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A numerical model of thermal bar in lake Baikal*

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A two-dimensional compressible nonhydrostatic numerical model is proposed for the simulation of thermal bar in a deep freshwater lake. An equation of state is used which shows a nonlinear dependence of density on temperature and pressure. Numerical experiments demonstrate the peculiarities of the temperature distribution and lake currents associated with the thermal bar. The two-cell convection divides the nearshore region into three zones in the direction of the temperature front movement: warm, transition, and cold regions. The temperatures of the transition zone are close to those of maximum density. Downward currents are developed between two cells in the transition zone. Their calculated values are rather large ($\max w = O(0.1)$–$O(1)$ cm/s). This indicates the intensive exchange between nearsurface and deep waters.

1. Introduction

The thermal bar is a natural phenomenon which takes place in freshwater temperate lakes in spring and autumn due to nonuniform heating of the shallow nearshore and the deep offshore regions. It looks like the alongshore hydrofront which propagates from the shore toward the center of the lake. The difference between the spring and autumn events is that nearshore is warmer than offshore region in spring, and vice versa in autumn.

From the physical point of view, its nature is the same in spring as in autumn and is connected with the anomalous behavior of water density. Namely, the water density reaches the maximum values at some temperatures. As the consequence of this, the mixing of two parcels of water, one of which is warmer and the other is colder than the temperature of maximum density (tmd), produces the resulting water mass which is denser than the both origins. In response to the formation of density gradients, dynamic processes occur in order to sink the heavier water. Thus, the temperature front is generated near the places where warm and cold waters mix. While the heavy water begins to sink, new portions of warm and cold water move towards the front from the opposite sides. For the compensation of sinking water, upstream flows are generated on the peripheries of the convective cells. This is, in general, a scheme for the thermal bar phenomenon.

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Forel was the first who described the thermal bar in his monography in 1895 [5]. His observations were carried out in lake Leman. Since his paper, a lot of descriptions of the phenomenon in different lakes were made. Tikhomirov [15, 16] analysed and generalized the majority of natural data. A part of the review [1] is devoted to the description of the phenomenon registered in Great Lakes. Theoretical studies were discussed in [20]. A few laboratory experiments were also conducted in order to verify the theoretical hypothesis, (see, for example, review in Kreiman [6]).

The thermal bar in Baikal causes the natural interest of the researches because of the lake's great depth [13, 14, 19]. The fact is that the tmd decreases with the increase of pressure (or depth). If a lake is very deep, the decrease of tmd is so significant that it might be essential for the conditions which control convection. The data recently published by Shimaraev et al [14] describe the observations of the spring thermal bar in lake Baikal in June 1991. The thermal bar manifests itself via two temperature stratification patterns: direct, in the nearshore, and inverse, in the offshore. Analysing the temperature distribution, the authors make the conclusion that the downward jet of cold water originates from the hypolimnion on the open-lake side of the thermal bar. Moreover, they suppose that very deep waters are renewed due to the mechanism of deep convection during the periods of migration of the thermal bar from shore to the center of the lake. The hypothesis about the influence of deep convection on the renewal of deep waters in the lake was also discussed in [19].

Doubtless, that it is desirable to develop a mathematical model which can help us to understand the phenomenon and check the validity of the hypothesis. In the literature there are some numerical models devoted to the thermal bar in shallow lakes. The majority of them are two-dimensional (2D) models (see review in [2] and [1]). The use of 2D models is motivated by the observational fact that the gradients normal to shore are much larger than gradients parallel to shore. It is therefore assumed that no major characteristic of the phenomenon is lost if the gradients parallel to shore are entirely neglected. We also accept the 2D simplification taking into account the possibility of numerical realization as well. The system of equations describing convective motions is well-known, that is why the formulations of the models developed by many authors are similar and have minor differences. As the essence of the phenomenon is connected with the special behaviour of water density, it is obvious that the nonlinear equations of state are exploited by all the models.

This paper reports the results of development and application of 2D nonhydrostatic finite-difference model for the simulation of thermal bar in a deep freshwater lake. The main difference of this model from other models is that the compressibility of water is taken into account.

2. A nonhydrostatic lake model

To simulate the motions with comparable horizontal and vertical scales in natural objects, a nonhydrostatic model with rotation is employed [17, 18]. The system of equations expresses the balance laws of momentum, mass and energy of the non-Boussinesq compressible fluid. The motion in a $x-z$ plane perpendicular to the front axis is governed by the momentum equations for velocities u, v, and w in the x, y, and z directions

$$\frac{du}{dt} - lv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\nu\frac{\partial u}{\partial z} + \frac{\partial}{\partial x}A\frac{\partial u}{\partial x}, \tag{1}$$

$$\frac{dv}{dt} + lv = \frac{\partial}{\partial z}\nu\frac{\partial v}{\partial z} + \frac{\partial}{\partial x}A\frac{\partial v}{\partial x}, \tag{2}$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g + \frac{\partial}{\partial z}\nu\frac{\partial w}{\partial z} + \frac{\partial}{\partial x}A\frac{\partial w}{\partial x}. \tag{3}$$

The continuity equation for the compressible flow is in the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho w}{\partial z} = 0. \tag{4}$$

Heat is advected and diffused with time by the temperature conservation equation

$$\frac{dT}{dt} - \Gamma\frac{dp}{dt} = \frac{\partial}{\partial z}\nu_T\frac{\partial T}{\partial z} + \frac{\partial}{\partial x}\mu\frac{\partial T}{\partial x} + q. \tag{5}$$

The nonlinear equation of state supplements the system

$$\rho = \rho(p, T, S_0). \tag{6}$$

Here l is the Coriolis parameter, which is prescribed as constant value at a mean latitude of lake Baikal, p is pressure, ρ is density, A, ν, μ, ν_T are turbulent momentum and thermal diffusivities in horizontal and vertical directions, respectively, T is temperature, Γ is adiabatic temperature gradient,

$$\Gamma = \frac{\alpha\bar{T}}{\rho c_p},$$

$\bar{T}$ is absolute temperature, c_p is specific heat at constant pressure, q is a source term which is responsible for the distributive heat income into the nearsurface layers, S_0 is salinity, α is the coefficient of thermal expansion. The special conditions of the heat absorption in lake Baikal were studied and parameterized by Zvonkova [21]. This parameterization is used in the model. It is supposed that solar radiation is absorbed in water masses according to the law $q = Q_0 B\exp(-\beta z)$, where β is extinction coefficient,

B is dimensional parameter, and Q_0 is the value of the solar radiation flux at the surface.

The governing system of equations (1)–(6) is considered in the domain $D_t = D \times [0, \bar{t}]$, where D is the domain of spatial variables (x, z) and $[0, \bar{t}]$ is the time interval.

The boundary and initial conditions are as follows:

at the lake surface z = 0:

$$\nu \frac{\partial u}{\partial z} = -\frac{\tau_x}{\rho}, \quad \nu \frac{\partial u}{\partial z} = -\frac{\tau_y}{\rho}, \quad \nu_T \frac{\partial T}{\partial z} = -\frac{Q}{\rho c_p}, \tag{7}$$

$$w = 0, \quad p = p_a(x, t);$$

at the bottom $z = H(x)$:

$$u = v = w = 0, \quad \frac{\partial T}{\partial N} = 0; \tag{8}$$

at the lateral boundaries

$$u = v = w = 0, \quad \frac{\partial T}{\partial N} = 0. \tag{9}$$

Initial conditions in D at $t = 0$ are:

$$\varphi = \varphi^0(x, z), \quad \varphi = (u, v, w, T, p, \rho). \tag{10}$$

Here $\partial/\partial N$ represents the derivatives with respect to the conormal

$$\frac{\partial}{\partial N} = \mu \cos(n, x) \frac{\partial}{\partial x} + \nu_t \cos(n, z) \frac{\partial}{\partial z}, \tag{11}$$

n is external vector normal to the boundary of the area, p_a is air pressure, τ_x, τ_y are wind stresses in x and y directions, Q is heat flux on the lake surface, $\varphi^0(x, y, z)$ are the given functions, $(\varphi = u, v, w, T, p, \rho)$.

In the given model, the state equation adapted to the limnological problems is used [3]. Due to this, density is the function of temperature, pressure and salinity. In this version of the model salinity is supposed to be the constant value of 0.098 g/kg. But, to be more objective, it is worthy of mention that the influence of salinity variations is not yet complitely understood. The data show very small changes of salinity in the lake. The rivers' incomes give the more pronounced difference of values. As the very sensitive phase of the year temperature cycle is considered, when the temperatures are close to those of maximum density and the variations are very small, pressure and salinity must play more important roles than they do in other circumstances. Here we take into account only pressure variations, and the salt influence will be the subject of further investigations.

There are some more parameters connected with the state equation. They are the adiabatic temperature gradient, the coefficient of thermal expansion and the specific heat at a constant pressure. All of them, when necessary, are calculated by the formulas given in [3], using the known values of temperature and pressure.

The problem of turbulence parameterization is really crucial for the simulation of the convective movement. Subgrid-scale turbulence is parameterized in the model with the help of the second order diffusive operators. The constant values were taken for the horizontal coefficients. The vertical ones were specially chosen in each experiment in order to compare the results and evaluate the sensitivity of the model. One way is to take the constant value coefficients and the other one is to calculate them in the accordance with some parameterization scheme. The parameterizations using the gradient Richardson number

$$Ri = N^2 \Big/ \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right],$$

where N^2 is the Brunt–Väisälä frequency

$$N^2 = -g\alpha \left[\frac{\partial T}{\partial z} - \Gamma g \rho\right]$$

are widespread in oceanic and lake applications, (see, for example, [9]). It is known that as long as Ri remains greater than 0.25, turbulence is supressed. If Ri is less than this critical value, the instability of Kelvin–Helmgoltz type occurs. For stable situations the variable coefficients can be defined as

$$\nu = \frac{\nu_0}{(1 + a\,\mathrm{Ri})^n} + \nu_b, \quad \nu_T = \frac{\nu}{(1 + a\,\mathrm{Ri})} + \nu_{Tb}.$$

Here ν_b and ν_{Tb} are background dissipation parameters and ν_0, a and n are adjustable parameters. Usually ν_0 is correlated with a neutrally stable fluid. If instability appears at some locations, $\nu = \nu_0$ there. There are some other parameterization formulas which we also use for different scenarious.

The model region is a cavity with the horizontal scale of 20 km and non-symmetric left and right bottom slopes. The maximum depth of the domain is 960 m, which is greater than the mean depth of lake Baikal (736 m). The region was covered by uniform grid in each direction. The grid with 49 levels in the vertical direction is used. There are 501 points in x-direction. The grid lengths are: in horizontal $\Delta x = 40$ m, in vertical $\Delta z = 20$ m, the time step $\Delta t = 60$ sec. In the realization, the bottom slopes are approximated by the linear segments parallel to the x and z axes. The chosen parameters of the domain D and discrete approximations of the model make it possible to describe the local fluid motions with the comparable scales in both directions.

The numerical realization is based on the splitting technique and the variational principle [7, 10]. According to the idea of splitting method, the solution of the whole problem is replaced by the succesive solution of a set of partial problems. Each of them is more simple than the original one. In practice, there are many different splitting schemes. The scheme used was described in [18] and is not given here. The fact that should be stressed is the use of monotonic and transportive scheme for the advection-diffusion stage [11] that is very important for the modelling of fronts. The second essential thing is the non-iterative technique for the solution of the second order partial differential equation for pressure.

3. Numerical experiments

A great number of numerical experiments were carried out with the help of the model in order to check its ability to work and to estimate its sensitivity to the change of the computational algorithm, approximations, parameters and parameterization patterns. In the geophysical sense, numerical experiments can be divided into two parts:

- generation of a thermal bar,
- dynamics of the developed thermal bar.

In the first case, the thermal bar is initiated from a winter temperature distribution. In the second case initial data contain the temperature distribution which is usual for the thermal bar pattern.

The results of only one numerical experiment are presented here. The choice is more or less arbitrary and it was dictated by the desire to illustrate the possibilities of the model and to give an idea of the character of the phenomenon. The other geophysical results will be generelized and discussed in the next publications.

Spring heating of lake Baikal begins with a winter temperature distribution which is characterized by the vertical profile with the maximum at the mid-depth. The temperature increases from the surface to some depth at which the local temperature is equal to that of maximum density for this depth. Below this depth, temperature slightly falls to the bottom. Such temperature profile is chosen as the initial distribution for the horizontally uniform temperature field in order to simulate the thermal bar generation. In the second case, which is discussed here, the initial temperature field is the same almost everywhere, except two parts of the domain near the left and right boundaries, where the depth is less than or equal to 200 m. The constant temperature of 4° C from surface to bottom is taken in the left and right nearshore regions. The initial pressure and density fields are adjusted to one another on the background of the prescribed temperature and salt distributions by means of the solution to the system of two equations. One of them is the state equation and the other is the hydrostatic balance

$$\frac{\partial p}{\partial z} = g\rho. \tag{12}$$

The system is solved by the forth-order Runge–Kutta scheme.

The model is driven from rest. This means that the components of vector velocity are zero. The stationary uniform surface heat flux Q is applied at the surface. The value is taken from the climatic data for June [12]. Although, in practice, the natural behaviour should not be thought of as independent of wind, it is one of the worthwhile exercises in the stage of developing an understanding of the phenomenon to consider the lake's response in the absence of wind.

The other parameters were: $\nu_0 = 50$ cm^2/s, $\nu_T = \nu$, $n = 1$, $a = 5$.

Representative results of the simulation of the developed thermal bar are given in Figures 1 to 9. The main features of the simulated phenomenon are discussed below.

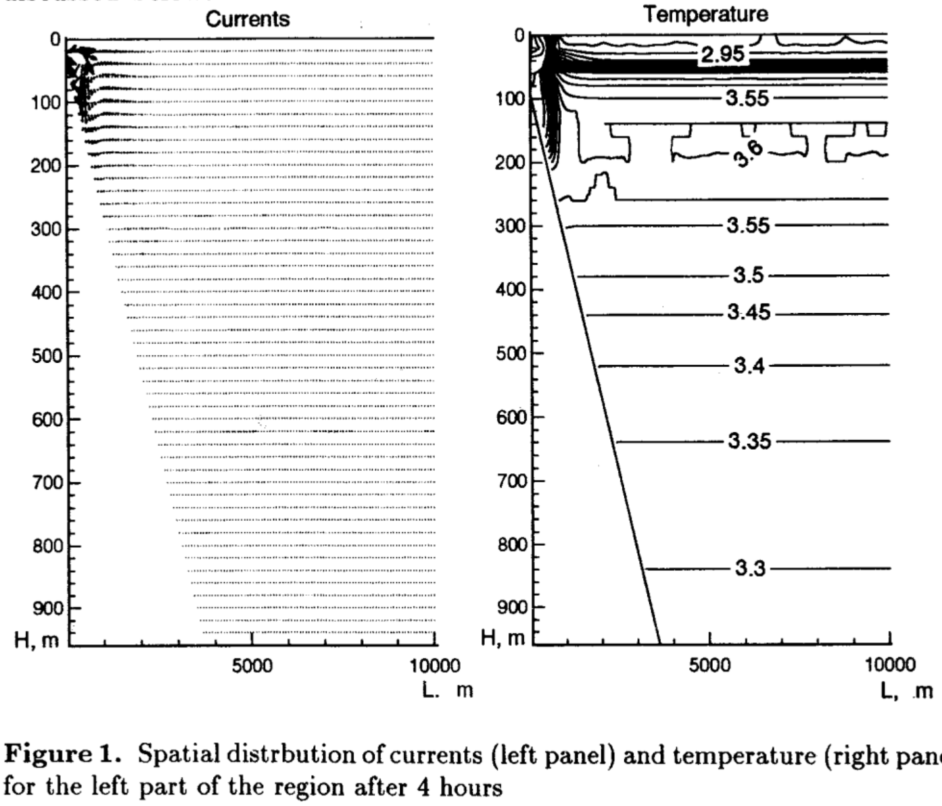

Figure 1. Spatial distrbution of currents (left panel) and temperature (right panel) for the left part of the region after 4 hours

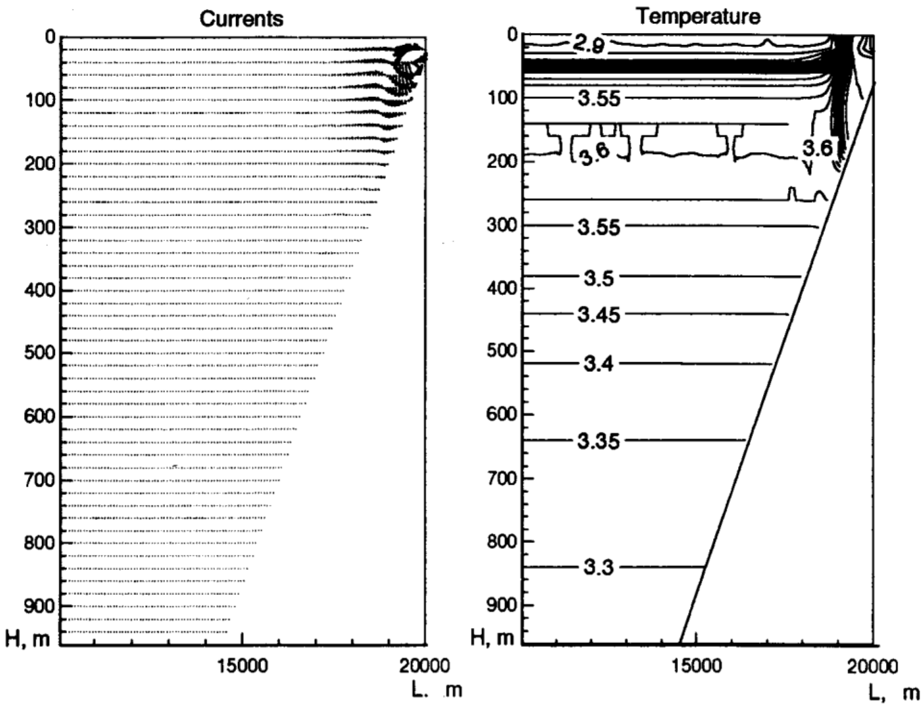

Figure 2. The same as in the Figure 1 but for the right part of the region

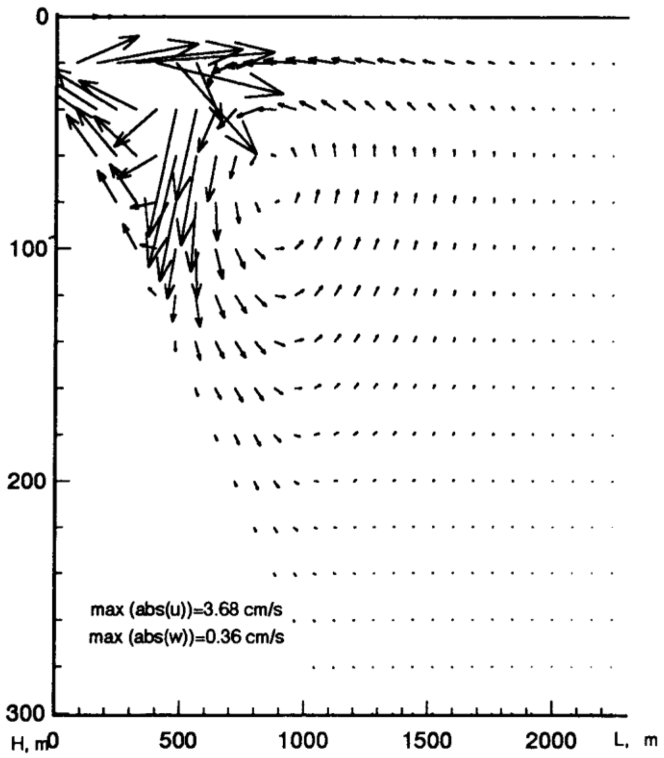

Figure 3. The currents associated with the thermal bar (t=4 hours)

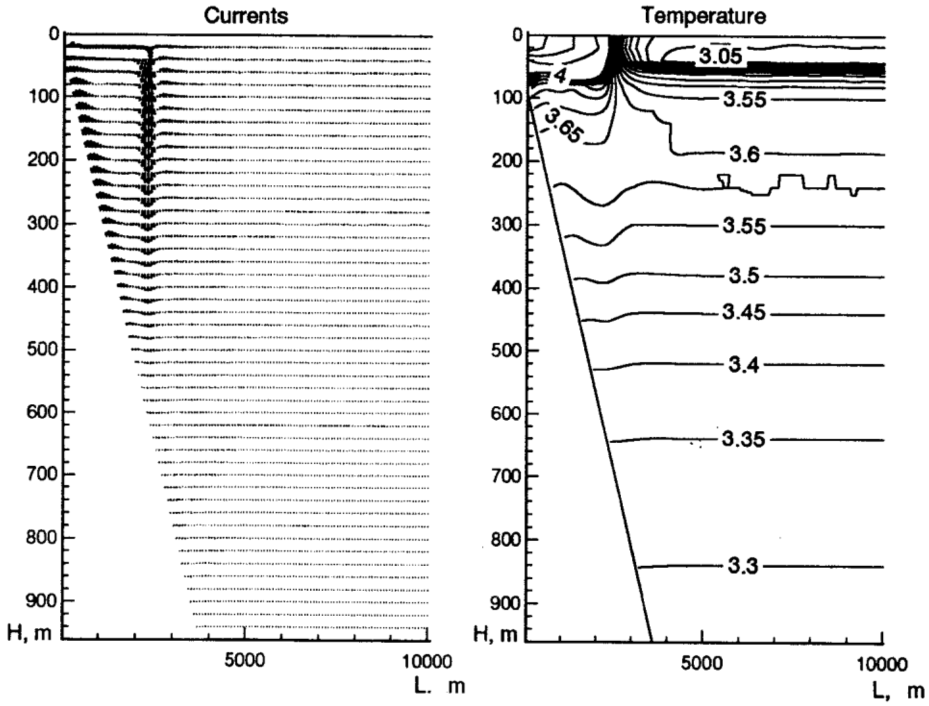

Figure 4. The same as in the Figure 1 but after 124 hours

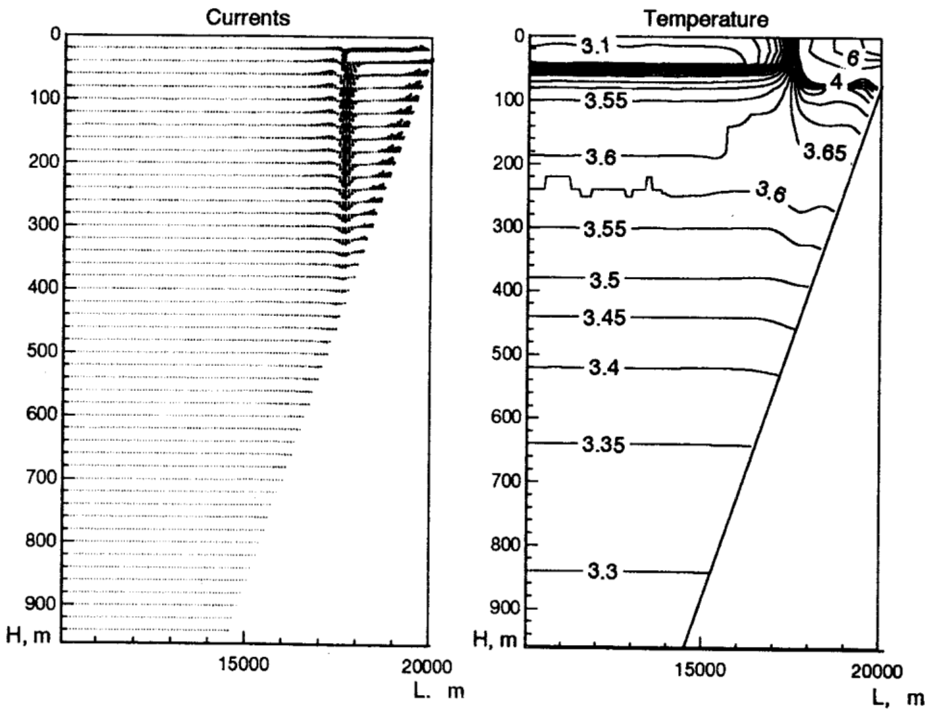

Figure 5. The same as in the Figure 2 but after 124 hours

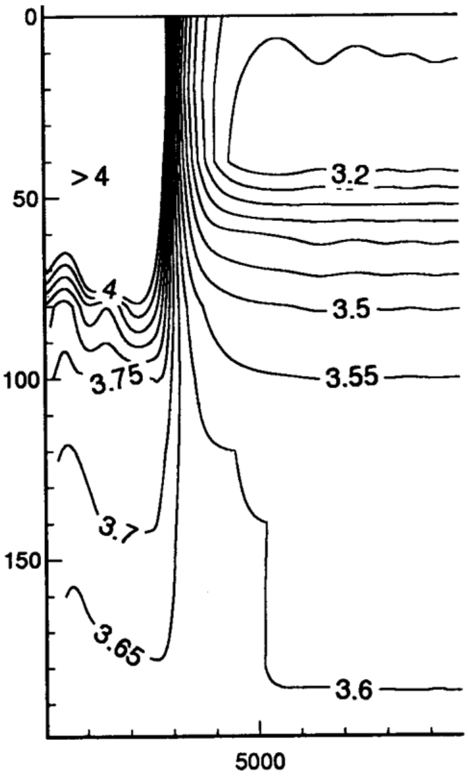

Figure 6. The temperature distrubution in the upper 200-m layer in the vicinity of the thermal bar (t=168 hours)

The temperature field is characterized by the co-existence of "summer", "spring" and "winter" typical climatic vertical temperature patterns simultaneously.

In the nearshore, the temperature decreases with depth showing the summer distribution. The isotherms are practically horizontal there that testifies the predominance of horizontal advection in comparison with vertical diffusion (Figures 4, 5, 8, 9).

The temperature front is placed in the intermediate zone (Figure 6). The isotherms are mainly vertically oriented and the temperatures are close to that of maximum density (3.6–4.0° C). As time elapses, the front configuration changes in space. Sometimes the *s*-form profile can be seen when warm water flows over cold water near the surface. Such configuration of front was observed in laboratory experiments [6].

In the open-lake side the winter temperature distribution with the maximum at the mid-depth is conserved for a long time. Accumulation of heat near the surface leads to the convective movement that results in the smoothing out of the temperature in vertical. The presence of the internal waves are seen everywhere.

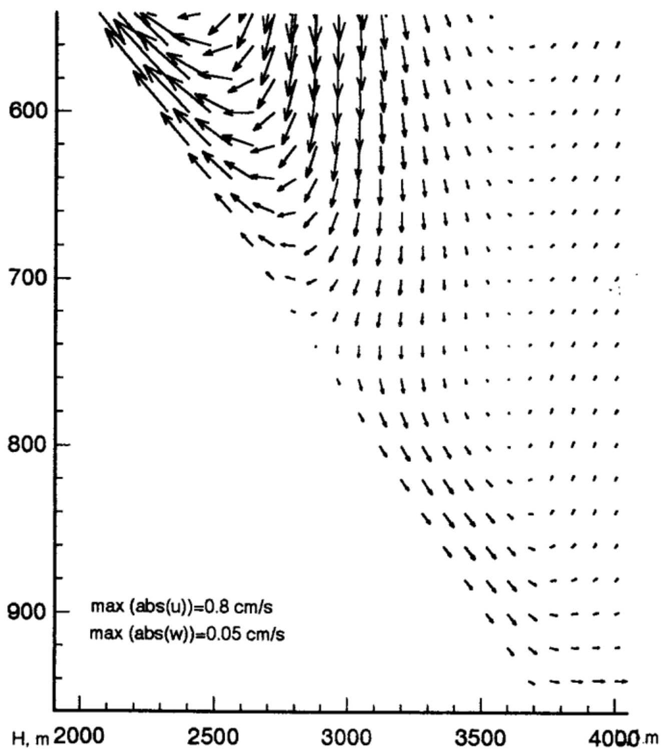

Figure 7. The circulation in the deep layers near the bottom (t=168 hours)

In all figures the temperature fields are displayed in the following way: the contours of values less than or equal to 4° C – with the step of 0.05° C, and the contours greater than 4° C – with the step of 1° C.

The pattern of currents associated with the thermal bar is a two-cell structure with a powerful downward jet between the cells. The two counter-rotating cells can be seen in all figures. As for the circulations in the cells, intensities of currents in them differ from each other. The influence of the lateral boundary and the bottom slope are expressed in the intensification of the upwelling near the shore. The open-lake cell is less intensive and more diffuse. It is interesting to analyse the jet behaviour. If it meets the bottom, further movement continues along the slope, as it is in Figure 7. The jet axis is displaced with time. Sometimes the spiral character of jet can be observed. The main question is how deep the jet spreads. As it is seen in Figures 7–9, the presence of the downward movement is distingwished till great depth although the values of the vertical velocity decrease. To compensate for the volume of sinking water, cold and warm currents move to meet each other in the thin surface layer. It is here, that the u-component of vector velocity reaches its maximum values. It is interesting to estimate

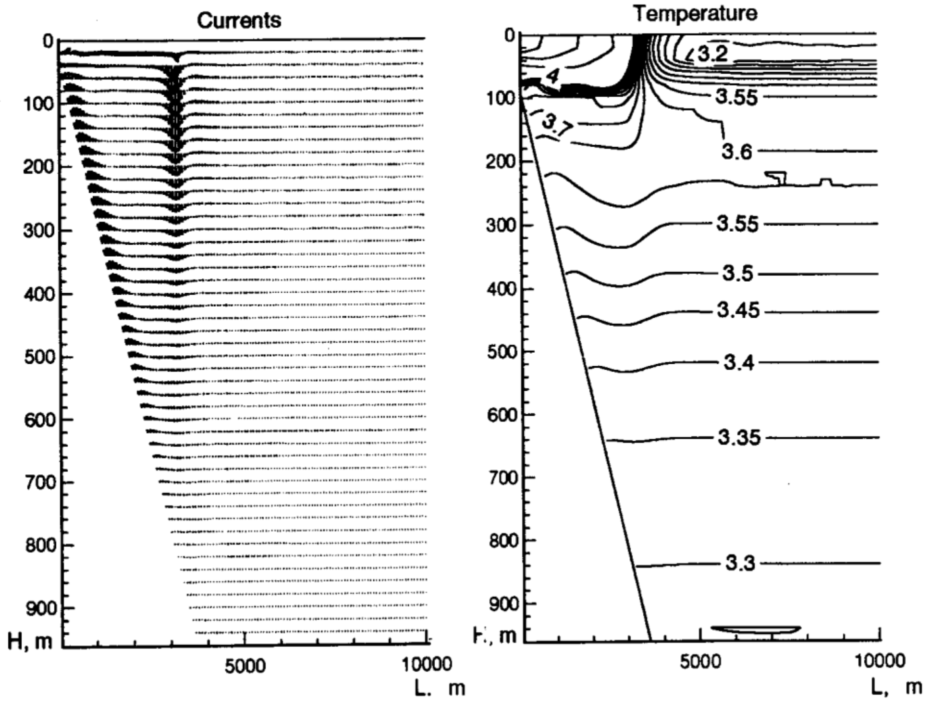

Figure 8. The same as in the Figure 1 but after 264 hours

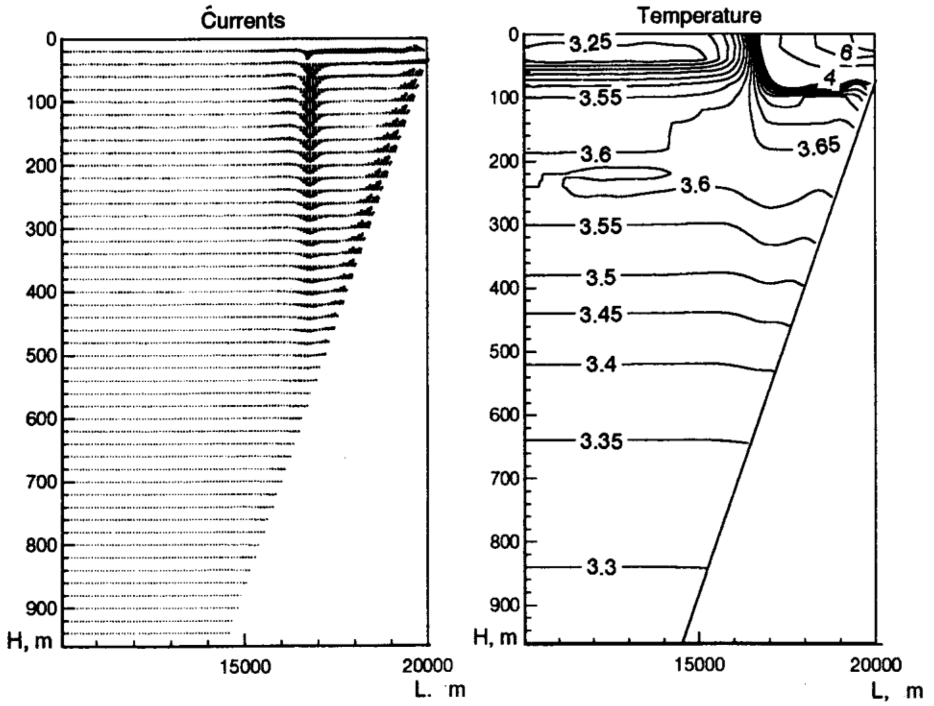

Figure 9. The same as in the Figure 2 but after 264 hours

the quantitative charactrstics of currents. It is important to evaluate the vertical component of vector velocity as a measure of the intensity of vertical redistribution of heat, oxygen and other chemical elements and pollutants. Some quantitative estimates were made for the thermal bar observed in June 1991 [14]. The first one was obtained from the analyses of the temperature series collected at the different depths. The temperature minimum appeared some 30 hours later at a depth of 580 m than at a depth of 290 m. A simple correlation between time and distance gave the value of 0.3 cm/s. The same value was calculated from the theoretical formula for the vertical velocity in the frontal zone [4]

$$w = \frac{g\nu d_s}{(l/2)^{3/2} B_m^2},$$

where B_m is the width of the mixing zone, d_s the difference of levels due to increasing density.

It seems strange but the values of vertical velocities calculated in the model agree with the above-mentioned estimates.

The mean values in the ratio between the components of vector velocity associated with the thermal bar were obtained in the model as $w : u : v \sim 1 : 10 : 20$–$100$. Approximately the same order of values was observed in June 1993 [8].

4. Conclusions

The motivation for the present study comes from the desire to construct a numerical model which could be used as a means for the understanding of the phenomenon. The model proposed is realized in such a way that the integration for many time steps is possible. The stable realization in a wide range of the input parameters is allowable. One of the main aims in the simulation of bar is the reproduction of temperature front and its propagation in time and space. The numerical experiments show that the temperature front is not diffused too much. Certainly, it is a positive feature of the model. Application of the monotonic and transportive schemes for advective-diffusive operators excludes the occurence of non-physical solutions.

Numerous calculations show that the model is sensitive to the parameterization of turbulence. This is understandable because the underlaying physics concerns with the nonlinear behaviour of the hydrodynamical system in the near-critical state. The model gives the general pattern of the thermal bar in a deep freshwater lake as a temperature front and two-cell convective structure moving from shore to the center of the lake. The movenemt of the thermal bar is accompanied by the generation and development of the internal waves on the background of stably stratified water strata in the surroundings. The results obtained seems to be plausible and, inspite of the known limitations of 2D models, we intend to use this model in further investigations.

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