Bull. Nov. Comp. Center, Math. Model. in Geoph., 6 (2000), 17–29

Optimization problem in vibroseismic

M.S. Khairetdinov

Currently, vibroseismic method for the Earth's Sounding (VES) is in most common use in solution of a great number of the geophysical tasks, such as vibroseismic monitoring of seismic-prone zones, seismic tomography, calibration sounding of seismic traces and seismic stations for increasing of the accuracy of definition of the coordinates of earthquakes and nuclear explosions, etc. The solution to such problems is connected with considerable expenditures of energy consumption exceeding 6×10^6 wt-hours with sounding at distances greater 1000 km. This follows from covering the given graph of damping of amplitudes of the waves P depending on the distance [1] with the values of the energy, consumed by the vibrator for separation of the wave P in the vibrational seismogram obtained for a distance of 320 km. The wave amplitude exceeds the mean quadratic value of noise by the factor of 5–6 in this case. Based on the well-known damping law it is possible to assess the consumed energy depending on distance for the proper separation of the wave P. This is illustrated in Figure 1. The minimization of energy consumption with the given quality of operation of the system VES is a permanently actual task. In this paper, the ways of solution of the formulated problem by methods of optimization of structure and parameters of the system VES are analyzed.

1. Statement of the problem

In the generalized form, the process of vibrosounding of the Earth can be described by the equation

$$U(x,y,z,t) = \int_0^T k(t,\tau,\bar{x},\bar{y},\bar{z})S(t-\tau)\,d\tau + n(t) = \nu(t) + n(t), \qquad (1)$$

here $k(t,\tau,\bar{x},\bar{y},\bar{z})$ is an impulse function of the medium for the coordinates of a radiator and the receiver $(\bar{x},\bar{y},\bar{z})$; $S(t)$ is a sounding signal, $n(t)$ is the noise at the recording a point, $\nu(t)$ is a response of the medium in the form of a seismogram to the sounding $S(t)$. The quality of measurement of the seismogram parameters against the multiply exceeding noise can be described by the signal/noise ratio in the form:

$$\gamma^2_{P,S} = \frac{\int_\Omega |\nu(\omega)|^2_{P,S}\,d\omega}{\int_\Omega N(\omega)\,d\omega} \qquad (2)$$

Here, the numerator represents the energy of the waves P and S in the sounding frequency band Ω, the denominator represents the energy of noise

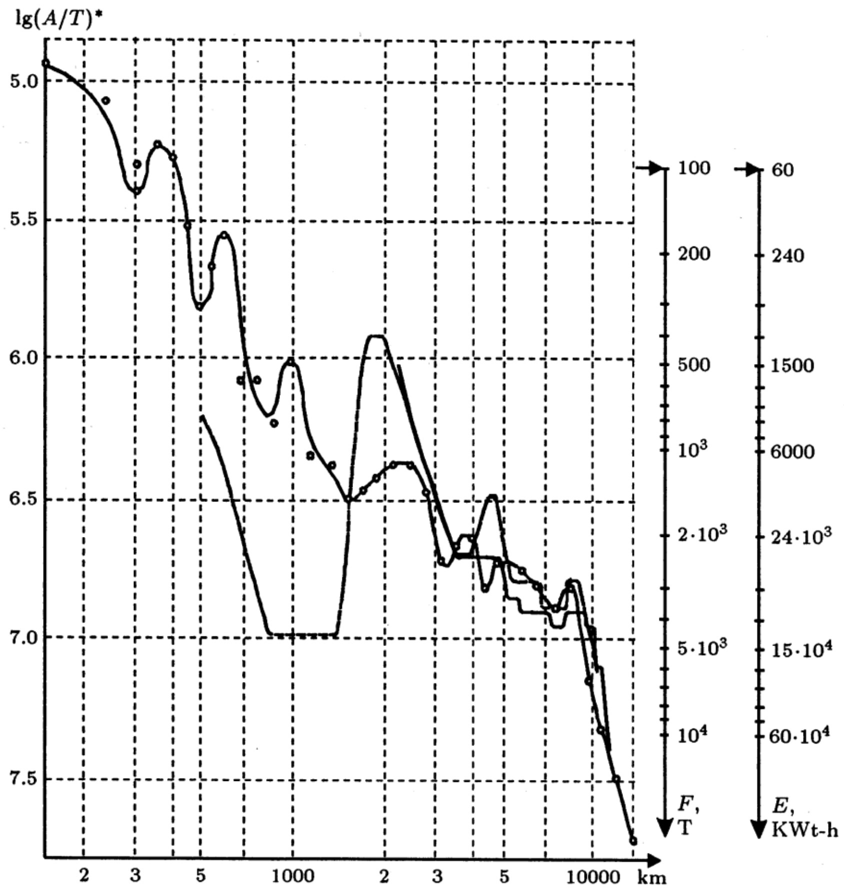

Figure 1. Selection of perturbing forces and energy for the vibrator with allowance for distance

in the same band. The parameter γ^2 is, in general case, a functional of a number of parameters of the system VES:

$$\gamma^2 = f\Big(E_{P,S},\ r,\ k(t,\tau),\ R_S(\tau),\ R_n(\tau),\ L[U(t)]\Big). \tag{3}$$

Here, $E_{P,S}$ are energies of the seismic waves P and S at the radiation point; r is the "radiator–receiver" distance, $R_S(\tau), R_n(\tau)$ are parameters of a sounding signal and noise represented as appropriate correlation functions, $L[U(t)]$ is the operator of the processing of recorded signals.

Mathematically, the problem of optimization is formulated as attaining a certain maximum:

$$\gamma^{*2} = \max(\gamma^2) \quad \text{for} \quad E = \text{const.}$$

This means obtaining the maximum of the noise stability of the system VES with constant of energy consumption E by the vibrator. In a general case, complexity of the solution of this optimization problem in the closed form is defined by complexity of a model of the 3D medium. The difficulties of calculation of a wave field (1) in this case are well-known.

The other simpler way of solving this problem is the following. From equation (1) follows that if we have the impulse function of the medium $k(t, \tau)$ for the given coordinates $(\bar{x}, \bar{y}, \bar{z})$ of "radiator-receiver", the response of the medium $\nu(t)$ to the effect of the long vibroseismic signal $S(t)$ can be defined as convolution (1). Thus, as $k(t, \tau)$, we may take a seismogram of an explosion which can be considered as the affect on the medium with δ-impulse. If we take into account the instrumental function of a source, the accuracy of determination of the response of the medium will be sufficiently high. High values of the coefficients of the spatial correlation of the waveform P, observed at large distances [2, 3], allow us to speak about generality of the given approach within the areas, where a system of sensors with linear sizes of tens km is located.

A simplified expression for γ^2 can be obtained for a model of sounding as a press affecting a quasihomogeneous half-space with the force F and the frequency ω_0. For this case, the closed expressions for powers of the seismic waves P, S, and R have been obtained, over which the above-said complete seismic power at the radiation point is distributed [4]. In view of the above-said, it is possible to determine the signal/noise ratio for certain types of waves at the recording points. In particular, for the waves P and S we obtain

$$\gamma = A_0 k_{p,s} r^n \sigma_n^{-1} F f_0 \sqrt{T} \exp(-\alpha r), \tag{4}$$

where A_0 is the signal amplitude in the point of sounding, σ_n is the mean square of the noise in the frequency band of sounding, $k_p = \sqrt{0.0852/(\pi\rho V_P^3)}$ for the longitudinal waves, $k_s = \sqrt{0.299/(\pi\rho V_S^3)}$ for the transverse waves [4], ρ is the density of the medium under a vibrator, T is the duration of sounding, $n = -1$ for a body wave, $\alpha = 2.5\times10^{-4} f_0\ \mathrm{s}^{-1}\mathrm{km}^{-1}$, V_P and V_S are velocities of the longitudinal and the transverse waves in the medium under a vibrator. Based on numerous data on recording the nuclear explosions [5] and signals of the VES [6] it is shown that equation (4) with a required approximation can be used for evaluation of the characteristics of the system of the VES at distances of recording up to 1000 km.

For a real medium, the maximization of functional (3) can be attained at the expense of the following resources:

- resonance coordination of a vibrator with a medium described by a multilayer model. It makes it possible to rise by one order and higher the efficiency of conversion of the energy of consumption to the energy of seismic waves. The experiments with the vibrator CV-100 have fully confirmed such a possibility;

- consideration of frequency dependent properties of the wave propagation medium and microseisms spectra;
- optimization of the algorithm of processing $L[u(t)]$;
- optimization of selection of the forms of sounding signals (with frequency and phase modulation, monochromatic signals).

Let us dwell on the enumerated possibilities and the ways of their implementation.

2. The resonance coordination of a vibrator with a medium

This problem has been studied since the beginning of development of vibrators for the deep seismic sounding of the Earth as in this case, as opposed to the exploratory seismic vibrators there arose a requirement to provide the increased radiation power in the seismological range of frequencies. One of the main reserves of rising the radiated seismic power is connected with implementation of the resonance generation scheme in the vibrator – medium circuit [7]. Theoretically, it was shown that at the expense of the choice of the underlying surface with the given characteristics of elasticity, the multiple increment of the radiated seismic power of a vibrator with constant consumption power is possible [8, 9]. In particular, the radiation of a stamp with the radius $r - 0$ on the two-layer half-space with the uniform distribution of the exciting vertical harmonic force F was considered. The medium represents a layer of thickness H with the parameters ρ_1, V_{P1}, V_{S1} located on the elastic half-space with the parameters ρ_2, V_{P2}, V_{S2}.

The full seismic power W_z, radiated by a stamp depending on dimensionless frequency of a layer is $\omega R_0/V_{S1}$ with $H/a = 1$ and $V_{S1}/V_{P1} = 0.5$ for different parameters of a layer and a half-space, is shown in Figure 2. Here the power is normalized to the coefficient $F^2/(2\rho_1 S V_{S1})$ which is equal to the power of a plane wave. Here $S = \pi a^2$ is the square of the stamp. As follows from a graph a soft stratum on a rigid half-space, the seismic power, radiated by the stamp in to the medium, has its maximum on the resonance frequency of the stratum equal to π. The radiated power one order increasing with the increase of the rigidity characteristic of the half-space.

With the increase of the layer thickness H/r_0, a maximum of the seismic power shifts to the area of lower frequencies and takes its place on the frequency $\pi/2$ for $H/r_0 = 2$, etc.

With the increase of number of stratums of spreading space under a vibrator, the curve of power of a radiation gains a multiresonance character.

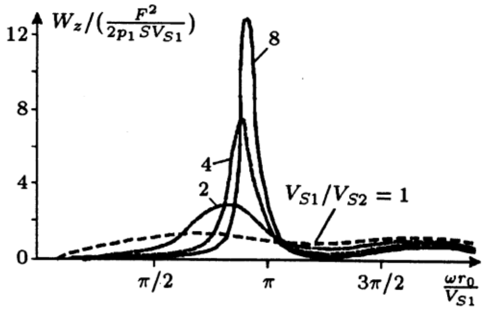

Figure 2. Calculated resonance characteristics of radiation witt allowance for the ratio between the transverse waves velocities of the underlying medium

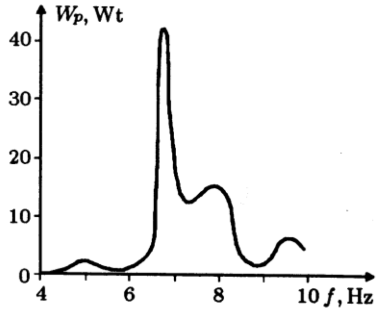

Figure 3. Resonance characteristics of the vibrator CV-100 on the Bystrovka polygon

In particular, for emanating a polygon Bystrovka with a successively increasing width of stratums $H_1 = 10$ m, $H_2 = 150$ m, $H_3 = 480$ m, $H_4 = 2000$ m, the curve of the radiation power W_p of waves P depending on frequency has a multiresonance character represented in Figure 3. The main peak of radiation corresponds here to 6.95 Hz and is determined by a set of all the layers [10].

3. The account of frequency dependent properties of the medium of the wave propagation and characteristics of microseisms

For the above mentioned model of a quasihomogeneous half-space, sufficient for obtaining approximate estimations of the oscillations levels at distances up to 1000 km, the frequency characteristic of the medium is described by the expression (cf. [11])

$$G(\omega) = \frac{A_0}{r} \exp\left(\frac{-\alpha r \omega}{2\pi}\right). \tag{5}$$

For distances of more than 1000 km, the frequency characteristic of the medium is approximately described by the Tseppritc–Vihert formula [5]. The natural seismic noise is characterized by the monotone increase towards low frequencies (Figure 4a). Based on the analysis of the power spectrum of noise in the frequency band 1–20 Hz at a number of seismostations its description was obtained in the form [12]:

$$N(\omega) = D \exp\left(\frac{-\beta\omega}{2\pi}\right)\left(1 + \frac{\omega}{2\pi}\right). \tag{6}$$

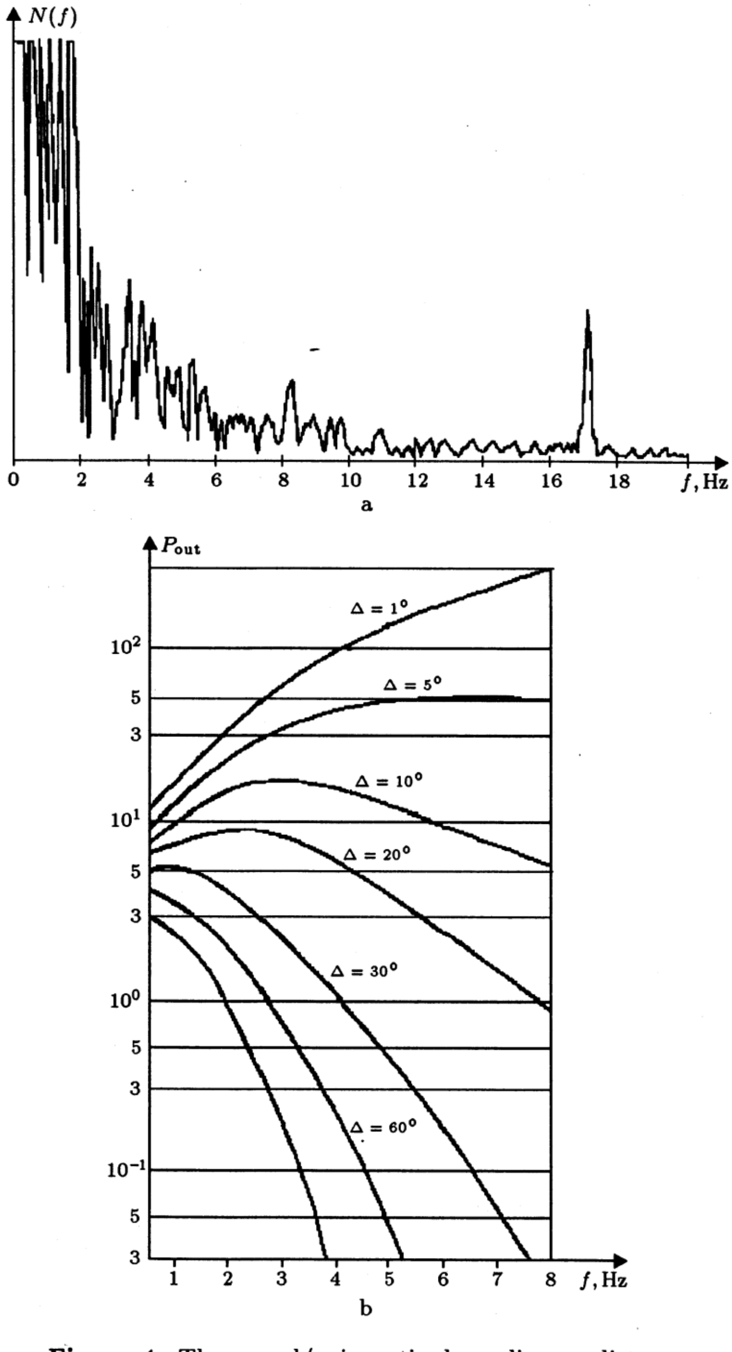

Figure 4. The sygnal/noise ratio depending on distance and sounding frequency

Here D is the dispersion of noise, β is the coefficient defining the steepness of the decay of the enveloping spectrum of noise, depending on the frequency. The experimentally defined value $\beta = 0.025$ was used in the calculation. In the calculations, the monochromatic signals of the form $S(t) = \exp(-\nu^2 t^2) \cos 2\pi f_0 t$ were selected as sounding vibroseismic signals.

According to (2), with the allowance for the given form of signals, the frequency characteristic of the medium $G(\omega)$, the spectrum of the noise $N(\omega)$, the signal/noise ratio as the function of the frequency f, and the epicentral distance Δ were calculated [12]. The graph of the obtained dependencies in the range of epicentral distances from 1 to 80° are presented in Figure 4b. The recommendations on the choice of the optimal range of frequencies of sounding signals with different epicentral distances can be found in the table

R, km	100	500	1000	2000	3500	6500–9000
Hz	8.0	5.0–8.0	2.0–4.0	1.5–2.0	0.5–1.0	0.5

In particular, as follows from the table, at distances of about 500 km the range of sounding should be selected within 5–8 Hz.

4. Optimization of the algorithm for processing of vibroseismic signals: the choice of an operator of processing $L[u(t)]$

As is known, the basic operation in processing of vibroseismic signals is the correlation convolution of a recorded signal with the basic signal, restored at the recording point according to the law of scanning a sounding signal. Some rise of noise stability here can be attained by introducing the weighed convolution, taking into account the spectral distinctions of the waves P and S:

$$R(m) = \frac{1}{N-m} \sum_{i=1}^{N-m} a_i u_i S_{i+m}, \qquad m = 1, \dots, L. \tag{7}$$

Here $a_i = G_{p,s}(f_i)/D_i$, where $G_{p,s}(f_i)$ is a spectrogram for the waves P and S; D_i is the dispersion of the noise on the frequency f_i; L is the number of countings-off of the received signal $u(t)$. The experimental results show that the gain obtained appears insignificant. In particular, in the experiments on sounding at a distance of 355 km a gain in the signal/noise ratio has made about one and a half time [13].

Another reserve of increasing the noise stability is connected with introduction of the spatial processing on a set of recording seismometers. As is known, under the condition of non-correlation of the noise between the adjacent seismometers, an expected gain is equal to $\sqrt{M}$ in the signal/noise ratio, where M is the number of seismometers. Actually, the gain appears

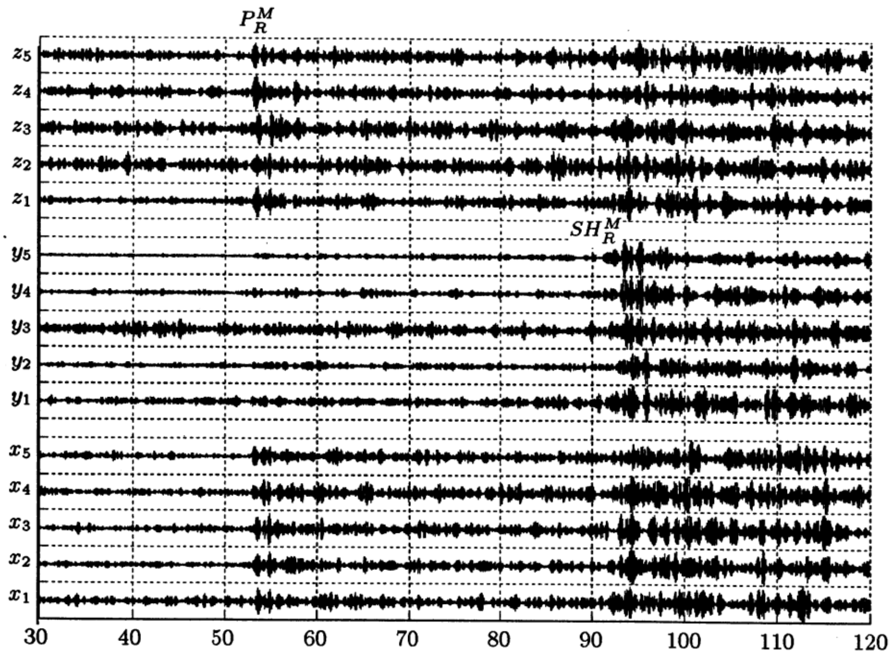

Figure 5. Vibrational seismograms of the components x, y, z at a distance of 355 km

lesser because of differences in the noise levels between the seismometers. The further increment of the noise stability can be attained by the weighed synchronous simulation over the set K of identical sessions of sounding [13]. Such an approach is possible due to the high recurrence of the form of signals of the VES. Theoretically, a total gain in the noise stability of vibroseismograms makes $\sqrt{MK}$. The experimentally measured gain at a distance "vibrator–receiver", equals to 355 km, with 5-point recording and 21 repeated sessions of sounding has made about 8.5 times [13]. As an example, Figure 5 presents the three component seismograms obtained by averaging over 21 sessions of sounding.

5. Optimization of the choice of the forms of sounding signals

The modern vibrators intended for the DSS are able to excite in the medium three forms of such signals: monochromatic, signals with frequency modulation (sweep signals), and phase manipulating signals. The vibroseismograms restored from such signals against the multiply exceeding noise have different noise stability estimated by criterion (2). It is theoretically shown and experimentally proved that for the classes of sounding signals $S(t)$ with the correlation function $R_s(\tau)$ criterion (2) will be equal to:

$$\gamma^2 = \frac{E_S \sigma_k^2}{N_0} \int_0^\infty \int_0^\infty R_k(\tau, \theta) R_S(\tau) R_S(\theta)\, d\tau\, d\theta, \tag{8}$$

where σ_k^2 is the seismogram dispersion, $R_k(\tau, \theta)$ is a normalized correlation function of the impulse seismogram, E_S is the energy of a sounding signal, N_0 is spectral of interference density. From (8) it follows that with the fixed energy E_S the highest noise stability (i.e., the maximum γ) with respect to such a complicated structure of wave propagation as the Earth, are the signals with the extended function $R_S(\tau)$. This requirement is met by narrow-band signals, in particular, monochromatic. Based on the results of the numerical simulation on evaluating the criterion as applied to harmonic, phase manipulation and sweep signals, it was shown that the obtained estimations of noise stability (2) are, approximately, in the ratio 1 : 0.4 : 0.1 [14]. In particular, it means, that the noise stability of monochromatic signals is one order higher than that sweep signals. Accordingly, there is an additional reserve of increasing the distance of sounding with constant energy consumed by a vibrator.

The result obtained at the stage of simulation have been completely confirmed by the results of experiments which were carried out at distances of hundreds kilometers. As an example, Figure 6 presents the results of simultaneous accumulation of a sweep-signal with the frequency band of sounding 5.5–8.5 Hz of 43 min 12 s duration, at a distance of 320 km (see Figure 6a at the top) and a monochromatic signal with frequency 7.23 Hz of 600 s duration (see Figure 6b). In the second case, the signal/noise for the wave P_n exceeds by one order the ratio obtained for the first case. In addition, Figure 6a (at the bottom) presents the form of the simulated according to (1) response of the medium to the effect of a monochromatic signal with the frequency 7.25 Hz. It should be noted that the obtained signal allows the evaluation of the form of the response of the medium to the effect by the monochromatic signals of different frequencies. As $k(t, \tau) = k(\tau)$ in (1), vibroseismogram, shown in Figure 6a for the component z, was taken. In this seismogram, the refracted wave P_n with the arrival time 48 s is clearly distinguished. The wave S with the arrival time 84 s is less distinct.

From the analysis of the considered reserves of increasing the noise stability of vibroseismograms, it follows that the most effective is the a resonance method of radiation of vibroseismic signals as monochromatic oscillations. Thus, the gain in power expenses for solution of the various tasks VES can reach two orders. With such a method of sounding it is possible to attain the increased accuracy of measurement of the dynamic characteristics of wave fields, first of all of the parameters of amplitudes and phases (see Figure 6c)

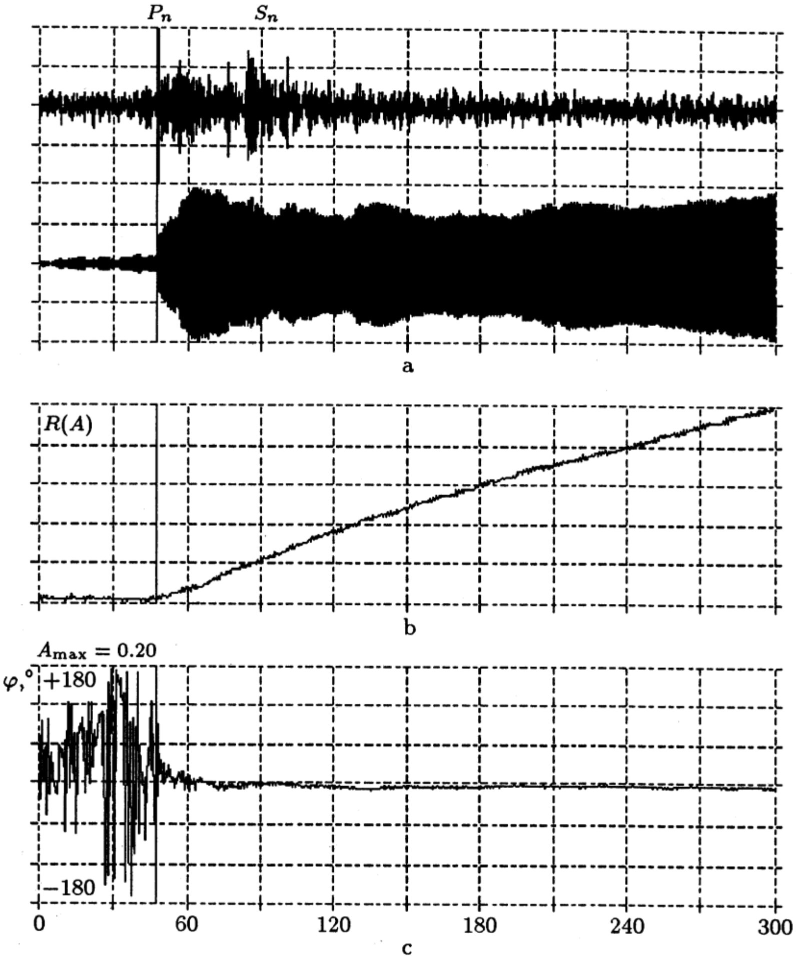

Figure 6. Vibrational seismograms for sounding regimes by linear frequency modulation and by monochromatic signals

of oscillations on a set of point of the space, where the units of a reception seismic array place. This will allow studying the solution of problems of seismic tomography [15], as well as studying the geodynamic processes in the crust and the upper mantle [16], etc. In particular, in this mode of sounding, high sensitivity of variations of parameters of amplitudes and phases of wave fields in connection with the Earth's tides [17]. This circumstance made it possible to obtain the new results in the indicated areas of research.

In connection with the use of the monochromatic mode of sounding, the question arises about the definition of seismic waves velocities. The solution to this problem is proposed employing the spatial–temporal method of decomposition of a wave field of a monochromatic source in a distant zone [18]. The method of decomposition is based on the well-known two-dimensional Fourier transform:

$$B(\omega, \nu_x) = \int_{\infty}\int_{\infty} U(x,t)f(x)f(t)\exp[-i(\omega t - x\nu_x)]\,dx\,dt. \tag{9}$$

For plane waves $U(x,t) = U(t - x/\nu)$, when using the discrete system of observations we obtain [18]:

$$B(\nu_j, \omega) = \sum_{k=1}^{n} S_k(\omega)U(k\Delta x)\exp(-i\nu_j k\Delta x), \tag{10}$$

where $S_k(\omega)$ is the spectrum of the signal $U(x_k, t)$ at the observation point x_k.

For the fixed sounding frequency ($\omega = \omega_0$), it is possible to accept $S(\omega) = S(\omega - \omega_0)$.

It is obvious that the decomposition of the wave field $U(x,t)$ with sounding by a monochromatic signal of the frequency ω_0 is reduced to the measurement of parameters as amplitudes and phases $\{\bar{A}, \bar{\varphi}\}$ of the wave field at the spatial points of the day surface of the Earth. The measurement of these parameters can be made with high accuracy exceeding the accuracy of the definition of wave arrival times. Thus, in the experiments on studying the Earth's tides, carried out at distances of 355 and 430 km, the accuracy of measurement of these parameters was within the limits of 3–5% and 1.5–2.0° respectively. To these estimates correspond variations of wave velocities at the level 10^{-5}.

6. Conclusion

The problem of increasing the sensitivity of the vibroseismic method for the solution of a wide spectrum of the tasks of seismology such, as studying fine geodynamic processes, the calibration of seismic traces and seismostations, studying the deep surface and the choice of monochromatic sounding signals. With such an approach, the increase of efficiency of vibrational sounding of up to two orders is possible with constant consumption energy by a vibrator. This circumstance is especially important when sounding is carried out at distances close to those teleseismic. It is proposed to solve the problem of definition of the seismic waves velocities originating with the monochromatic mode of sounding with the use of the two-dimensional Fourier transform in the coordinates "time–space". Thus, the measurable dynamic characteristics of seismograms will be the fields of amplitudes and phases of the monochromatic oscillation spaces, measured at selected points. The experimentally obtained estimations of the accuracy of measurements of the given parameters make up 3–5% in amplitude and 1.5–2° in phase. To these evaluations correspond the variations of velocities of the waves P at the level of 10^{-5}.

References

[1] Antonova L.V., Apticaev F.F., Kurochkina R.I., et al. Experimental Seismic Investigations of the Earth's Interior. – Moscow: Nauka, 1978 (in Russian).

[2] Archambean C.B., Bradford J.C., Broome P.W., Dean W.C., Flin E.A., Sax R.L. Data processing techniques for the detection and interpretation of teleseismic signals // Processing of the IEEE. – December, 1965. – Vol. 53, № 12. – P. 2071–2081.

[3] Khairetdinov M.S., Krivoputsky V.S., Driahlov S.I. Mathematical simulation of the system for the vibrational Earth's sounding // Problem-oriented computer complexes. – Novosibirsk, 1985. – P. 68–73 (in Russian).

[4] Gushchin V.V., Dokuchaev V.P., Zaslavsky Yu.M., Konyukhova L.D. On the distribution of power between different types of radiated waves in the half-bounded elastic medium // Study of the Earth by nonexplosive seismic sources. – Moscow: Nauka, 1981. – P. 113–117 (in Russian).

[5] Pasechnik I.P. The Characteristics of Seismic Waves with Nuclear Explosions and Earthquakes. – Moscow: Nauka, 1970 (in Russian).

[6] Khairetdinov M.S., Krivoputsky V.S. The analysis of spatially frequency of parameters of the environment with reference to the tasks of vibrational sounding of the Earth // Problem oriented computer complexes. – Novosibirsk, 1990. – P. 42–53 (in Russian).

[7] Ivashin V.V., Miloradov I.A., Simkin S.A., Chichinin I.S. The resonance schemes of the coordination of a vibrator with ground // Problems of vibrational of the Earth. – Moscow: Nauka, 1977. – P. 115–128 (in Russian).

[8] Boev S.I., Rumyancev A.I., Seleznev M.G. Solution of the energizing problem of waves in an elastic two-layer half-space // Methods of the extension of a frequency range of vibroseismic oscillations. – Novosibirsk, 1987. – P. 30–38 (in Russian).

[9] Kuzmenko A.P. Average offset and resistance of radiation of a stamp with vertical orientation of force operating on the two-layer medium // A radiation and reception of vibroseismic signals. – Novosibirsk, 1990. – P. 41–54 (in Russian).

[10] Voronina T.A., Dobrinsky V.I. Influence of discontinuities of Earth crust on power of vibroseismic waves // Mathematical simulation in Geophysics: Proceedings of Computer Center SD RAS. – Novosibirsk, 1993. – № 2. – P. 35–39.

[11] Kondratiev O.K. Seismic Waves in the Absorbing Media. – Moscow: Nedra, 1986 (in Russian).

[12] Khairetdinov M.S., Krivoputsky V.S. Results of mathematical simulation of the system VES based on impulse seismograms // Problem oriented Computer Complexes. – Novosibirsk, 1988. – P. 88–99 (in Russian).

[13] Glinsky B.M., Rodionov Y.I., Sedukhina G.F., Khairetdinov M.S., Shorokhov M.N. Results of experimental investigations on rising of noise stability of vibrational seismograms // Materials of international conference: Methods of study, structure and monitoring of the lithosphere, Novosibirsk, 6–13 September, 1998. – P. 50–55.

[14] Khairetdinov M.S., Krivoputsky V.S., Senin A.G. On noise stability of vibrational for DSS of the Earth // Problem oriented Computer Complexes.— Novosibirsk, 1991. – P. 28–40 (in Russian).

[15] Alekseev A.S., Meerson A.E., Jerniak G.F., et al. Seismic holography and photographing – methods and results of research // Problems of vibrational sounding of the Earth. – Moscow, 1977. – P. 31–52 (in Russian).

[16] Alekseev A.S., Glinsky B.M. Active seismology with powerful vibrator // NCC Bulletin, Series Mathematical Modeling in Geophysics. – Novosibirsk: NCC Publisher, 1998. – Issue 3. – P. 1–9.

[17] Glinsky B.M., Kovalevsky V.V., Khairetdinov M.S. Relationship between the wave fields of powerful vibrators and Atmospheric and Geodinamic Processes // Geology and Geophysics. – 1999. – Vol. 40, № 3. – P. 422–431.

[18] Khairetdinov M.S. Expansion of harmonics source seismic field // NCC Bulletin, Series Mathematical Modeling in Geophysics. – Novosibirsk: NCC Publisher, 1996. – Issue 2. – P. 37–47.