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Expansion of a harmonic source seicmic field

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A vibroseismic method of studying the Earth's deep structure has been most intensively used in the last fifteen years. Much progress has been made toward the creation of powerful seismic vibrators with precise amplitude-frequency-phase characteristics as well as toward methods and instruments for measuring the wave field parameters in the near and far zones. The present method is characterized by high logical purity, which opens up fresh opportunities for the use of the vibroseismic method when solving a number of actual seismological problems such as the vibroseismic monitoring of seismic-prone zones, vibra-tional seismotomography, vibrational seismic zoning, etc. In the solution of these problems of practical significance is the harmonic oscillation seismics. The wave field of a harmonic source represents the steady-state interference picture of different types of waves: pressure, shake and surface waves with the parameters $\bar{A}$, $\bar{\varphi}$ on the Earth's day surface, where $\bar{A}$ and $\bar{\varphi}$ are, respectively, vectors of amplitudes and phases at points of the space. It is theoretically shown and experimentally verified that the attainable accuracies and the noise-proof feature in the measurement of these parameters are higher than in the conventional impulse seismics. This allows vibrational sounding of the Earth at large depths (up to hundreds of kilometers) and at extended seismic traces (longer than 1.000 km). Estimations of the parameters $\bar{A}$, $\bar{\varphi}$ can be used for the solution of a number of practical tasks, the above-mentioned included. There is, for example, a much used method of seismic tomography on time delays. It is based on measuring travel times of the waves T_i along the ray trajectory S_i [1].

$$\delta T_i = -\int\limits_{S_i} \frac{\delta V(r)}{V_0(r)^2} dS, \qquad i = 1, \ldots, N. \tag{1}$$

In contrast to it, of higher resolution is the method of vibroseismic tomography based on harmonic oscillations. In this method a reconstructed image is described by equation:

$$U(x,y,z) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} U_0(x_0,y_0) \cdot k(x_0,y_0,x,y,z)\,dx_0 dt_0. \tag{2}$$

Here $U_0(x_o, y_0)$ is the field of complex amplitudes on the Earth's day surface ($z = 0$), $k(x_0, y_0, x, y, z)$ is the transformation kernel function, $V(r)$ is the velocity of wave propagation in the Earth, $V_0(r)$ is the simulated velocity of wave propagation in the model, $\delta V(r) = V(r) - V_0(r)$.

Taking into account the fact that $U_0(x_0, y_0) = |U_0(x_0, y_0)| \cdot e^{i\varphi_0}$, it is obvious that realization of (2) is connected with measuring the field of amplitudes and phases of steady-state oscillations on the Earth's day surface. As distinct from the method (1), waves in method (2) are not classified with respect to their arrival times. Therefore the problem arises of preliminary expansion of the interference wave field $U_0(x_0, y_0)$ in components of simple waves.

The present paper puts forward the methods of measuring the parameters $\bar{A}$, $\bar{\varphi}$ of the wave field of harmonic oscillations and proposes using these parameters in determining velocities of individual seismic waves and directions of their arrivals. An equation is presented to determine a connection between the main parameters of the system of vibrational sounding of the Earth nd characteristics of traces of seismic wave propagation. For profile observations, i.e., when $y =$ const, it is possible to expand the wave field $U(x, y)$ on the basis of two-dimensional (2D) transformation along the time and spatial coordinates of the function $U(x, t)$ [3]. In this case the wave field $U(x, t)$ is expanded in the infinite system of planar monochromatic waves $B \exp[i \cdot (\omega t - x\nu_x)]$, each one propagating with the velocity ω/ν_x in x-direction. Here ν_x is the spatial frequency.

A 2D Fourier spectrum can be written down as:

$$B(\omega, \nu_x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x, t) \cdot f(x) f(t) \exp[-i(\omega t - x\nu_x)]\, dx\, dt. \tag{3}$$

Here $f(\cdot)$ is the weight function.

At $U(x, t) = U(t - x/\nu)$ and when measuring its values at n-discrete points of the space, the discrete Fourier transform will be of the following form:

$$B(\nu_j, \omega) = \sum_{k=1}^{n} S_k(\omega) U(\Delta x \cdot k) \exp(-i\nu_j \cdot \Delta x \cdot k), \tag{4}$$

where $S(\omega)$ is the spectrum of the signal $U(x, t)$ at the point x.

For the fixed sounding frequency $\omega = \omega_0$

$$S(\omega) = \delta(\omega - \omega_0), \qquad U(\Delta x \cdot k) = A_k \cdot \cos\varphi_k + iA_k \sin\varphi_k,$$

therefore instead of 2D Fourier transform (3) we can apply a transformation along the coordinates $\{A_k, \varphi_k\}$. For the sum of n-planar waves in discrete measurements on the basis of $l = n \cdot \Delta x$ the spatial spectrum is of the following form:

$$B(\nu_x) = \sum_{k=1}^{n} B_k \frac{\sin[1/2(\omega/V_k - \nu_x)n \cdot \Delta x]}{1/2(\omega/V_k - \nu_x) \cdot \Delta x}, \tag{5}$$

where B_k is the amplitude of the k-th planar wave.

The global maxima of this function are located at the points $\nu_x = \frac{\omega_0}{V_k}$, where V_k is the propagation velocity of the planar wave with the amplitude proportional to nB_k.

The maxima on the spectral curve $B(\nu)$ correspond to apparent velocities of the planar wave. Parameters $\bar{A}$, $\bar{\varphi}$ can be estimated on the basis of the processing of seismic signals of the following form:

$$U(t) = S(t, \bar{A}, \bar{\varphi}) + n(t), \tag{6}$$

where $n(t)$ is the noise at the recording points and $S(t, \bar{A}, \bar{\varphi})$ are harmonic signals described at each spatial point in the following form:

$$S(t, A, \varphi) = F(t, A)\cos[\omega_0 t + \psi(t, A) + \varphi]. \tag{7}$$

Here $F(t, A)$ is the enveloping of a signal, φ is its initial phase, $\psi(t, A)$ is the term taking into account fluctuations of a narrow-band signal phase.

Bearing in mind the fact that levels of useful vibroseismic signals $S(t)$ are essentially (about two orders in amplitude) lower than the level of the noise $n(t)$, the parameters $\bar{A}$, $\bar{\varphi}$ can be measured on the basis of the quadratic processing algorithm [4], which is connected with the calculation of the statistics

$$R(A) = \sqrt{x^2(A) + y^2(A)}. \tag{8}$$

For the processing time $T \gg t_e$, where t_e is the time of establishing the response to the sounding of a harmonic signal at the output of the medium, the following relationships are true:

$$\begin{aligned} x(A) &= 1/T \int_0^T U(t) \sin \omega_0 t \, dt, \\ y(A) &= 1/T \int_0^T U(t) \cos \omega_0 t \, dt. \end{aligned}$$

It is easy to see that in this case

$$R(A) = A, \qquad \varphi = \operatorname{arctg} \frac{x(A)}{y(A)}.$$

Errors in determining the parameters $\{\bar{A}, \bar{\varphi}\}$ can be characterized by the following estimation dispersions:

$$\sigma_A^2 = G_0/T, \qquad \sigma_\varphi^2 = 1/\gamma_0^2 = \frac{G_0}{2E}. \tag{9}$$

Here G_0 is the spectral density of noise, γ_0 is the signal/noise ratio at the processor's output, E is the energy of a sounding signal at the sensor's input.

As an example, Figure 1 graphically shows the result of calculation of the parameters A, φ of the vibroseismic signal which was recorded from a centrifugal vibrator CV-100 at the sounding frequency $f_0 = 6.95$ Hz at a distance of 60 km from the vibrator. As is seen from the figure, the phase of the response according to (9) is stabilized at the time of signal arrival. Signals start to accumulate up to the amplitude of the response.

It is a well-known fact that the use of the discrete Fourier transform with the rectangular weight function results in increased side maxima of the spectral function which can be only 14 db different from the global maximum. Therefore, if two planar waves differ considerably in amplitude, the side maxima of the spectrum of the larger wave distort the value of the global maximum of the smaller wave spectrum. In order to eliminate the effect of the side maxima in DFT the Hahn window was used.

In this case the ratio between the global spectrum maximum and the side maxima reaches 40 db. The application of the Hahn window, however, doubles the width of the global maximum, which naturally leads to a decreased resolution of the spectral method. In this case DFT is used to increase the reliability of determining the maxima of the spectral function, whose locations are then defined more exactly by application of DFT with the rectangular weight function. As an illustration, Figure 2 shows the result of calculation of the spatial spectrum for the sum of two planar waves which propagate along the axis x with the velocity of 500 and 5000 m/sec. The amplitude of the second wave is half the amplitude of the first wave. Here the wave field has been given at $n = 14$ points. Let us determine the conditions under which the planar wave model can be employed:

1. The observation basis should be essentially smaller than the distance to the source. Then the damping of the wave amplitude at this basis can be neglected.
2. Travel time curves of interference waves at the observation basis should be nearly rectilinear.

These conditions are met for refracted waves. If the second condition is not fulfilled, as it is the case with the parabolic travel time curve which is symmetric to the observation basis, the maximum value of the spatial spectrum will correspond to the apparent velocity at the extreme point of the basis, i.e., the minimum velocity at this basis [3]. In order to estimate the resolution of the method under consideration, field experiments were carried out, in which the parameters $\{\bar{A}, \bar{\varphi}\}$ were measured at 14 points

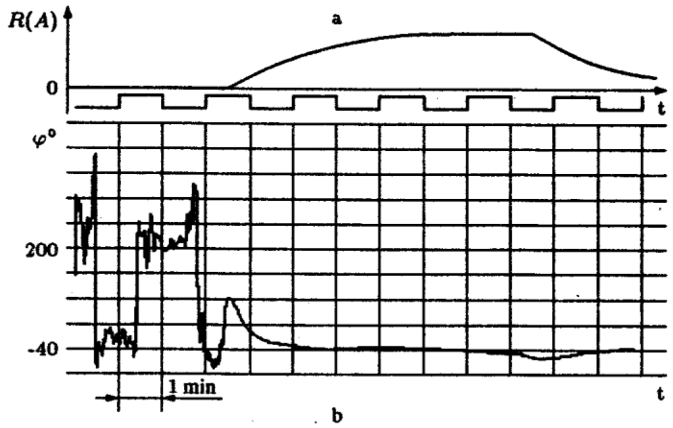

Figure 1

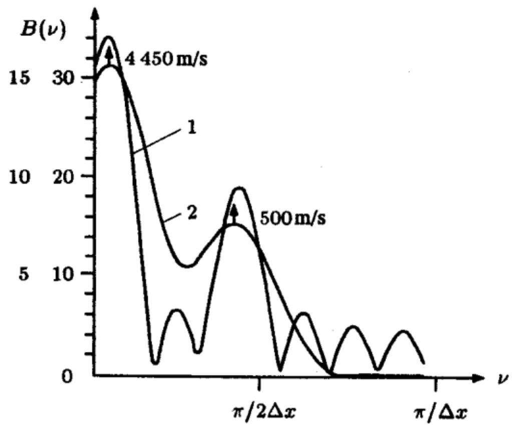

Figure 2

of a 700-meter profile with respect to x, y and z components, respectively. The axis of the profile was directed towards the vibrator which operated at the frequency $f = 2.08$ Hz. Figures 3, 4, 5 show the respective spatial spectra for the components x, y and z. The spectra were calculated based on DFT (4) with the rectangular weight function (the curve 1) and the Hahn weight function (the curve 2). It follows from the analysis of the plots that the spatial spectrum of z-component has three maxima corresponding to the waves with the velocities of 5000, 720 and 430 m/sec; the spectrum of x-component (source-directed) has two maxima – 5000 and 370 m/sec and the y-component spectrum – 2200 and 440 m/sec. Waves with the velocities of 370, 430 and 440 m/sec are surface waves. It should be emphasized that the wave velocity estimates which were obtained for the experimental data in question are approximate because of the limitedness of the observation basis

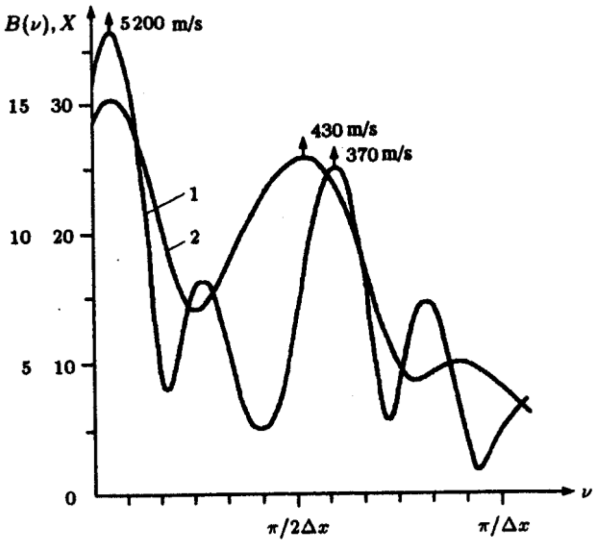

Figure 3

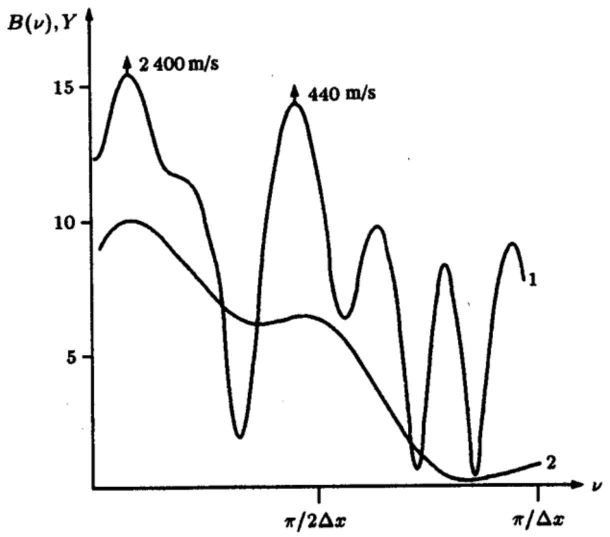

Figure 4

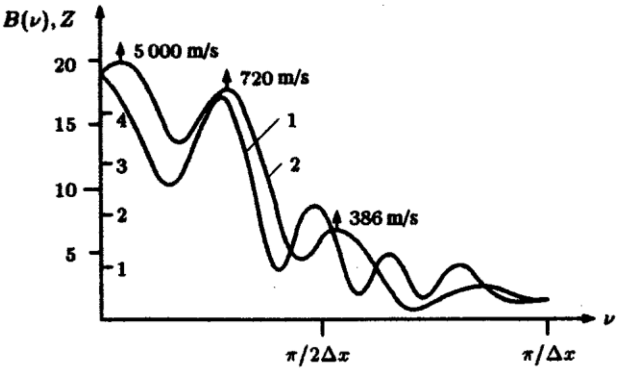

Figure 5

and the number of points at which measurements were taken. An increase of these parameters will provide a more detailed analysis of the wave field picture from a harmonic source. Resolution of the neighbouring waves with the velocities V_1, V_2 in the spatial spectrum is determined by the following condition:

$$l \geq \frac{2\pi}{\omega_0} \cdot \frac{V_1 \cdot V_2}{V_1 - V_2}.$$

Consideration of the effect of seismic noise and selection of parameters of sounding signals with allowance for their damping along the propagation trace are of great importance. The first of the above factors is particularly important because the levels of seismic signals are much lower than the noise level at recording stations. Examination of results of numerous measurements shows that the noise model can be represented in the following form:

$$n(t) = n_1(t) + \sum_{i=1}^{L} A_i(t) \cos[\omega_i t + \varphi_i(t)], \tag{10}$$

where $n(t)$ is a wide-band random noise with the monotonously decreasing spectrum in the band of 0–10 Hz. The second term represents the concentrated noise caused by rotation of different mechanisms connected with the Earth via their footings (water turbines, machine tools, compressors, etc.).

The concentrated noise level exceeds that of the wide-band noise by one order or more. Most frequently occurs the concentrated noise at frequencies 0.8, 1.4, 2.08, 3.3, 7.6–7.7, 8, 10 Hz. Taking into account this fact, a preliminary analysis of parameters of noise at measurement stations is necessary in order to prevent the overlapping of the frequency operation range on the concentrated noise frequencies. As an example, Figure 6 presents a spectrum of seismic noise near the seismic station Novosibirskaya, which illustrates main features of model (10). Figure 6a gives a steady-state spectrum, Figure 6b shows the same spectrum with overlapping noise from a

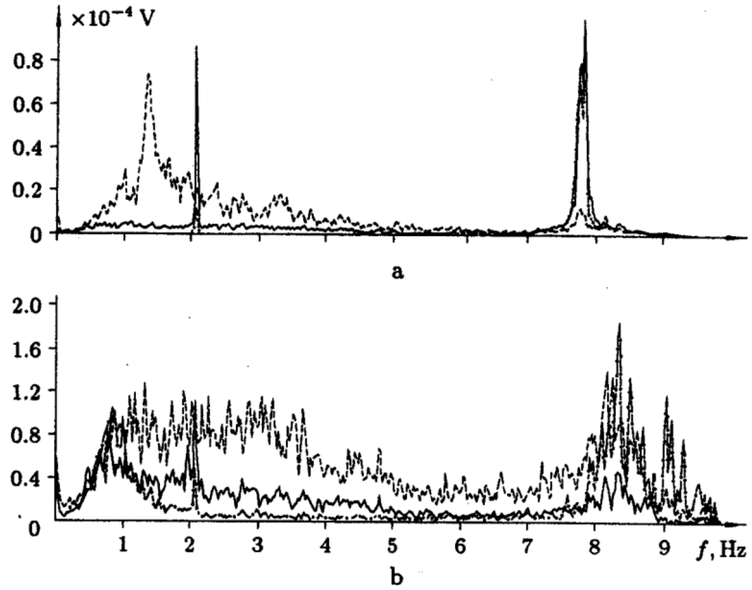

Figure 6

moving vehicle. The dashed line in the graph corresponds to the component y, the dash-dotted and continuous lines – to x and z, respectively. The concentrated noise at frequencies 2.08 Hz, 7.6–7.7 Hz is clearly distinguished in the graph. The duration of a realization of noise in the processing made 20 min.

As has already been noted, errors of measurement of the parameters are determined by the signal/noise ratio (9) at the recording points. We can show that for a model of a quasi-homogeneous medium with allowance for the seismic power distribution of surface vibrations with respect to main types of waves [3], the signal/noise ratio in the amplitude after processing will be of the following form:

$$\gamma = A_0 k r^{-n} \sigma_n^{-1} F f_0 \sqrt{T} \exp(-\alpha r). \tag{11}$$

Here we make use of the following notations:

A_0 – the level of a signal at the sounding point;

$k = \sqrt{0,0852\pi\rho v_p^3}$ – for pressure waves;

$k = \sqrt{0,299\pi\rho v_s^3}$ – for shake waves;

ρ – density of the medium under the vibrator;

V_p, V_b – velocities of pressure and shake waves, respestively, under the vibrator;

$n = 1$ – for the pressure wave;

r – distance between the vibrator and the recorder;

σ_n – the level of noise at the recording point;

F – the perturbing force of the vibrator;

f, T – frequencies and time of sounding, respectively;

α – the coefficient of absorption of the seismic wave energy.

For the mantle $\alpha \simeq 2.5 \cdot 10^{-4} f_0$ 1/km. Relation (11) can be used for approximate selection of parameters of sounding signals with allowance for characteristics of wave propagation traces, the medium at the radiation point and the level of noise at measurement points.

Conclusions

1. The procedure is proposed for measuring the parameters of amplitudes and phases of the wave field of a harmonic source on the Earth's day surface. The expansion in individual harmonic waves of the wave field of the harmonic source on the Earth's day surface is consideted which is possible due to high stability of the frequency-phase-time characteristics of powerful seismic vibrators. The proposed approach is illustrated on the basis of measurement of parameters of the wave field generated by the centrifugal vibrator CV-100 and a high-pressure compressor. The developed procedure allows covering a large area on the Earth's day surface at the expense of subsequent measurements of the parameters in the space in relation to the standard reference signal restored at the measurement station.
2. The procedure proposed can be used for the solution of seismic tomography problems, for the control of rheological characteristics of a medium by phase characteristics of waves in seismology, in deep seismic sounding, etc.
3. Errors in measurement of parameters can be made at the level of 1° for φ and 5% for A when optimizing the parameters of sounding signals and when taking account of noise characteristics at measurement points. The corresponding recommendations on the selection of parameters are presented in the paper.

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