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An inverse problem of electromagnetoelasticity: simultaneous determination of elastic and electromagnetic parameters and the unknown source of elastic oscillations*

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This paper describes a numerical method for the solution to the inverse problem for the equations of electromagnetoelasticity. We consider the case when electromagnetic waves are generated by elastic deformations. At the same time, we neglect the reverse influence of the electromagnetic field on the elastic oscillations.

We focus our attention on one of the simplest versions of the system of the equations of electromagnetoelasticity which, at the same time, preserves the basic properties of more adequate model. The solution to the inverse problem (the unknown elastic and electromagnetic medium parameters) is sought by means of minimization of the data misfit functionals which are the mean square deviations of the registered fields from the fields calculated for some "test" medium models.

Results of numerical experiments are given to illustrate the efficiency of the method.

Introduction

We consider one of the possible statements of inverse problems connected with electrodynamics of vibrating elastic media. In order to introduce corresponding equations, we need some preliminary discussion.

The motion of an elastic conductive medium in the electromagnetic field is described by two sets of equations: that of elasticity and that of electrodynamics. Just as in magnetic hydrodynamics [1], these equations are interdependent due to the presence of additional terms that account for effects related to the motion of the elastic conductive medium in the electromagnetic field. The waves arising in the result of this interaction are usually referred to as electromagnetoelastic. The first attempts to apply the theory of electromagnetoelasticity to the study of the process of wave propagation in elastic conductive media were made by L. Knopoff [2], P. Chadwick [3], J. Dunkin

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and A. Eringen [4], V.I. Keilis–Borok and A.S. Monin [5], V.I. Kalinin [6], and G. Paria [7]. L. Knopoff studied the influence of electromagnetic fields on the propagation of elastic waves and arrived at the conclusion that in the class of geophysical problems the effect of electromagnetic phenomena on the process of elastic waves propagation is negligible, at least in the case of not too large electromagnetic disturbances.

1. Basic equations

We assume that the model under consideration satisfies the basic hypotheses of continuum mechanics: continuity, euclidity, and absoluteness of time. The first hypothesis means that an uninterrupted continuum is considered, the second one implies the possibility to introduce a Cartesian frame of reference for all points, and according to the third hypothesis relativistic effects are not taken into account. Moreover, the model is inapplicable in the case of strong magnetic fields. We also assume that electromagnetoelastic waves arise under the action of mechanical perturbations, and that one can neglect the effect of electromagnetic waves on the process of propagation of elastic oscillations and also neglect the displacement currents as compared with conduction currents. Lastly, we will consider the fields of small perturbations.

Now we can write down our equations. The assumption that we consider the fields of small perturbations allows us to consider the linearized statement of the problem when the displacement vector $\boldsymbol{\mathcal{U}}$, the vector of the electric field intensity $\boldsymbol{\mathcal{E}}$, and the vector of the magnetic field intensity $\boldsymbol{\mathcal{H}}$ can be represented in the form

$$(\boldsymbol{\mathcal{U}}, \boldsymbol{\mathcal{E}}, \boldsymbol{\mathcal{H}}) = (\mathbf{0}, \mathbf{0}, \boldsymbol{H}^0) + (\boldsymbol{u}, \boldsymbol{E}, \boldsymbol{H}), \tag{1}$$

where $(\mathbf{0}, \mathbf{0}, \boldsymbol{H}^0)$ is the value related to the unperturbed state of the medium ($\boldsymbol{H}^0$ is a constant vector); and the vectors $\boldsymbol{u} = (u_1, u_2, u_3)$ (displacement of the points of the medium from the reference configuration), $\boldsymbol{E} = (E_1, E_2, E_3)$ (intensity of the electric field), and $\boldsymbol{H} = (H_1, H_2, H_3)$ (intensity of the magnetic field) correspond to small perturbations of the elastic and electromagnetic fields. Besides, in view of our assumptions one can consider that the process of elastic waves propagation is governed by the usual system of differential equations of the theory of elasticity:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sum_{j=1}^{3} \frac{\partial T_{ij}(\boldsymbol{u})}{\partial x_j}, \qquad i = 1, 2, 3, \tag{2}$$

where the stress tensor $T_{ij}(\boldsymbol{u})$ is defined in terms of the components u_i of the displacement vector and in the case of an isotropic magnetoelastic medium has the form

$$T_{ij}(\boldsymbol{u}) = \kappa\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \delta_{ij}\lambda \operatorname{div} \boldsymbol{u}, \qquad i,j = 1,2,3. \tag{3}$$

Here ρ, λ, κ denote the density of the medium and the Lamé coefficients respectively, and δ_{ij} is the Kronecker symbol.

The process of electromagnetic waves propagation through an elastic conductive medium is described in our case by the following system of equations:

$$\operatorname{rot} \boldsymbol{H} = \boldsymbol{J}, \qquad \frac{\partial \boldsymbol{B}}{\partial t} = -\operatorname{rot} \boldsymbol{E}, \qquad \operatorname{div} \boldsymbol{B} = 0, \tag{4}$$

where, in virtue of our assumptions, the constitutive relations are written as

$$\boldsymbol{B} = \mu(\boldsymbol{H}^0 + \boldsymbol{H}), \qquad \boldsymbol{J} = \sigma\left(\boldsymbol{E} - \mu\frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{H}^0\right). \tag{5}$$

Here μ is the magnetic permeability, and σ is the conductivity of the medium. Such are, in general outline, the differential equations describing the process of interaction of electromagnetic and elastic waves in our case.

Now we proceed to the statement of the direct problem for differential equations (2)–(5). Consider the rectangular Cartesian frame of the reference $(x_1, x_2, x_3) = \mathbf{x}$. Let the plane $x_3 = 0$ be the interface of two media of the types "air" ($x_3 < 0$) and "conductive ground" ($x_3 > 0$). Electromagnetic and elastic characteristics of the ground are described by piecewise constant functions with break planes parallel to the plane $x_3 = 0$. Let us introduce the notation

$$[f]_\Gamma = f|_{\Gamma^+} - f|_{\Gamma^-},$$

i.e., the symbol $[f]_\Gamma$ denotes the jump of the function f on the oriented surface Γ in the direction from the inner to the outer side of Γ. We assume that elastic oscillations arise under the action of a force source concentrated at the origin of coordinates

$$T_{k,3}(\boldsymbol{u})|_{x_3=0} = \delta_{k,3} f(t)\delta(x_1, x_2), \qquad k = 1,2,3, \tag{6}$$

where $\delta(\cdot)$ is the generalized Dirac delta.

As concerns the force source and initial data, we assume that the function $f(t)$ and the electromagnetoelastic field are absent before the moment $t = 0$, i.e.,

$$(f, \boldsymbol{u}, \boldsymbol{E}, \boldsymbol{H})|_{t<0} \equiv 0. \tag{7}$$

To single out the unique solution to the direct problem, one has to require the fulfillment of the radiation condition at infinity:

$$\lim_{|\mathbf{x}|\to\infty} (\boldsymbol{E}, \boldsymbol{H}) = 0. \tag{8}$$

Moreover, on the planes where the coefficients of the problem have breaks we require the fulfillment of standard consistency conditions

$$[u_m] = [E_k] = [H_k] = [T_{m,3}(\boldsymbol{u})] = 0, \qquad k = 1,2, \quad m = 1,2,3. \tag{9}$$

Thus, the direct problem consists in finding the vector functions $\boldsymbol{u}$, $\boldsymbol{E}$, $\boldsymbol{H}$ satisfying equations (2)–(9) providing that we know the elastic and electromagnetic characteristics of the medium and the constant vector $\boldsymbol{H}^0$ characterizing the magnetic field of the Earth.

Our main task will consist in showing the possibility of applying the optimizational approach to the simultaneous determination of electromagnetic and elastic characteristics of the medium and the function $f(t)$ from the system (2)–(9) basing on some additional information on the components of the vector functions $\boldsymbol{u}$, $\boldsymbol{E}$. We will study a special case of the formulated problem which, however, will reflect many principal points of the more general case.

It should be noted that the problem of determination of elastic and electromagnetic characteristics of a medium with account of interaction of two fields was considered also in the works by A. Lorenzi and V.G. Romanov [8], M.M. Lavrent'ev (jr.) and V.I. Priimenko [9], A. Lorenzi and V.I. Priimenko [10], V.G. Romanov [11], and I.Z. Merazhov and V.G. Yakhno [12]. One should also mention the work by O.A. Klimenko [13], where a numerical algorithm to determine the coefficient of the electromagnetoelastic coupling was proposed.

As concerns the form of the sensing signal (i.e., the function $f(t)$), in most of cases of real geophysical investigations it is either unknown or is given only approximately, while its accurate estimate is necessary for practical solution to many inverse problems. Note that problems of simultaneous reconstruction of the structure of the medium and the form of the sensing signal were studied theoretically by A. S. Blagoveschenskij [14] and K. G. Reznickaya [15]. The most complete bibliography on the questions of numerical solution can be found in the work by P. Carrion *et al* [16].

2. Statement of the inverse problem

Now let us state the inverse problem that will be studied in the present paper. Let z denote the variable x_3. Consider the functions

$$V(z) = \left(\frac{\lambda + 2\kappa}{\rho}\right)^{1/2}, \qquad c(z) = \left(\frac{1}{\sigma\mu}\right)^{1/2},$$

where $V(z)$ is the speed of longitudinal waves and $c(z)$ is that of the diffusion process of electromagnetic waves.

Definition. We will say that the functions $V(z)$, $c(z)$, and $f(t)$ belong to the class $\mathcal{M}$, if there exist positive constants V_m, c_m, f_m, z_m, z'_m, t_m such that

$$V(z) = \begin{cases} V_m, & z \in (z'_{m-1}, z'_m), \quad m = \overline{1, k+1}, \\ V_{k+1}, & z > z'_{k+1}, \end{cases} \tag{10a}$$

$$\mathrm{c(z)} = \begin{cases} c_m, & z \in (z_{m-1}, z_m), \quad m = \overline{1, n+1}, \\ c_{n+1}, & z > z_{n+1}, \end{cases} \tag{10b}$$

$$\mathrm{f(t)} = \begin{cases} f_m, & t \in (t_{m-1}, t_m), \quad m = \overline{1, l+1}, \\ 0, & t > t_{m+1}, \end{cases} \tag{10c}$$

where $z_0 = z'_0 = t_0 = 0$; $n, k, l \in \boldsymbol{N}$.

Henceforth we will always assume that the functions $V(z)$, $c(z)$, $f(t)$ belong to the class $\mathcal{M}$.

Consider the functions

$$u(z,t) = \operatorname{Re} F_{x_1x_2}(u_3)|_{\nu_1=\nu_2=0}, \tag{11a}$$

$$E(z,t) = \operatorname{Re} F_{x_1x_2}(E_1)|_{\nu_1=\nu_2=0}, \tag{11b}$$

where $F_{x_1x_2}(\cdot)$ denotes the generalized Fourier transform in the variables x_1, x_2; and (ν_1, ν_2) are the variables dual to them. Starting from equations (2)–(9), we can write down the system of relations for the functions u, E in the domain $z \geq 0$

$$\frac{\partial^2 u}{\partial t^2} = V^2(z)\frac{\partial^2 u}{\partial z^2}, \qquad (t,z) \in \boldsymbol{R} \times \Omega', \tag{12a}$$

$$u|_{t<0} \equiv 0, \tag{12b}$$

$$\left.\frac{\partial u}{\partial z}\right|_{z=0} = F(t), \tag{12c}$$

$$[u]_{z=z'_m} = \left[\frac{\partial u}{\partial z}\right]_{z=z'_m} = 0, \qquad m = \overline{1, k+1}, \tag{12d}$$

$$\frac{\partial E}{\partial t} = c^2(z) \cdot \frac{\partial^2 E}{\partial z^2} - \mu H \frac{\partial^2 u}{\partial t^2}, \qquad (t,z) \in \boldsymbol{R} \times \Omega, \tag{13a}$$

$$E|_{t<0} \equiv 0, \qquad \lim_{z\to\infty} E = 0, \tag{13b}$$

$$\left.\frac{\partial E}{\partial z}\right|_{z=0} = 0, \tag{13c}$$

$$[E]_{z=z_m} = \left[\frac{\partial E}{\partial z}\right]_{z=z_m} = 0, \qquad m = \overline{1, n+1}, \tag{13d}$$

where

$$\Omega' = \boldsymbol{R}_+ \setminus \{z = z'_m, \, m = \overline{1, k+1}\},$$

$$\Omega = \boldsymbol{R}_+ \setminus \{z = z_m, \, m = \overline{1, n+1}\},$$

$F(t) = (\lambda(0) + 2\kappa(0))^{-1} f(t)$, and H is the constant characterizing the magnetic field of the Earth.

Now we formulate the inverse problem that will be studied below.

Inverse problem. Find the functions $V(z), c(z), f(t) \in \mathcal{M}$ (i.e., the set of numbers c_m, V_m, f_m), if the following additional information on the solutions to the problems (12), (13) is known:

$$u|_{z=0} = u_0(t), \tag{14}$$

$$E|_{z=0} = E_0(t), \qquad t \in [0, +\infty), \tag{15}$$

and the numbers μ, H are known also.

Remark. Without any loss of generality we will assume that $\mu = \mu_0$, where μ_0 is the magnetic permeability of vacuum.

3. An optimizational method of solution to the inverse problem of electromagnetoelasticity

To numerically solve the inverse problem stated above an optimizational approach was employed based on minimizing the objective residual functionals of observed data and data computed in solving "test" direct problems.

3.1. The first stage

At the first stage the initial-boundary value problem (12) describing the propagation of elastic waves in a vertically inhomogeneous medium was considered.

It should be noted that in spite of its simplicity the vertically inhomogeneous model of the medium not only allows one to study the basic characteristic features of the processes of arising and propagation of geophysical fields, but also in a number of cases describes rather well real geophysical conditions (e.g., the presence of a layered sediment cover). At the same time, when employing this model, one encounters main mathematical difficulties, the change-over to more realistic models (media with absorption, with anisotropy, multi-dimensionally inhomogeneous media, etc.) being impossible without their resolution.

In this model a medium is modelled by a stack of homogeneous layers lying on the homogeneous half-space. A semi-analytic method described in [17] allows one to obtain the exact solution in this case and, what is especially important, to organize the process of constructing the solution to the inverse problem in the most efficient way.

As concerns the set of equations (12), we considered the inverse problem of reconstructing the functions $V(z), f(t) \in \mathcal{M}$ by the additional information (14).

Applying the Fourier transform in the variable t, we rewrite the original statement (12), (14) in the following form:

$$\frac{d^2}{dz^2}u(z,\omega) + \nu^2 u(z,\omega) = 0, \quad z \in \Omega', \tag{16a}$$

$$\left.\frac{du(z,\omega)}{dz}\right|_{z=0} = F(\omega), \tag{16b}$$

$$[u(z,\omega)]_{z=z'_m} = \left[\frac{du(z,\omega)}{dz}\right]_{z=z'_m} = 0, \qquad m = \overline{1,k+1}, \tag{16c}$$

where $\nu^2 = \omega^2 V^{-2}(z)$,

$$F(\omega) = \int\limits_0^{+\infty} F(t)\exp(-i\omega t)dt.$$

To single out the unique solution, we assume that the principle of the limit absorption is satisfied, i. e.,

$$u(z,\omega) = \lim_{\varepsilon\to+0} u(z,\omega - i\varepsilon), \tag{17}$$

where, in its turn,

$$\lim_{z\to+\infty} u(z,\omega - i\varepsilon) = 0. \tag{18}$$

The additional information (14) is rewritten as follows:

$$u(z,\omega)|_{z=0} = u_0(\omega). \tag{19}$$

As was shown in [17], realization of the process of the search for the minimum point of the objective functional in the frequency domain ω allows one to substantially reduce the computational resources necessary for multiple solution of the direct problem and, moreover, to carry out a detailed analysis of the spectra of wave fields at each stage of calculations. In view of this result we will look for the solution to the inverse problem as the minimum point of the functional

$$\Phi_1[n(z), F(\omega)] = \int\limits_{\omega_1}^{\omega_2} |u_0(\omega) - B_1[n(z), F(\omega)](\omega)|^2\, d\omega, \tag{20}$$

where (ω_1, ω_2) is the range of temporal frequencies defined by the spectral contents $F(\omega)$ of the sensing signal, and $B_1[n(z), F(\omega)]$ is a nonlinear operator mapping the functions $n(z) = V^{-2}(z)$ and $F(\omega)$ into the solution to the direct problem (16)–(18) at $z = 0$.

One can prove the Frechét differentiability of the functional (20) with respect to its arguments $n(z)$ and $F(\omega)$ and then obtain the following expressions for its gradients:

$$\nabla_{n(z)}\Phi_1\,[n(z),F(\omega)](\xi) = \\ -2\mathrm{Re}\int\limits_{\omega_1}^{\omega_2}(\omega+i\varepsilon)^2 F(\omega)[u_0(\omega)-B_1[n(z),F(\omega)](\omega)]\times\overline{\mathcal{G}}_1(\xi,\omega)d\omega, \quad (21)$$

$$\nabla_{F(\omega)}\Phi_1[n(z),F(\omega)](\omega) = \\ -2\,\mathrm{Re}\,[u_0(\omega)-B_1[n(z),F(\omega)](\omega)]\times\overline{\mathcal{G}}_1(\xi,\omega) \\ -2i\,\mathrm{Im}\,[u_0(\omega)-B_1[n(z),F(\omega)](\omega)]\times\overline{\mathcal{G}}_1(\xi,\omega), \quad (22)$$

where $\mathcal{G}_1(\xi,\omega)$ is the solution to problem (16)–(18) for $F(\omega)\equiv 1$, and the bar over the symbol of the function denotes the complex conjugation.

Let there exist a point (n_s,F_s) at which the gradients of the functional vanish. Then from (21), (22) one can easily obtain the following expression:

$$F_s(\omega) = \frac{\overline{\mathcal{G}}_1(0,\omega)\cdot u_0(\omega)}{|\mathcal{G}_1{}^2(0,\omega)|^2}, \quad (23)$$

where $\mathcal{G}_1(0,\omega)$ is the solution to problem (16)–(18) for $F(\omega)\equiv 1$ and $n(z) = n_s(z)$ taken at the point $z=0$.

V.A. Cheverda and T.A. Voronina [18] proposed to use a formula analogous to (23) for computing the impulse $F_k(\omega)$ on the k-th iteration when solving the inverse problem of VSP (vertical seismic profiling). A.V. Avdeev and E.V. Goruynov [19] described the application of this algorithm for the solution to the inverse dynamical problem of seismics with unknown source in the case when entire wave field is registered on the free surface $z=0$.

Using the obtained expressions (21) and (23), we can apply the optimizational methods of descent of the first order for the search of the minimum point of the functional (20), i.e., for reconstruction of the unknown functions $V(z)$ and $F(t)$. If we succeed in reconstructing these functions, then, having solved the direct problem (16)–(18), we can determine the spectrum of the wave field $u(z,\omega)$ in the whole of the half-space under study, i.e., find the right-hand side in the differential equation for the electric field in problem (13).

3.2. The second stage

On the second stage the initial-boundary value problem (13) is considered which after application of the Fourier transform in the variable t can be written as

$$\frac{d^2}{dz^2}E(z,\omega) + \eta^2(z)E(z,\omega) = -i\omega\mu_0 H\eta^2(z)u(z,\omega), \quad z \in \Omega, \tag{24a}$$

$$\left.\frac{dE(z,\omega)}{dz}\right|_{z=0} = 0, \tag{24b}$$

$$[E(z,\omega)]_{z=z_m} = \left[\frac{dE(z,\omega)}{dz}\right]_{z=z_m} = 0, \qquad m = \overline{1, n+1}, \tag{24c}$$

where $\eta^2(z) = -i\omega c^{-2}(z)$.

The additional information (15) is rewritten as

$$E(z,\omega)|_{z=0} = E_0(\omega). \tag{25}$$

We will look for the solution to the inverse problem (24), (25) as the minimum point of the objective functional

$$\Phi_2[\sigma(z)] = \int_{\omega_1}^{\omega_2} |E_0(\omega) - B_2[\sigma(z)](\omega)|^2 d\omega, \tag{26}$$

where $B_2[\sigma(z)]$ is a nonlinear operator mapping the function $\sigma(z)$ (the "test" value of conductivity) into the solution to the direct problem (24) at $z = 0$.

The expression for the gradient of the objective functional (26) with respect to conductivity is written as follows:

$$\nabla_\sigma \Phi_2[\sigma(z)](\xi) = A_1(\xi) + A_2(\xi), \tag{27}$$

where

$$A_1(\xi) = 2\mu_0^2 H \mathrm{Re} \int_{\omega_1}^{\omega_2} \omega^2 [E_0(\omega) - B_2[\sigma(z)](\omega)] \times \overline{\mathcal{G}}_2(\xi,\omega)\overline{u}(\xi,\omega) d\omega, \tag{28a}$$

$$A_2(\xi) = 2\mu_0^2 H \mathrm{Im} \int_{\omega_1}^{\omega_2} \omega^3 [E_0(\omega) - B_2[\sigma(z)](\omega)] \times \overline{\mathcal{G}}_2(\xi,\omega) \times \int_0^{+\infty} \sigma(\tau) \times \overline{\mathcal{G}}_2(\tau,\omega)\overline{u}(\tau,\omega) d\tau \, d\omega, \tag{28b}$$

and $\mathcal{G}_2(\xi,\omega)$ is the Green function of problem (24).

Using formulas (27), (28), we can apply the optimizational methods of descent of the first order for the search for the minimum point of the functional (26), i. e., for reconstruction of the unknown conductivity $\sigma(z)$.

4. Numerical experiments

To carry out numerical experiments a software package was written on the language Watcom C++ with enhanced graphical interface, this package permitting to reconstruct the functions $V(z)$, $\sigma(z)$, and $f(t)$ in interactive mode employing the optimizational approach.

To organize the interactive process of the search for the minimum points of objective functionals, the conjugate gradient method in the following interpretation was used:

$$\begin{aligned} & f_{j+1}(z) = f_j(z) - \alpha_j P_j(z), \\ & \alpha_j = \arg\min_{\alpha\geq 0} \Phi[f_j(z) - \alpha P_j(z)], \\ & P_0(z) = \nabla_f \Phi[f_0(z)], \qquad P_j(z) = \nabla_f\Phi[f_j(z)] - \beta_j P_j(z), \quad j \geq 1, \\ & \beta_j = \big(\nabla_f\Phi[f_j(z)], \nabla_f\Phi[f_{j-1}(z)] - \nabla_f\Phi[f_j(z)]\big), \end{aligned}$$

where the step α_j is chosen according to the "golden section" method.

To carry out numerical experiments a rather complex model of a vertically inhomogeneous medium was chosen, this model incorporating sharp changes of the values of parameters. The reconstruction of the medium was carried out up to the depth of 1.75 km. The medium below this depth was assumed to be homogeneous. All the medium from the surface to the 1.75 km depth was partitioned into 9 layers of equal width.

As a sensing signal the impulse with a "bell-shaped envelope" was chosen (the dominating frequency $f = 20$ Hz):

$$F(\omega) = \left[\exp\left(-\left(\frac{\omega - 2\pi f}{\pi f}\right)^2\right) + \exp\left(-\left(\frac{\omega + 2\pi f}{\pi f}\right)^2\right)\right] \times \exp(-i \cdot 1.75 \cdot \omega/f). \tag{29}$$

Computations were made for temporal frequencies from 5 to 40 Hz.

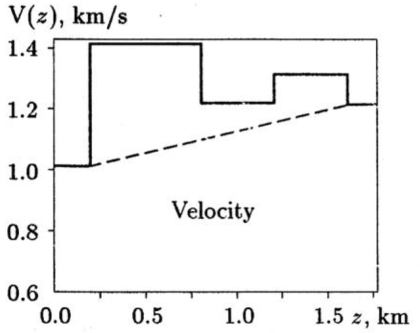

Figure 1

To compute the entire wave field $u(z,\omega)$ and the electrical field intensity $E(z,\omega)$, the semi-analytical method described in [17] was used.

To calculate the impulse $F_j(\omega)$ on the j-th iteration, we used the condition of vanishing of the gradient of the functional (20) with respect to the function F_j on the current velocity $V_j(z)$, i. e., the expression (23). In Figure 1 the velocity model of the medium (solid line) and the initial approxima-

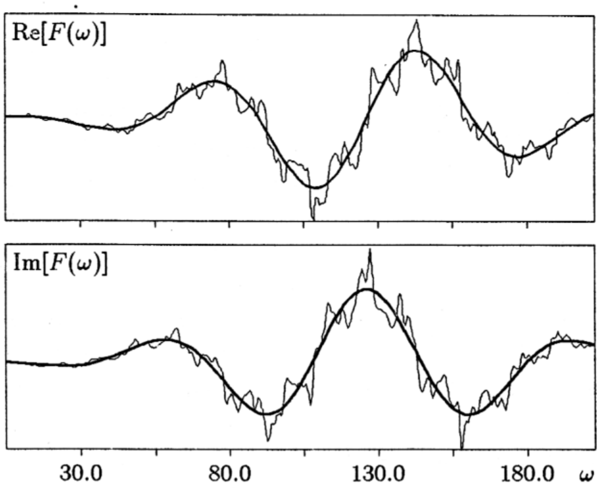

Figure 2

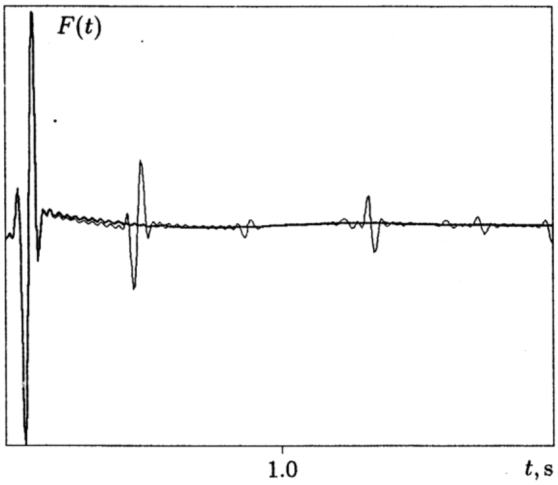

Figure 3

tion (dashed line) are shown. In Figure 2 the spectrum $F(\omega)$ of the input signal impulse $F(t)$ (thick line) and the first approximation to it (thin line) obtained from formula (23) are shown. In Figure 3 the function $F(t)$ (thick line) and the first approximation to it (thin line) are shown. In the result of 35 iterations by the method of conjugate gradients we succeeded in reconstructing with good accuracy both the velocity distribution for this medium and the functions $F(\omega)$, $F(t)$. The results of calculations are plotted in Figures 4–6 (dashed line in Figure 4, thin lines in Figures 5, 6; exact functions are plotted in thick lines).

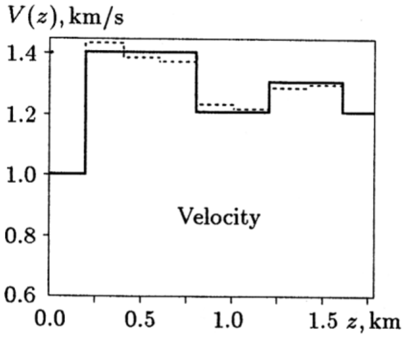

Figure 4

Here and below, pointing out the number of iterations made, we mean a practically complete stop of the iteration process on the considered stage. The quality of the obtained approximations was estimated by the closeness of the values of the corresponding functional to zero.

On the next stage, using the reconstructed functions $V(z)$ and $F(t)$, we calculated the spectrum of the wave field $u(z,\omega)$ in the whole of the half-space under study, i.e., the right-hand side in problem (24) was determined.

In Figure 7 the plot of the "true" function $\sigma(z)$ (solid line, see also the same solid line in Figure 8) and the initial approximation to it (dashed line) are shown. The final approximation computed by 68 iterations of the conjugate gradient method is plotted in Figure 8 (dashed line). As one could expect (see [20]), the deeper layers were reconstructed much worse than those lying close to the day surface.

It is conceived that one of the ways out of such a situation is the search for good initial approximations (i.e., the ones containing low-frequency components of the parameters of the medium) with the aid of an unequally scaled basis as was done in [18]. Another possible approach is the consideration of combined inverse problems [21, 22]. In this case the solution is sought as the minimum point of a comprehensive objective functional which takes into account *a priori* connections between the parameters of a medium under investigation.

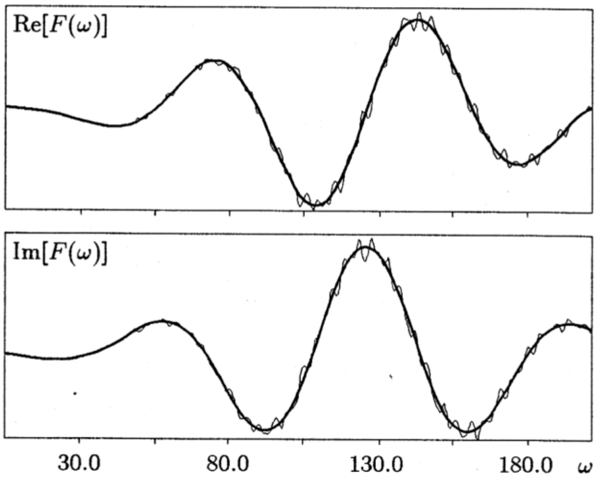

Figure 5

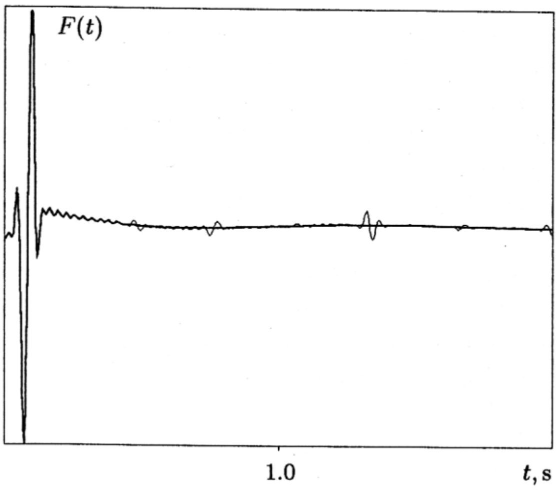

Figure 6

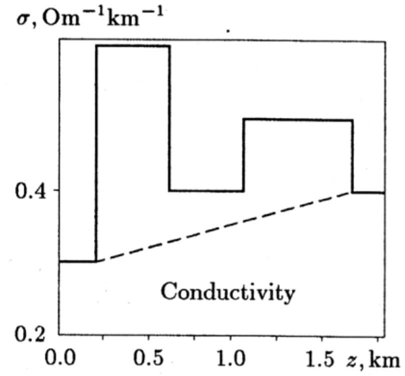

Figure 7

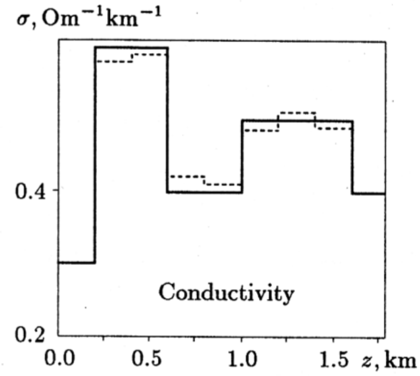

Figure 8

5. Conclusion

In the last years the optimizational approach, i. e., the search for the minimum point of the objective residual functional of observed and computed data, became one of the most popular methods of numerical solution to various statements of inverse problems. The popularity of this approach can be attributed to its universal character, the ability to take into account all the *a priori* information on the solution on each stage of computations, and to the development of computers making it possible to solve direct problems multiply in acceptable time.

The aim of this work was to demonstrate the possibility to effectively apply the optimizational approach to solving inverse problems of electromagnetoelasticity.

A rather simple (vertically inhomogeneous) model of a medium was considered. However the investigation of such models provides us with a possibility to apply the developed techniques to the numerical solution to more complex formulations of inverse problems (simultaneous determination of the speeds of longitudinal and transversal waves and conductivity, etc.). The authors consider the solution to these problems as further stages of their research.

To conclude with, we would like to emphasize once again that the algorithms for solution to inverse problems of electromagnetoelasticity that were proposed till now were not realized as working software packages and were not tested in numerical experiments.

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