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Three-factors analysis of the computer palette and compression of the colour images

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The aim of this paper is to introduce the new kind of decomposition for computer colour images based on the three-factors analysis of the palette and to demonstrate the improvement of the compression coefficients in the comparison with the well-known PAL television oriented decomposition standard.

1. Factor analysis based on apriori given palette

Let the restricted palette π_N be given in the red-green-blue coordinates

$$\pi_N = \{P_i = (R_i, G_i, B_i),\ i = 1, 2, \ldots, N\}, \tag{1}$$

and R_i, G_i, B_i be non-negative integers. Let us determine the "center of gravity" as the vector P^0 with the coordinates

$$R^0 = \Big(\sum_{i=1}^{N} R_i\Big)\Big/N, \quad G^0 = \Big(\sum_{i=1}^{N} G_i\Big)\Big/N, \quad B^0 = \Big(\sum_{i=1}^{N} B_i\Big)\Big/N. \tag{2}$$

At first we find the "main direction" ν in the palette π_N, i.e., the straight line L of the form

$$L = \{P \in R^3 : P = P_0 + t\nu,\ -\infty < t < +\infty\}, \tag{3}$$

where ν is a vector with the unit Euclidian length, and the functional

$$\Phi(\nu) = \sum_{i=1}^{N} \rho^2(L, P_i) \tag{4}$$

to be minimal; here $\rho(L, P_i)$ means the Euclidian distance between the line L and the palette point P_i. After the simplest linear transformation $P' = P - P^0$ we have

$$\rho^2(L, P_i) = \rho^2(L', P_i') = \|P_i'\|^2 - (P_i', \nu)^2.$$

Then we have the minimization problem

$$\Phi(\nu) = \sum_{i=1}^{N} \left[\|P_i'\|^2 - (P_i', \nu)^2\right] \to \min_{\nu} \tag{5}$$

under the natural constraint $\|\nu\|^2 = 1$, or the maximization problem arises

$$\psi(\nu) = \sum_{i=1}^{N} (P_i', \nu)^2 \to \max_{\nu}, \quad \|\nu\|^2 = 1. \tag{6}$$

Using the Lagrange function

$$L(\nu, \lambda) = \psi(\nu) + \lambda(1 - \|\nu\|^2) \tag{7}$$

after the differentiation we obtain

$$\frac{1}{2}\frac{\partial L}{\partial \nu_k} = \Big(\sum_{i=1}^{N} x_i^{(k)} P_i', \nu\Big) - \lambda \nu_k = 0, \quad k = 1, 2, 3, \tag{8}$$

where $x_i^{(1)} = R_i - R^0$, $x_i^{(2)} = G_i - G^0$, $x_i^{(3)} = B_i - B^0$, $i = 1, 2, \ldots, N$. If we denote by B the rectangular $N \times 3$–matrix

$$B = \{x_i^{(k)}\}_{i=\overline{1,N},\, k=\overline{1,3}},$$

then the relations (8) can be rewritten in the matrix form

$$B^T B \nu = \lambda \nu, \tag{9}$$

i.e., ν is the eigenvector of $B^T B$, and $\|\nu\|^2 = 1$. There are three orthonormalized eigenvectors $\nu^{(1)}$, $\nu^{(2)}$, $\nu^{(3)}$ for problem (9), and let $B^T B \nu^{(l)} = \lambda_l \nu^{(l)}$, $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0$, $l = 1, 2, 3$. Let us find $\psi(\nu^{(l)})$,

$$\psi(\nu^{(l)}) = \sum_{i=1}^{N} \Big(\sum_{k=1}^{3} x_i^{(k)} \nu_k^{(l)}\Big)^2 = (B^T B \nu^{(l)}, \nu^{(l)}) = \|B\nu^{(l)}\|^2 = \lambda_l^2. \tag{10}$$

Thus, the maximal value of $\psi(\nu)$ or minimal value of $\phi(\nu)$ take place on the eigenvectors $\nu^{(1)}$ which corresponds to the maximal eigenvalue λ_1 of the matrix $B^T B$.

If we construct now the plane which includes "the center of gravity" P^0 of the palette π_N and is orthogonal to the eigenvector $\nu^{(1)}$, and find in this plane the best direction, then it corresponds to the second eigenvector $\nu^{(2)}$ for the matrix $B^T B$. The last eigenvector $\nu^{(3)}$ determines the least important direction in the palette π_N. Thus, the eigenvectors $\nu^{(1)}$, $\nu^{(2)}$, $\nu^{(3)}$ determine three factors in the order of "importance decay" with respect to the *apriori* given palette π_N, and the levels of "importance", as we suppose, are regulated by the ratio $\lambda_1 : \lambda_2 : \lambda_3$.

According to this 3-factors analysis of the palette the colour RGB-image can be decomposed into 3 components (3FA-decomposition) F_1, F_2, F_3 by the following rule:

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} \nu_1^{(1)} & \nu_2^{(1)} & \nu_3^{(1)} \\ \nu_1^{(2)} & \nu_2^{(2)} & \nu_3^{(2)} \\ \nu_1^{(3)} & \nu_2^{(3)} & \nu_3^{(3)} \end{pmatrix} \cdot \begin{pmatrix} R - R^0 \\ G - G^0 \\ B - B^0 \end{pmatrix}, \qquad (11)$$

or the matrix form

$$F = V(P - P^0).$$

The inverse transformation is also very simple

$$P = P^0 + V^T F,$$

because V is an orthogonal matrix.

This kind of decomposition for the colour image we call 3FA standard.

2. Comparison of PAL and 3FA standards with respect to $\Sigma\Pi$-compressions

PAL is the famous television oriented standard [1] based on transformation

$$\begin{pmatrix} Y \\ U \\ V \end{pmatrix} = \begin{pmatrix} 0,30 & 0,59 & 0,11 \\ -0,15 & -0,29 & 0,44 \\ 0,62 & -0,51 & -0,1 \end{pmatrix} \cdot \begin{pmatrix} R \\ G \\ B \end{pmatrix},$$

which tends to the forthcoming compression. As the compression algorithm we use here the so called $\Sigma\Pi$-approximation of the scalar function $f(x, y)$ with two independent variables by the sum of products of the functions with one variable. The algorithm for the optimal determination of $\Sigma\Pi$-approximation in the Hilbert cross-norms is described in [2], [3] and based on one-dimensional polynomial splines, which are continuous or discrete, like in the pixel image case. If the number of terms in $\Sigma\Pi$-approximation is equal to $\min(NX, NY)$, where NX and NY are the pixel sizes of the image, then we have the exact restoration of image. In practice the number of these terms is essentially less, and we have fine compression effect under high quality requirements.

We execute the comparison of PAL and 3FA standards in this context with two kinds of images: usual life colour photos and computer generated images like geographic maps with the artificial colouring.

Unfortunately, it is impossible here to print colour pictures, and we describe separate half-tone components only. For the assembled compressed images the expert evaluation of quality will be presented in the Table by four-level scale: excellent, good, fair, bad. For convenience we introduce the compression ratio. The image in 3FA-decomposition and in PAL-decomposition will be compressed with the same compression coefficients. To provide the suitable quality laser printing the pictures are given in the reversed form.

Example 1. Marsh frog (photo).

Initial image is given in the IBM-standard 256 colours palette at 320×200 pixels screen. Three-factor analysis of this palette gives the following results:

$$P^0 = (24, 25, 19),$$

$$\lambda_1 : \lambda_2 : \lambda_3 = 17.7 : 2.87 : 1,$$

$$V = \begin{pmatrix} .5997605E+00 & .6662923E+00 & .4431049E+00 \\ .7984537E+00 & -.4619793E+00 & -.3860659E+00 \\ .5252742E-01 & -.5853458E+00 & .8090804E+00 \end{pmatrix}.$$

Three components in 3FA and PAL standards in the initial (upper part of screen) and compressed (bottom part of screen) forms are presented in Figures $1 \div 6$.

Example 2. Skier (photo).

Three-factor palette analysis gives the following results:

$$P^0 = (36, 34, 39),$$

$$\lambda_1 : \lambda_2 : \lambda_3 = 207.34 : 3.29 : 1,$$

$$V = \begin{pmatrix} .5348475E+00 & .6074323E+00 & .5873365E+00 \\ -.7594221E+00 & .4084173E-01 & .6493151E+00 \\ ..3704271E+00 & -.7933209E+00 & .4831416E+00 \end{pmatrix}.$$

The components of 3FA and PAL standards in the initial and the compressed forms are presented in Figures $7 \div 12$.

Example 3. Map (artificial colouring).
Palette analysis gives

$$P^0 = (31, 30, 31),$$

$$\lambda_1 : \lambda_2 : \lambda_3 = 1.99 : 1.16 : 1,$$

$$V = \begin{pmatrix} .5210022E+00 & .5927272E+00 & .6141915E+00 \\ -.8461934E+00 & .4529785E+00 & .2806551E+00 \\ -.1118636E+00 & -.6659467E+00 & .7375646E+00 \end{pmatrix}.$$

In Figures $13 \div 18$ 3FA and PAL components are presented for the various compression coefficients.

The results of compression for 3FA and PAL standards and their expert evaluation for the various compression ratios are presented in the Table.

Expert evaluations of the quality

Figure	Compression ratio	Expert evaluation for	
		3FA standard	PAL standard
frog	100 : 17 : 6	Excellent	Excellent
	100 : 7 : 3	Good	Good
	40 : 7 : 3	Fair	Fair
skier	100 : 7 : 3	Excellent	Good
	100 : 2 : 1	Good	Fair
	40 : 7 : 3	Fair	Fair
	40 : 2 : 1	Fair	Bad
map	100 : 28 : 24	Excellent	Excellent
	100 : 14 : 12	Excellent	Good
	100 : 6 : 5	Good	Fair
	40 : 6 : 5	Fair	Bad

3. Conclusion

In our experiments with the decomposition and compression of the colour images we have not the aim to get the exact answer to the question: what is better, the well-known PAL-standard or new 3FA-standard? In fact, our experiments show that 3FA-standard is usually better, and it is not surprising because the additional information on the palette was used in decomposition. On the other hand PAL-standard is more universal and has the same coefficients for all pictures and palettes.

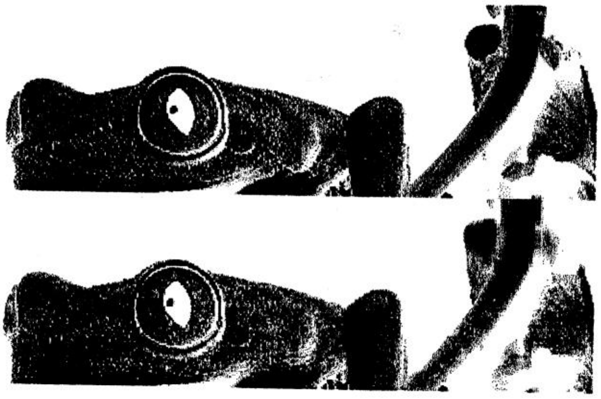

Figure 1. F_1-component, compression coefficient $k = 0.4$

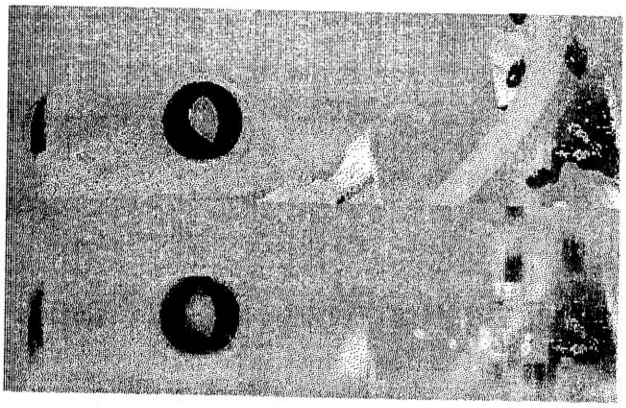

Figure 2. F_2-component, compression coefficient $k = 0.17$

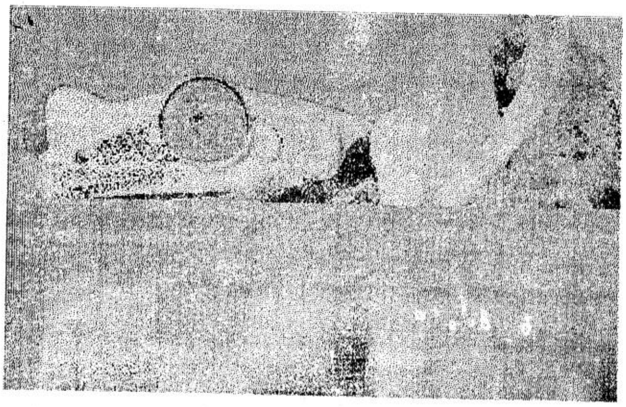

Figure 3. F_3-component, compression coefficient $k = 0.06$

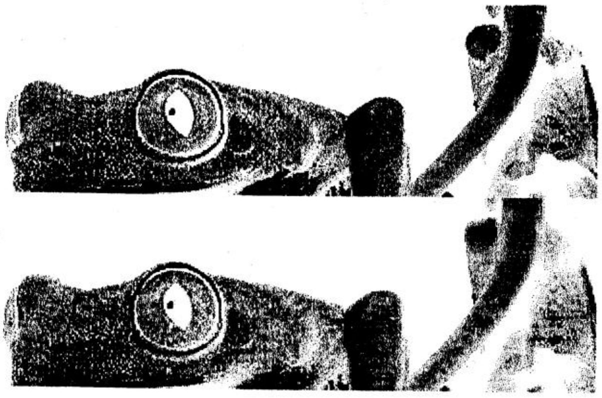

Figure 4. Y-component, compression coefficient $k = 0.4$

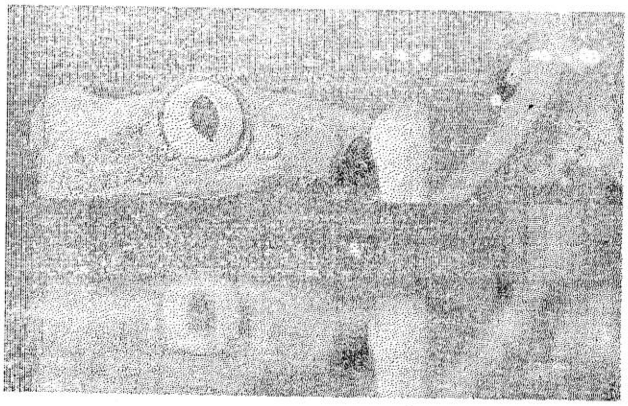

Figure 5. U-component, compression coefficient $k = 0.17$

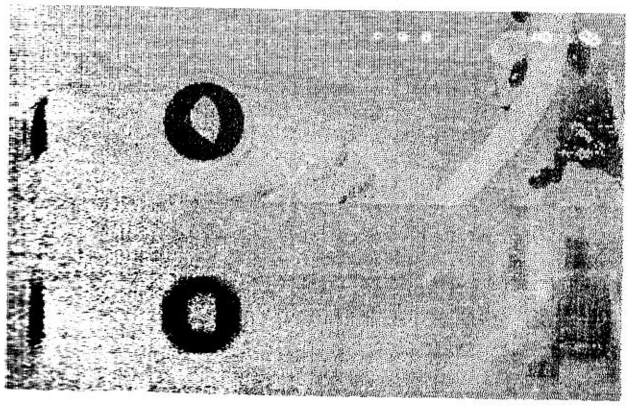

Figure 6. V-component, compression coefficient $k = 0.06$

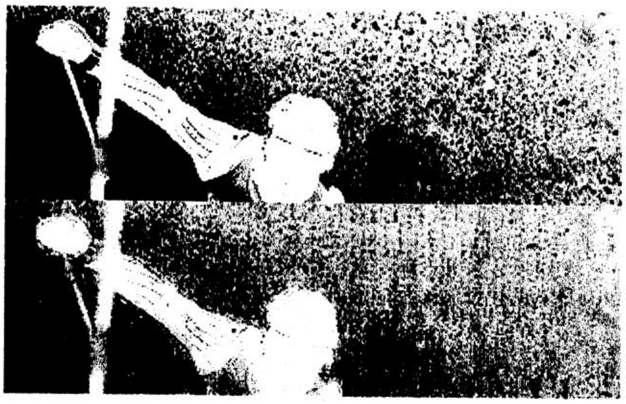

Figure 7. $F1$-component, compression coefficient $k = 0.4$

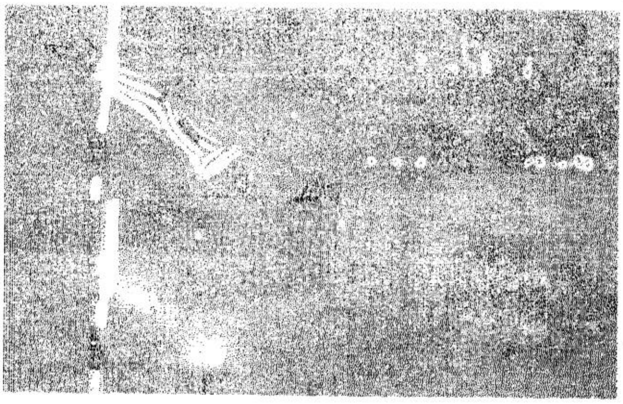

Figure 8. $F2$-component, compression coefficient $k = 0.07$

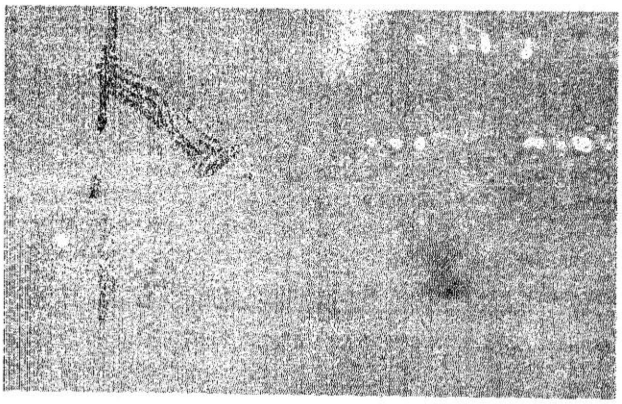

Figure 9. $F3$-component, compression coefficient $k = 0.03$

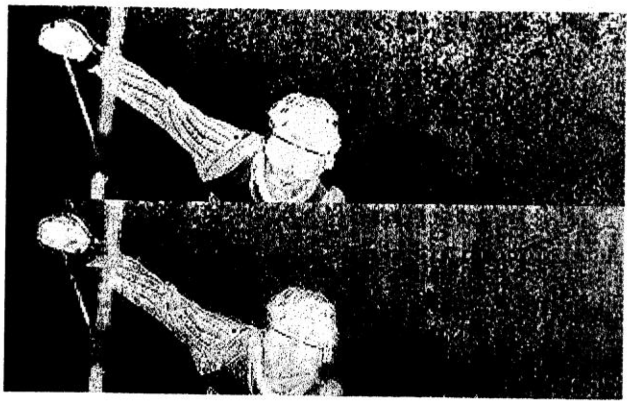

Figure 10. Y-component, compression coefficient $k = 0.4$

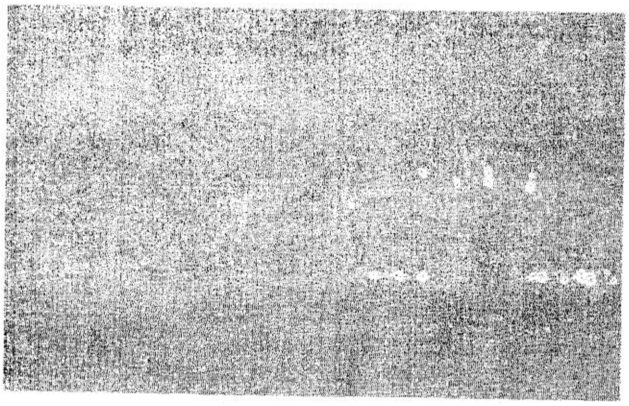

Figure 11. U-component, compression coefficient $k = 0.07$

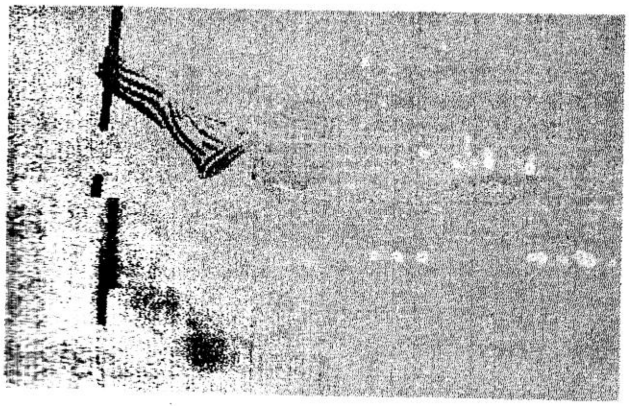

Figure 12. V-component, compression coefficient $k = 0.03$

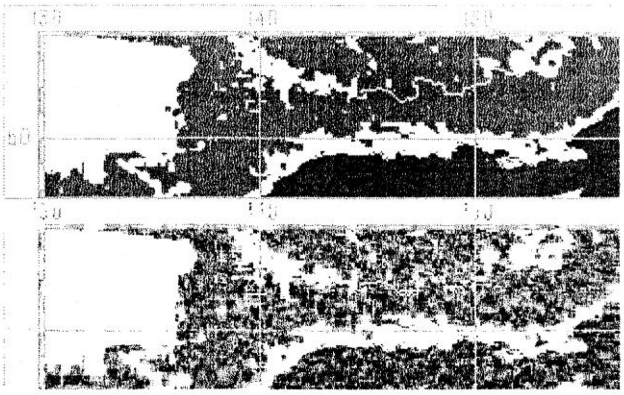

Figure 13. $F1$-component, compression coefficient $k = 0.4$

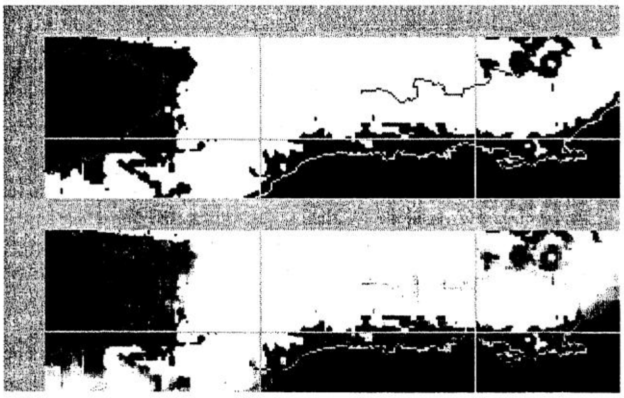

Figure 14. $F2$-component, compression coefficient $k = 0.14$

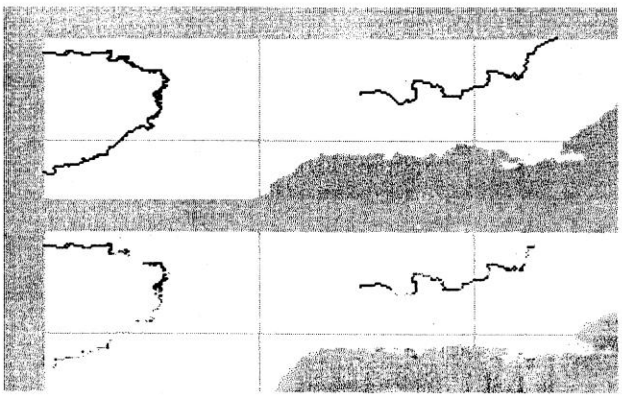

Figure 15. $F3$-component, compression coefficient $k = 0.12$

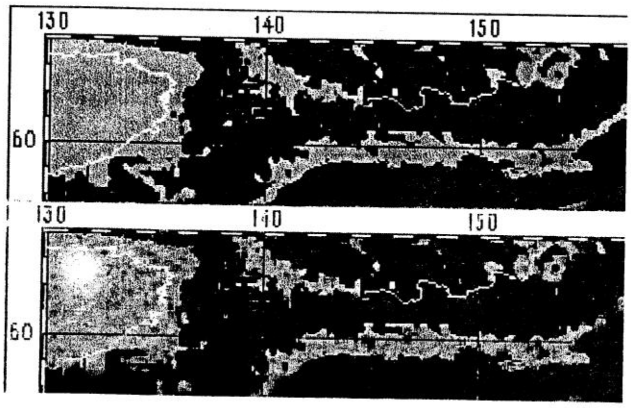

Figure 16. Y-component, compression coefficient $k = 0.4$

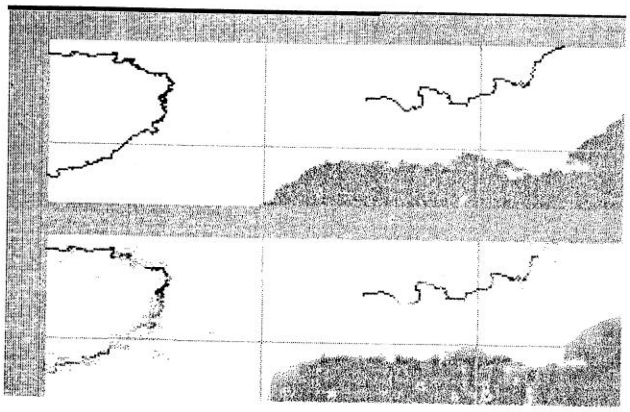

Figure 17. U-component, compression coefficient $k = 0.14$

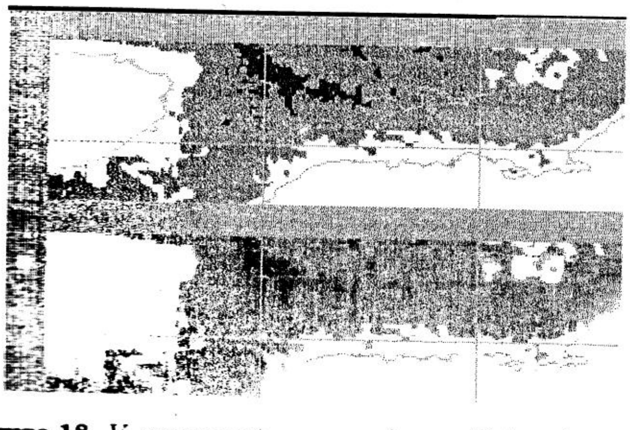

Figure 18. V-component, compression coefficient $k = 0.12$

It is dangerous to compress the main components F1 and Y both in 3FA and PAL standards but the rest components can be essentially compressed without loss of quality. The components F1 and Y are quite informative in the colour image, and the rest components which provide the beautiful colouring of the world around us, are not essentially important. Are the fine woman's lips more important thing than the colour of the lipstick? Coauthors of this paper have no common opinion on this point.

References

[1] D.H. Pritchard, U.S. Color television fundamentals, A Review, IEEE Transactions on Consumer Electronics, CE-23(4), Nov. 1977, 467–478.

[2] V.A. Vasilenko, The best finite dimensional ΣΠ-approximation, Sov. J. Num. Anal. Math. Mod., 5, 1990, No.4/5, 435–443.

[3] O.E. Baklanova, V.A. Vasilenko, Data compression with ΣΠ-approximations based on splines, NCC Bulletin, series "Numerical Analysis", issue 2, 1993, 1–7.