient of the sound speed in the air is 0.59 m/s·deg, and the velocity c_1 at 0 degrees is 331 m/s. It is seen from the table that with a temperature variation of 5–10 degrees, the angle of limiting reflection is 8–11 degrees, and the wave channel can include from 13% to 20% of the acoustic radiation power. This is essential if we take into account the fact that this power is retained in the channel. It follows from the geometry of the problem and expressions (1) that the acoustic power value in the layer does not depend on its thickness. It is determined from a difference in velocities (or temperatures). It should be noted that the ratio between the amplitudes at the recording point in the presence or absence of the wave channel has larger values at a smaller thickness of the layer. This is explained by the fact that the smaller the layer thickness, the shorter the distance to the channel formation point (R_0). Therefore, at smaller distances the law of a decrease in amplitudes that corresponds to the cylindrical geometry of the channel ($1/\sqrt{R}$) becomes valid, instead of $1/R$ for the spherical geometry without channel. This explains the well known from acoustics fact of a rapid appearance of a near-surface wave channel with insignificant cooling at the surface. The same fact is observed in the experiments with the above presented vibrators.

3. Modeling of acoustic wave propagation in a near-surface wave channel

The propagation of acoustic waves from a vibrator to large distances is associated with the existence of waves in a channel that conserve their energy without radiation into the above half-space. The problem of modeling is simplified by the fact that at large distances from the source the spherical wave field is locally plane and admits 2D modeling.

We consider a plane problem for the above model of a gas half-space with a low-velocity layer on a rigid half-space. The gas medium occupies the upper half-space $0 < z$ and contains a low-velocity layer $0 < z < h$ of thickness h. The velocity of sonic waves in the layer is c_1, and in the half-space above the layer c_2, and the gas densities in the layer and half-space are the same, ρ.

The wave equations for the pressure in the layer and the half-space, as well as the relation between the velocities and the pressures have the following form:

$$\begin{aligned} &\frac{1}{c_1^2}\frac{\partial^2 p_1}{\partial t^2} - \Delta p_1 = 0, \quad \rho\frac{\partial \vec{u}_1}{\partial t} + \nabla p_1 = 0, \\ &\frac{1}{c_2^2}\frac{\partial^2 p_2}{\partial t^2} - \Delta p_2 = 0, \quad \rho\frac{\partial \vec{u}_2}{\partial t} + \nabla p_2 = 0, \end{aligned} \tag{4}$$

where $p_1(z,x,t)$ and $p_2(z,x,t)$ are the pressure in the layer and the half-space, and $\vec{u}_1(z,x,t)$ and $\vec{u}_2(z,x,t)$ are the velocity vectors in the layer and the half-space. Boundary conditions are as follows: the equality to zero of the normal velocity at the boundary with the rigid half-space; and the equality of pressures and normal velocities in the first and the second media:

$$\begin{aligned} &u_{z1}(z,x,t)\big|_{z=0} = 0, \\ &u_{z1}(z,x,t)\big|_{z=h} = u_{z2}(z,x,t)\big|_{z=h}, \\ &p_1(z,x,t)\big|_{z=h} = p_2(z,x,t)\big|_{z=h}, \end{aligned} \tag{5}$$

where $u_{z1}(z,x,t)$ and $u_{z2}(z,x,t)$ are the vertical velocity components.

The solution can be represented in the form of the superposition of two plane homogeneous waves in the layer with real wave vectors and an inhomogeneous wave in the upper half-space with a real wave vector in the direction of x-axis and an exponential decrease in the amplitude along z-axis:

$$\begin{aligned} &p_1(z,x,t) = \exp(i\omega t - ikx)\left(a_1\exp(-ik_1 z) + a_2\exp(ik_1 z)\right); \\ &p_1(z,x,t) = b_1\exp(i\omega t - ikx)\exp(-\alpha z), \end{aligned} \tag{6}$$

where a_1 and a_2 are the plane waves amplitudes in the layer, b_1 is the amplitude of the wave in the half-space $h < z$, ω is frequency, k is the projection of the wave vector to x-axis, k_1 is the projection of the wave vector to z-axis, and α is the attenuation coefficient in the direction of the z-axis in the upper half-space.

From wave equations (4), with allowance for the boundary conditions (5), and the form of the solution, we obtain the relation between the wave vectors and the frequency (the dispersion equations being typical of wave channels):

$$\frac{\omega^2}{c_1^2} = k^2 + k_1^2, \quad \frac{\omega^2}{c_2^2} = k^2 - \alpha^2, \quad \frac{k_1}{\sqrt{\frac{\omega^2(c_2^2-c_1^2)}{c_1^2c_2^2} - k_1^2}} = \operatorname{ctg} k_1 h. \qquad (7)$$

Equation (7) is numerically solved. The results of calculations for the parameters of the layer from Table 1 and a frequency of 6 Hz are presented in Table 2.

Table 2

ΔT, °C	c_2	h, m	k_1, m^{-1}	α, m^{-1}	π/k_1, m	$1/\alpha$, m
5	334	10	0.015	0.002	417	441
5	334	25	0.014	0.005	440	188
5	334	50	0.012	0.008	506	113
5	334	100	0.009	0.012	681	82
10	337	10	0.021	0.004	300	225
10	337	25	0.019	0.009	330	102
10	337	50	0.015	0.014	408	67
10	337	100	0.010	0.018	595	53

It is seen from Table 2 that a nearly plane wave is formed in the layer with small velocity differences. The values of the wave number k_1 and the attenuation coefficient α are much smaller than the horizontal wave number $k = 0.113$ m^{-1}, and are close in magnitude to each other. The corresponding wavelengths and typical sizes also significantly differ. On the whole, the following regularities are observed: the higher the velocity difference in the layer and space and the thicker the layer, the less the penetration of an inhomogeneous wave into the half-space (the parameter $1/\alpha$). The quantitative estimates show that with the considered temperature variations of 5–10 degrees, the acoustic field energy is concentrated in an area of 100–200 m above the Earth's surface, in the low-velocity layer, and in the half-space near the layer interface. The maximum energy density is concentrated in the low-velocity layer, and the maximum amplitude of pressure in the acoustic wave is attained at the lower boundary of the layer on the elastic half-space surface.

4. Modeling of the acoustoseismic induction process

Solution (6) shows that a nearly plane wave propagates in the low-velocity layer with the velocity close to the sound speed in the air. The sonic wave action in the layer on the underlying elastic half-space implies that a pressure wave runs along the interface $z = 0$ (see (6)), thus causing a deformation wave. The problem of excitation of a surface seismic wave in an elastic half-space under the action of the acoustic wave in the air can be considered in the elastic half-space model with a free boundary, at which the normal stresses in the form of a running pressure wave are specified [5].

Let us consider a plane problem for a homogeneous isotropic elastic half-space $z > 0$, with the parameters λ, μ, and ρ. The acoustic wave that propagates along the boundary in the direction of x-axis is taken into account in the form of the boundary conditions for normal stresses on the surface of the elastic half-space ($z = 0$), and the harmonic acoustic wave with a constant velocity c, equal to the sound speed in the air, is described by the amplitude of the pressure p and the frequency ω.

Let us solve the Lame equations with the boundary conditions

$$(\lambda + \mu)\,\mathrm{grad} \cdot \mathrm{div}\, u + \mu \Delta u - \rho \frac{\partial^2 u}{\partial t^2} = 0, \tag{8}$$

$$t_{xz}\big|_{z=0} = 0, \quad t_{zz}\big|_{z=0} = p \exp i(\omega t - kx), \tag{9}$$

where $k = \omega/c$ is the wave number of the acoustic wave.

The solution to problem (8) with boundary conditions (9) can be represented in the form of plane waves. With the notation $\gamma = v_s/v_p$ and $\theta = c/v_s$, we can write down the solution for the components of the displacement field u_x and u_z in the following form:

$$\begin{aligned} u_x &= -ikC\Big[(2-\theta^2)e^{-kz\sqrt{1-\gamma^2\theta^2}} - \\ &\qquad 2\sqrt{1-\theta^2}\sqrt{1-\gamma^2\theta^2}\,e^{-kz\sqrt{1-\theta^2}}\Big]e^{i(\omega t - kx)}, \\ u_z &= kC\sqrt{1-\gamma^2\theta^2}\Big[(\theta^2-2)e^{-kz\sqrt{1-\gamma^2\theta^2}} + 2e^{-kz\sqrt{1-\theta^2}}\Big]e^{i(\omega t - kx)}, \end{aligned} \tag{10}$$

where

$$C = \frac{p}{k^2 \rho v_s^2 R(\theta)}, \quad R(\theta) = (2-\theta^2)^2 - 4\sqrt{1-\gamma^2\theta^2}\sqrt{1-\theta^2}, \tag{11}$$

$$k_{\varphi z} = k\sqrt{\frac{c^2}{v_p^2} - 1}, \quad k_{\psi z} = k\sqrt{\frac{c^2}{v_s^2} - 1}. \tag{12}$$

The solution (10) depends on the relation between the velocities of longitudinal and transverse waves in the solid Earth and the velocity of the acoustic wave. We can distinguish three domains of parameters that determine different types of solutions.

Domain I. $0 < c < v_s$, $\gamma\theta < \theta < 1$ is the acoustic wave propagation above a half-space with greater velocities of longitudinal and transverse waves than the sound speed in the air. In this case, a surface wave is induced in the half-space, which propagates with the acoustic wave velocity. The amplitudes of the displacement field exponentially decrease at $z > 0$, and there is no energy flow in the direction of the axis $z > 0$. The induced surface wave is elliptically polarized. In this domain of parameters, there are values of v_s and v_p, at which the sound speed coincides with the velocity of the surface Rayleigh wave. The solution (10) has a singularity at this point, because the Rayleigh function in the denominator of the coefficient C becomes equal to zero. As the parameters of the half-space approach these values, an unlimited increase in the amplitude of the displacement field takes place. Physically, this corresponds to the resonant excitation of the surface wave with a constant energy replenishment from the acoustic wave.

Domain II. $v_s < c < v_p$, $\gamma\theta < 1 < \theta$: the wave number $k_{\psi z}$ is real. The solution (10) is the superposition of the following two wave processes: a surface wave that propagates with the velocity c along x-axis and has exponential attenuation of amplitude with depth, and a transverse wave of constant amplitude with the wave vector $(k, k_{\psi z})$ that moves downward. The polarization at the surface of the half-space remains elliptic with a variable inclination of the ellipse.

Domain III. $v_p < c$, $1 < \gamma\theta < \theta$: the wave numbers $k_{\varphi z}$ and $k_{\psi z}$ are real, and the solution (10) is a superposition of the longitudinal and the transverse waves of constant amplitudes with the wave vectors $(k, k_{\varphi z})$ and $(k, k_{\psi z})$, which propagate at different angles to the free surface and transfer the energy in the direction of the wave vectors. The polarization of the displacement field on the half-space surface represents degenerate ellipses with a variable inclination.

It can be concluded that as the acoustic wave propagates above a "rigid" half-space (with high values of the velocities of longitudinal and transverse waves with respect to the velocity c), a surface wave propagating with the sound speed in the air is induced. For a half-space with the velocity value of the Rayleigh wave equal to the sound speed in the air, the resonant absorption of the acoustic wave energy and resonant swinging of the amplitude of the surface wave propagating along x-axis, take place. In the case of a "soft" half-space (with low values of v_p and v_s with respect to c), both the

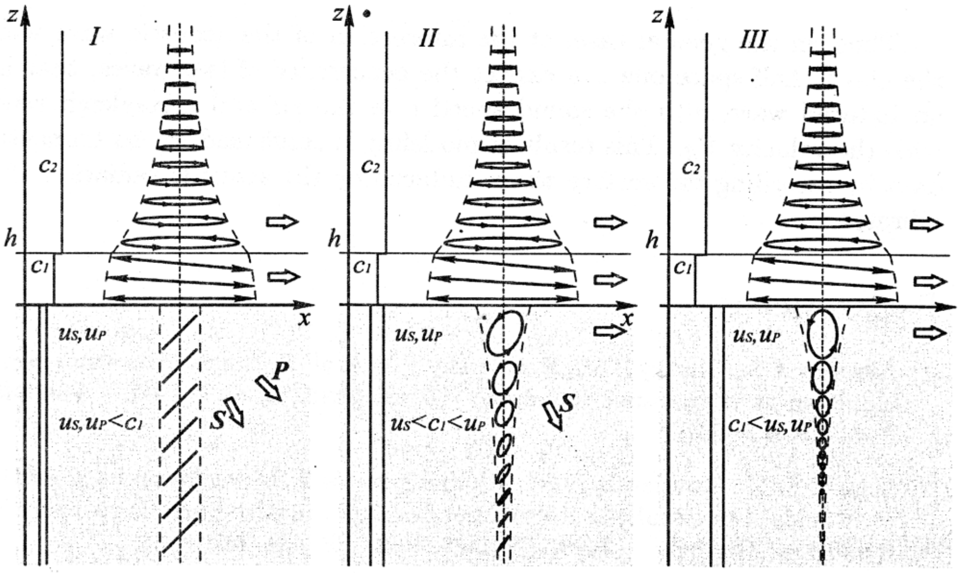

Figure 2. Polarization curves in the air with a low-velocity layer near the surface and inside the elastic half-space

surface wave and the waves propagating at an angle to the free surface and transferring the energy from the acoustic wave to the half-space are induced.

Polarization curves in the air with a low-velocity layer on the surface obtained from the solution (6) and in the elastic half-space obtained from the solution (10) for the three considered domains of the relation between the sound speed in the layer and the velocities of longitudinal and transverse waves are shown in Figure 2. A plane wave with the linear polarization propagates in the low- velocity layer. In the upper half-space, polarization is elliptic with a small value of Z-axis of ellipses and the exponential vertical attenuation. In the elastic half-space, polarization changes from elliptic (Domains I and II) to linear (Domain III). Only Domain I corresponds to the stationary wave in the direction X. It can be considered as a version of the Stoneley wave in the presence of a low-velocity gas layer at the boundary of the elastic half-space. In the two other regions, there are plane waves propagating downward at different angles.

In conclusion, it should be noted that the superposition of the solutions of the inhomogeneous system of equations with a non-zero right-hand side and the homogeneous one with a zero right-hand side is the solution to problem (8) with the boundary conditions (9). The solution of the inhomogeneous system is given above. It describes an induced surface wave propagating with the sonic wave speed c along the surface. It is known that the Rayleigh wave is the solution to the homogeneous system of equations.

Thus, in the general case, of the interaction of the acoustic wave with the elastic half-space one can expect the occurrence of two waves, that is, an induced wave with the sound speed c in the air and a Rayleigh wave with the velocity V_R. This result of modeling is confirmed by experimental data by recording the surface waves induced by the acoustic radiation of a vibrator.

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