Bull. Nov. Comp. Center, Math. Model. in Geoph., 8 (2003), 27–34

Mathematical simulation of invariant seismic experiments for research into requirements of equivalence conditions of sounding signals

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Modern technology of seismic survey is based on assumptions about the use of sounding signals distinguished by their characteristics. Thus, the experimentally obtained seismograms have a different form, which appears to be an objective hindrance when comparing and interpreting the obtained seismograms. An invariant seismic experiment is understood as seismic survey of the Earth's crust, whose result – a seismogram – does not depend on the kind of a sounding signal. As a procedure of reduction of a seismogram to an invariant form it is offered to use a deconvolution operation (Wiener filtration). The mathematical simulation of invariant seismic experiments with the use of the Laplace transform displays a possibility of employing different sources of seismic waves. A monochromatic sounding signal serves as an example of the formatting a correcting filter whose application allows attaining invariance of seismic experiments.

1. Statement of the problem and methodology of solution

The technology of seismic survey is based on the use of active sounding signals. Originally, in seismology, (one of the founders of which is Academician Boris Borisovich Golitsin (1862–1916), the Russian physicist and geophysicist), seismic waves from earthquakes were used as a sounding signal. This circumstance causes additional difficulties in carrying out geophysical experiments as compared to purely physical investigations, where a physicist-researcher sets up an experiment so that it would be convenient for him to study a considered phenomenon from a certain view point. In geophysics, this is nature, as it is, which sets up an experiment. It is natural that a question arises: why it is impossible to carry out such a systematic experiment in geophysics, as this is done in physics? These ideas underlie the research by Academician G.A. Gamburtsev (1903–1955), who aimed at proceeding from a natural experiment to a simulated one. He proposed and developed a method of sounding the Earth' s surface layer with the help of seismic waves, generated by large charges of chemical explosions, and shortly before his death G.A. Gamburtsev suggested that Professor Y.P. Bulashevich consider the problem of the development of mechanical sources for excitation of seismic waves (vibrators).

Since that time the technology of vibroseismic experiment has gained certain progress in studying the Earth' surface layer. However the energy released due to natural deformations, many thousands times exceeds the energy of simulated sources (including nuclear explosions) and makes it possible to sound the Earth as a whole. A conventional technology of seismic survey, as well as research and development of promising technologies in this direction, are connected with the use of sources of sounding signals, whose characteristics differ from each other. Thus, there is a necessity in reduction of the obtained seismograms to the form enabling one to successfully compare and reliably interpret seismic waves arrivals with different sources of a sounding signal. In the present paper, the mathematical simulation of invariant seismic experiments is considered and the conditions, in which different sounding signals gain the equivalence property. By the invariant seismic experiment the author means such an experiment, when application of different sounding signals results in obtaining invariable seismograms. In this paper, the mathematical simulation is based on the assumption that the properties of the Earth' surface adequately correspond to a linear system. In the structure of the technology of seismic experiments there are three elements whose essence determines the efficiency of the solution of "geodynamic" problems: the source of seismic waves, the explored area of the Earth's crust, an intellectual receiver of a sounding signal. In addition to the recording function, the intellectual receiver includes hardware and algorithms of processing (optimal filters). The structure of a mathematical model of seismic experiments is shown in Figure 1.

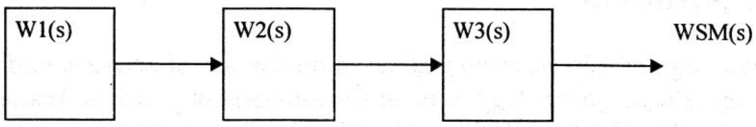

Figure 1. The structure of the mathematical model of invariant seismic experiments

The structure contains the following blocks: a source of a sounding signal $W1(s)$, the Earth's crust $W2(s)$, the receiver $W3(s)$. When describing the properties of separate blocks of the mathematical model we make use of the Laplace transform. Let us consider a limiting case, when a signal source is a delta-impulse. Such a model of a seismic experiment is an ideal one and in further considerations about the equivalence of sounding signals we accept it as a basis. An ideal model of a seismic experiment includes a sounding delta-impulse, the explored area of the Earth's crust and a receiver. The receiver in this model is a simple recording device without "intelligence", and the mathematical model of the Earth's crust represents a variety of lines of delays with certain factors determining attenuation of waves in a medium. The ideal model of seismic experiments is described by the following expressions:

$$\mathrm{W1}(s) = 1, \tag{1}$$

$$\mathrm{W2}(s) = \sum_{i=0}^{n} A_i e^{-s\tau_i}, \tag{2}$$

$$\mathrm{W3}(s) = 1, \tag{3}$$

where A_i is the wave amplitude, τ_i is the time of wave propagation, n is the number of waves at a recording point.

In this model of seismic experiments, a source of a sounding signal is a unit in the pulse function $\delta(t)$, which is also called a continuous delta-function. The unit delta-function can be considered as an input signal of zero width and infinite height:

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0; \end{cases} \qquad \int_{\infty}^{-\infty} \delta(t)dt = 1; \qquad L\{\delta(t)\} = 1, \tag{4}$$

where $L\{\}$ is the Laplace transform.

The receiver in the ideal model degenerates to a simple recording device of a seismic record without "intelligence", i.e., $\mathrm{W3}(s) = 1$. On the receiver output, the researcher will observe a record, representing a seismogram. The form of this seismogram will be defined by the expression $\mathrm{WSM}(s)$ (5), in which all parameters of the delay τ of a signal, radiated by a source of waves, are clearly distinguished. In a temporal seismogram $wsm(t)$, the wave arrivals will be marked with integral delta-functions:

$$\mathrm{WSM}(s) = \mathrm{W1}(s) \times \mathrm{W2}(s) \times \mathrm{W3}(s) = \sum_{i=0}^{n} A_i e^{-s\tau_i}; \tag{5}$$

$$\mathrm{wsm}(t) = \sum_{i=0}^{n} A_i \delta(t - \tau_i). \tag{6}$$

Now let us consider the case, when the sounding seismic waves differ in their shape from the delta-function. In this case, a sounding signal will have the form of the Laplace function $\mathrm{W1}(s) \neq 1$. Let us designate it as $\mathrm{W1}^0(s)$. For obtaining (with a new sounding signal) a seismogram, invariant to a seismogram of the ideal model of seismic experiment, let us carry out the following transformations. Let us introduce a function $\mathrm{W1}^1(s)$ for correction of a source of waves in the mathematical model. The new structure of the mathematical model of seismic experiment is presented in Figure 2.

This structure contains the following blocks: the corrected source of a sounding signal $\mathrm{W1}^0(s) \times \mathrm{W1}^1(s)$, the Earth's crust $\mathrm{W2}(s)$, the receiver

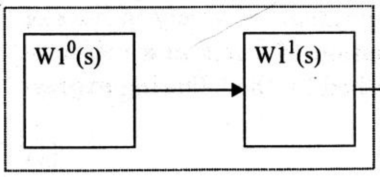

Figure 2. The structure of the mathematical model of seismic experiments with a source after correction

W3(s). For obtaining an invariant seismogram in relation to the ideal model of seismic experiment, the correcting function $W1^1(s)$ should satisfy the following condition:

$$W1^0(s) \times W1^1(s) = 1. \tag{7}$$

From expression (7), determine the form of the correcting function

$$W1^k(s) = \frac{1}{W1^0(s)}. \tag{8}$$

Formula (8) represents, in fact, a characteristic of the inverse filter, being a particular case of a Wiener forming filter [2]. The operation of this filter corresponds to the compensation of the influence of the characteristic $W1^0(s)$ in a general scheme of transformations for the simulation of seismic survey, and therefore is termed as inverse convolution or deconvolution.

However it, generally, appears impossible to correct the characteristic of a source if this is a natural phenomenon, or the possibilities of correction are strictly limited if a source is an industrial explosion or a vibrator. Therefore the correction operation in the mathematical model should be made in the receiver. Then the receiver will gain another characteristic (it becomes optimal in view of obtaining an invariant seismogram) $W3^k(s) = W1^k(s) \times W3(s) = 1/W1^0$. Now, on the receiver output (Figure 3) a researcher will observe a new record, which will be a seismogram from a new sounding signal.

The form of this seismogram will be determined by the formula $WSM^1(s)$ (9), in which all the parameters of the delay τ of a signal, radiated by an

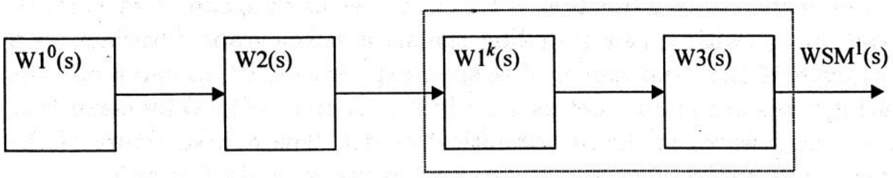

Figure 3. The structure of the mathematical model of seismic experiments after transformation of the receiver

imperfect source of waves are also clearly seen. The obtained seismogram has not changed in relation to the seismogram of an ideal seismic experiment. Thus, we have obtained an invariant seismogram, according to expressions (5) and (6):

$$\mathrm{WSM}^1(s) = \mathrm{W1}^0(s) \times \mathrm{W2}(s) \times (\mathrm{W1}^0(s))^{-1} = \sum_{i=0}^{n} A_i e^{-s\tau_i}; \tag{9}$$

$$\mathrm{wsm}^1(t) = \sum_{i=0}^{n} A_i \delta(t - \tau_i). \tag{10}$$

Of practical importance is a case, when instead of a sounding signal as delta-function a monochromatic signal, restricted in time, is used. The receiver in this case should gain "intelligence" in the form of a correcting filter. "Intelligence" of the receiver should recognize and form in a seismogram the clear indications to the arrival time of monochromatic waves. Let us define the expression for the optimal filter using monochromatic sounding signals. For the mathematical model of seismic experiment with a monochromatic sounding signal, let us obtain the following expressions:

$$\mathrm{w1}(t) = (\Phi(t) - \Phi(t - T_0)) \cos \omega t; \tag{11}$$

$$\mathrm{W1}(s) = \frac{s}{s^2 + \omega^2} - \exp(-T_0 s) \frac{s \cos T_0\omega - \omega \sin T_0\omega}{s^2 + \omega^2}; \tag{12}$$

$$\mathrm{W2}(s) = \sum_{i=0}^{n} A_i e^{-s\tau_i}; \tag{13}$$

$$\mathrm{W3}(s) = \frac{1}{\mathrm{W1}(s)}; \tag{14}$$

$$\mathrm{WSM}^m(s) = \mathrm{W1}(s) \times \mathrm{W2}(s) \times \mathrm{W3}(s); \tag{15}$$

$$\mathrm{wsm}^m(t) = \sum_{i=0}^{n} A_i \delta(t - \tau_i), \tag{16}$$

where, $\mathrm{w1}(t)$ is a monochromatic sounding signal with the time of radiation $\mathrm{T_0}$; $\Phi(t)$ is the Heaviside function, defined by the following conditions:

$$\Phi(t) = \begin{cases} 1, & t \geq 0; \\ 0, & t < 0. \end{cases} \tag{17}$$

Analysis of the obtained formulas shows that for obtaining an invariant seismogram with a monochromatic sounding signal the receiver should have a correcting filter with the following characteristic:

$$\mathrm{W3}(s) = \frac{s^2 + \omega^2}{s - \exp(-T_0 s)(s \cos T_0\omega - \omega \sin T_0\omega)}. \tag{18}$$

This formula (delta-function) defines the equivalence condition of a monochromatic signal and a sounding signal as delta-function. Similarly it is possible to construct the optimal filter for a monochromatic (or another) signal by selecting it as basic seismic experiment, which can be an explosive (chemical or natural) sounding signal, i.e.,

$$\mathrm{W1}^0(s) \times \mathrm{W1}^1(s) \neq 1. \tag{19}$$

2. Some comments to the obtained results and conceptual potentialities of the proposed approach

On the basis of analytical transformations (1)–(16), the key possibility of forming the correcting filters for seismic receivers with the use of sounding signals with different characteristics is shown. The correcting filter of the receiver is a necessary condition for obtaining invariant seismograms and attaining the equivalence of sounding signals. Therefore it is necessary to consider the problem of equivalence of sounding signals only in combination of three elements: two sources of sounding signals with different characteristics and the correcting filter, whose characteristic results in equivalence of a considered pair of sounding signals (19). The basic sounding signal $\mathrm{W1}(s)$, in relation to which the correcting filter $\mathrm{W1}^k(s)$ is formed, can be any one from sounding signals $\langle \mathrm{W1}^0(s), \mathrm{W1}(s)\rangle$. The selection of such a signal is determined by a researcher, based on the most suitable frequency characteristics of a signal, which affect the form of a seismogram

$$\mathrm{W1}^0(s) \times \mathrm{W1}^k(s) = \mathrm{W1}(s). \tag{20}$$

The proposed approach for obtaining an invariant seismogram of a monochromatic sounding signal can be useful, first of all, in vibroseismic survey. The present approach enables us to increase a distance from a source to the recording point when sounding the Earth's crust up to 2000 km.

This approach is confirmed by experimental data, when using the vibrator of 100 tons and the time of radiation 40 min which allows a sounding signal of considerable power for the recording point at a distance up to 2,000 km (10), (11). The form of the function W1(s) is not in principle, restricted by (19). The above-said allows one to use seismic noise both of natural and technogenic character, as well as seismic waves generated under the effect of stable astronomical phenomena, such as lunar-solar tides. For the definition of the function $\mathrm{W1}(s)$, it is necessary to have a seismic receiver with synchronization of data with absolute time. It is natural that data at a remote recording point should be synchronized in time as well. Conclusions and suppositions stated in this paper are based on using the correcting filters, simulated by means of deconvolution. However the author admits the possibility of using other correcting filters or algorithms for seismic data processing for forming the equivalence conditions of sounding signals.

Acknowledgements. The author would like to express his gratitude to his colleagues from the Department of Geophysical Informatics for valuable discussions.

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