Bull. Nov. Comp. Center, Math. Model. in Geoph., 8 (2003), 7–18

On the calculation of parameters for CDP observation system

A.S. Alekseev, V.A. Tsetsokho

1. The problem of inhomogeneity of the observation system in the Common Depth Point Method (CDPM) is discussed in [1]. It results from the following. At the construction of the CDP seismograms by using the data obtained by the method of multiple overlaps (MMO), there appear the CDP configurations whose parameters, generally speaking, depend on the location of the picket over which the seismogram is formed. It is shown in [2] that practicing geophysicists working with the CDP method have always been aware of this fact. In the available literature, however, there are no general formulas or algorithms for the calculation of these parameters. The method of graphical display of the MMO schemes on the so-called generalized plane is, however, well-known. It makes it possible to determine these parameters for each specific configuration of side (or central) arrangements of the Common Blast Point (CBP) from which an MMO array is formed.

The authors of [1] propose a purely logical method based on a "superposition principle". It allows us to determine the parameters of the CDP groups for any configurations of the initial CBP arrangements. Although this reasoning is recurrent, no general recurrent formulas are given. In our consideration of these questions, we managed to find explicit expressions for the parameters of the CDP groups depending on the picket number and at any parameters of the initial CBP arrangements on a rectilinear profile. The formulas are so simple that they can be included not only in reference books but also in textbooks on the CDP method.

2. First, let us consider the case of side CBP arrangements. It is known that the quantities $(\ell, \Delta x, n)$ "form" the configuration of (side) SBP arrangement. Here, ℓ is the least distance between the arrangement and the blast, $\Delta x = \Delta\text{RP}$ is the spacing between receiving points (RP), and n is the number of the RP in the arrangement. Field arrangement of the CBP (FCBP) requires the specification of one more parameter, $\tilde{x}$, which denotes the coordinates of the blast point in the linear profile, along which the arrangement is oriented. Hence, the FCBP is determined by $(\tilde{x}, \ell, \Delta x, n)$. A finite set of n points with the profile coordinates $x_1, x_2, \dots, x_n$ is connected with each FCBP. These coordinates are determined by the formula

$$x_k = \frac{\tilde{x} + (\tilde{x} + \ell + (k-1)\Delta x)}{2} \equiv \tilde{x} + \frac{\ell}{2} + (k-1)\frac{\Delta x}{2}, \quad k = 1, 2, \ldots, n. \quad (1)$$

These points are called pickets of this FCBP. It follows from (1) that the k-th picket is the middle of a section with ends in the BP and the k-th RP.

The MMO data that form the CDP seismograms consist of a finite number of m seismograms of field arrangements of the CBP located in a common rectilinear profile with a spacing $\Delta\tilde{x}$ between the BP. That is, the coordinates $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_m$ of BPs are given by the following formula:

$$\tilde{x}_i = \tilde{x}_1 + (i-1)\tilde{\Delta} x, \qquad i = 1, 2, \ldots, m. \quad (2)$$

The following conditions must be satisfied:

1. The condition of multiplicity of the spacing $\Delta\tilde{x}$ and $\Delta x/2$ given by the relation
$$\Delta\tilde{x} = Q\frac{\Delta x}{2}, \qquad Q = 1, 2, 3, \ldots;$$
2. The condition of overlap between neighboring arrangements. It consists in the following: the least distance between the pickets of the i-th and $(i+1)$-th arrangements is zero, or $\Delta x/2$. It can be easily calculated that this condition is reduced to the inequality $Q \leq n$.

Let us write Conditions 1 and 2 in the form

$$\Delta\tilde{x} = Q\frac{\Delta x}{2}, \qquad Q = 1, 2, \ldots, n. \quad (3)$$

When (3) is satisfied, the totality of all FCBP pickets $(\tilde{x}_i, \ell, \Delta x, n)$, $i = 1, 2, \ldots, m$, forms a set of N points called the CDPM pickets located in the MMO profile with a spacing $\Delta x/2$. Let us denote the profile coordinates of the CDP pickets by p_r, $r = 1, 2, \ldots, N$, in ascending order, that is,

$$p_r = p_1 + (r-1)\frac{\Delta x}{2}, \qquad r = 1, 2, \ldots, N. \quad (4)$$

The first CDP picket is the first FCBP picket $(\tilde{x}_1, \ell, \Delta x, n)$. Therefore,

$$p_1 = \tilde{x}_1 + \frac{\ell}{2}. \quad (5)$$

The last CDP picket is the last FCBP picket $(\tilde{x}_m, \ell, \Delta x, n)$. Therefore

$$p_N = \tilde{x}_m + \frac{\ell}{2} + (n-1)\frac{\Delta x}{2},$$

or, with allowance for (2) at $i = m$ and (3),

$$p_N = \tilde{x}_1 + \frac{\ell}{2} + [(m-1)Q + n - 1]\frac{\Delta x}{2}. \tag{6}$$

We derive from (6) and (5) that the number N of all CDPM pickets is expressed by the formula

$$N = (m-1)Q + n. \tag{7}$$

Let us enumerate the FCBP by natural i-numbers from 1 to m in the ascending order of the BP coordinate and, in each FCBP, numerate the receiving points by natural k-numbers from 1 to n in the ascending order of their distance from the BP. Then, let us numerate the CDPM pickets by natural r-numbers from 1 to N in the ascending order of their profile coordinate. Let T_{ik} denote the trace corresponding to the i-th arrangement and k-th receiver in it, and r_{ik} the picket number of this trace. The trace picket T_{ik} means the average point between the i-th BP and k-th RP. Its profile coordinate x_{ik} is expressed by formula (1), in which x_k must be replaced by x_{ik}, and $\tilde{x}$ must be replaced by $\tilde{x}_i$. Similarly to conclusion (7), we find that

$$r_{ik} = (i-1)Q + k, \qquad 1 \le i \le m, \quad 1 \le k \le n. \tag{8}$$

Now, we can formulate our problem as follows:

The family S_r of the traces $T_{ik}(r)$ corresponding to the r-th CDPM picket is called a CDP seismogram over the r-th picket, $r = 1, 2, \ldots, N$, and the totality of the BP–RP pairs in this seismogram is called a CDP group over the r-th picket. It is necessary to find the parameters of CDP group depending on the picket number r.

These parameters are as follows:

- ℓ_r is the least distance between the group and the blast equal to the least distance between the BP and the RP in the BP–RP pairs;
- h_r is the spacing between the BP–RP pairs in the group;
- κ_r is the number of the BP–RP pairs in the group, which is also called group multiplicity.

The CDP group spacing. The quantity h_r is determined if the multiplicity $\kappa_r > 1$. Then the following equality is valid:

$$h_r = 2\Delta\tilde{x} \equiv 2\Delta\mathrm{BP}, \qquad r = 1, 2, \ldots, N. \tag{9}$$

This follows from the fact that the neighboring BP–RP pairs in the CDP group are chosen from the neighboring FCBP, in which the distance between blasting points is equal to $\Delta\tilde{x}$.

The CDP group multiplicity. It follows from the definition of the CDP group over the r-th picket that its multiplicity κ_r is the number of ordered pairs (i, k) of natural numbers that satisfy the condition $r = r_{ik}$, where r_{ik} are determined by equality (8). In other words, κ_r is the number of integer solutions i, k of the equation

$$(i-1)Q + k = r, \tag{10}$$

which satisfy the conditions

$$1 \le i \le m, \qquad 1 \le k \le n. \tag{11}$$

Relation (10) is a first-order Diophantine equation with two unknowns. It is known (see, for instance, [3, p. 284]), that the totality of all its solutions is infinite and expressed by the following formulas:

$$i = i_0 - j, \qquad k = k_0 + Qj, \tag{12}$$

where (i_0, k_0) is one of solutions to equation (10), and j is any integer number.

Let r' be the remainder from the division of r by Q, that is,

$$r = \left[\frac{r}{Q}\right]Q + r', \qquad 0 \le r' < Q, \tag{13}$$

where, as usual, the symbol $[\,\cdot\,]$ denotes the integer part of the number.

Let us write (13) in the form

$$r = \left(\left(\left[\frac{r}{Q}\right] + 1\right) - 1\right)Q + r'.$$

Then we see that the numbers

$$i_0 = \left[\frac{r}{Q}\right] + 1, \qquad k_0 = r'. \tag{14}$$

can be taken as the particular solution (i_0, k_0) of equation (10).

Let us substitute (12) into (11), and solve the inequalities obtained for j. Then, to determine all possible integer values for j in (12), we obtain the system

$$i_0 - m \le j \le i_0 - 1, \qquad \frac{1 - k_0}{Q} \le j \le \frac{n - k_0}{Q},$$

or, after the substitution of i_0 and k_0 values in accordance with (14), we have the system

$$\left[\frac{r}{Q}\right] - (m-1) \le j \le \left[\frac{r}{Q}\right], \tag{15a}$$

$$\frac{1-r'}{Q} \le j \le \frac{n-r'}{Q}. \tag{15b}$$

In principle, it remains for us to take the common part of integer intervals (15a) and (15b). First, however, let us simplify, to some extent, inequalities (15b). By virtue of (13), the left term in (15b) satisfies the inequalities

$$-1 < \frac{1-r'}{Q} \le 0 \quad \text{at } r' = 1, 2, \ldots, Q-1.$$

Therefore, the left inequality in (15b) in integer j is equivalent to the inequality $\underline{j} \le j$, where

$$\underline{j} = \begin{cases} 1 & \text{at } r' = 0, \\ 0 & \text{at } r' = 1, \ldots, Q-1. \end{cases} \tag{16}$$

We write the right inequality in (15b) for integer j in the following equivalent form:

$$j \le \left[\frac{n-r'}{Q}\right] \equiv \overline{j}.$$

Representing n in the form

$$n = \left[\frac{n}{Q}\right] Q + n', \qquad 0 \le n' < Q, \tag{17}$$

we find that

$$\overline{j} = \left[\left[\frac{n}{Q}\right] + \frac{n'-r'}{Q}\right] = \begin{cases} [n/Q], & r \le n', \\ [n/Q]-1, & r' > n'. \end{cases} \tag{18}$$

Thus, condition (15b) is equivalent to the condition

$$\underline{j} \le j \le \overline{j}, \tag{19}$$

where the integer $\underline{j}$ and $\overline{j}$ are determined by formulas (16) and (18).

Let us assume that

$$1 \le r < mQ. \tag{20}$$

Then the left term in (15a) is non-positive. Hence, the solution to system (15) (with allowance for the equivalence (15b) $\Leftrightarrow$ (19)) is written in the following form:

$$\underline{j} \le j \le \min\left\{\left[\frac{r}{Q}\right], \bar{j}\right\}. \tag{21}$$

It has been proved that at condition (20) each solution to problem (10), (11) is uniquely determined by integer j (by using (12)) satisfying (21). Therefore, for the number κ_r of these solutions, we obtain the expression

$$\kappa_r = 1 + \min\left\{\left[\frac{r}{Q}\right], \bar{j}\right\} - \underline{j} = 1 + \min\left\{\left[\frac{r-1}{Q}\right], \bar{j} - \underline{j}\right\},\ 1 \le r < mQ. \tag{22}$$

Now, let $r \ge mQ$, that is,

$$r = mQ + s, \qquad s = 0, 1, \ldots, n - Q. \tag{23}$$

It can be easily seen that (23) describes the numbers of all remaining (from the Q-th to the N-th) pickets (see (7)). In this case, relation (15a) has the following form:

$$\left[\frac{s}{Q}\right] + 1 \le j \le \left[\frac{s}{Q}\right] + m.$$

This, together with (19) equivalent to (15b), allows us to write the solution to system (15) in the form ($\underline{j} \le 1$),

$$\left[\frac{s}{Q}\right] + 1 \le j \le \min\left\{\left[\frac{s}{Q}\right] + m, \bar{j}\right\}. \tag{24}$$

Hence

$$\kappa_r = \min\left\{\left[\frac{s}{Q}\right] + m, \bar{j}\right\} - \left[\frac{s}{Q}\right] = \min\left\{m, \bar{j} - \left[\frac{s}{Q}\right]\right\}, \tag{25}$$
$$r = mQ + s, \quad s = 0, \ldots, n - Q.$$

Combining (22) and (25), we obtain

Preliminary result. *For any $r = 1, 2, \ldots, N \equiv (m-1)Q + n$, the multiplicity κ_r of the CDP group over the r-th picket is calculated by the formula*

$$\kappa_r = \begin{cases} 1 + \min\left\{\left[\dfrac{r-1}{Q}\right], \bar{j} - \underline{j}\right\}, & 1 \le r < mQ, \\ \min\left\{m, \bar{j} - \left[\dfrac{s}{Q}\right]\right\}, & r = mQ + s, \quad s = 0, 1, \ldots, n - Q, \end{cases} \tag{26}$$

where

$$\bar{j} = \begin{cases} [n/Q], & r' \le n', \\ [n/Q] - 1, & r' > n', \end{cases} \qquad \underline{j} = \begin{cases} 1, & r' = 0, \\ 0, & r' > 0, \end{cases} \tag{27}$$

$$r' = r - \left[\frac{r}{Q}\right]Q, \qquad n' = n - \left[\frac{n}{Q}\right]Q. \tag{28}$$

It seems reasonable to present other forms of representation of the result obtained. In particular, we can assume from graphic representations that the function $r \to \kappa_r$, $1 \le r \le N$, must have symmetry of the form

$$\kappa_r = \kappa_{N-r+1}, \qquad r = 1, 2, \ldots, N. \tag{29}$$

However, the form of expressions (26)–(28), which determine κ_r, does not have such symmetry.

Let us show that property (29) really exists, and transform the formulas for κ_r to a form symmetric in terms of (29).

Let us begin by showing that the expression $\bar{\jmath} - [s/Q]$, where $s = r - mQ$, can be represented in the form

$$\bar{\jmath} - \left[\frac{s}{Q}\right] = 1 + \left[\frac{N-r}{Q}\right], \qquad r = 1, 2, \ldots, N. \tag{30}$$

In fact, in accordance with (7) and the equality $r = mQ + s$, we have

$$\frac{N-r}{Q} = \frac{-Q+n-s}{Q} = -1 + \frac{n-s'}{Q} - \left[\frac{s}{Q}\right],$$

where s' is the remainder from the division of s by Q. We notice that $s' = r'$, and can write

$$\left[\frac{N-r}{Q}\right] = -1 + \left[\frac{n-r'}{Q}\right] - \left[\frac{s}{Q}\right],$$

or, in accordance with the formula without a number located between (16) and (17),

$$\left[\frac{N-r}{Q}\right] = -1 + \bar{\jmath} - \left[\frac{s}{Q}\right],$$

which proves equality (30).

Now, let us find out at what values of r the least elements under the "min" sign in (26) are changed. For this purpose, we first define all r at which these elements are equal. It turns out that in both terms with "min" this takes place if and only if r takes Q-values successfully:

$$r := n - p, \qquad p = 0, 1, \ldots, Q - 1.$$

This can be easily seen when each of the equations

$$\left[\frac{r-1}{Q}\right] = \bar{\jmath} - \underline{\jmath}((27)) \equiv \begin{cases} [n/Q], & (r-1)' \le n' - 1, \\ [n/Q] - 1, & (r-1)' > n' - 1, \end{cases}$$

is solved in integer r. Here, $(r-1)'$ is the remainder from the division of $r - 1$ by Q,

$$m = \overline{j} - \left[\frac{s}{Q}\right]((30)) \equiv 1 + \left[\frac{N-r}{Q}\right].$$

On the basis of this and from the character of monotony of integer-valued expressions $[(r-1)/Q]$, $[(N-r)/Q]$ with respect to r, we conclude that

$$\min\left\{\left[\frac{r-1}{Q}\right], \overline{j} - \underline{j}\right\} = \begin{cases} [(r-1)/Q], & r < n - Q + 1, \\ [(r-1)/Q] = \overline{j} - \underline{j}, & n - Q + 1 \le r \le n, \\ \overline{j} - \underline{j}, & n < r \le N, \end{cases} \quad (31)$$

$$\min\left\{m, \overline{j} - \left[\frac{s}{Q}\right]\right\} = \begin{cases} m, & r > n - Q + 1, \\ m = 1 + [(N-r)/Q], & n - Q + 1 \le r \le n, \\ 1 + [(N-r)/Q], & n < r \le N. \end{cases} \quad (32)$$

Let us use formulas (31) and (32) in the right-hand side of (26). To do this, we have to consider the cases of $n < mQ$ and $n \ge mQ$ separately.

In the case of $n < mQ$, we have

$$\kappa_r \begin{cases} \left.\begin{array}{l} \text{at } r \le n - Q + 1 \\ \text{at } n - Q < r < mQ \end{array}\right\} ((26),(31)) = \begin{cases} 1 + [(r-1)/Q], \\ 1 + \overline{j} - \underline{j}, \end{cases} \\ \text{at } mQ \le r \le N\}((26),(32)) = 1 + [(N-r)/Q]. \end{cases} \quad (33)$$

In the case of $n \ge mQ$, we have

$$\kappa_r \begin{cases} \text{at } 1 \le r < mQ\}((26),(31)) = 1 + [(r-1)/Q], \\ \left.\begin{array}{l} \text{at } mQ \le r \le n \\ \text{at } n < r \le N \end{array}\right\} ((26),(32)) = \begin{cases} m \\ 1 + [(N-r)/Q]. \end{cases} \end{cases}$$

Notice that, at $(m-1)Q < r \le mQ$,

$$1 + \left[\frac{r-1}{Q}\right] = 1 + m - 1 = m.$$

Therefore, the last expression for κ_r is written in the final form, which is symmetric in terms of (29),

$$\kappa_r = \begin{cases} 1 + [(r-1)/Q], & 1 \le r \le (m-1)Q, \\ m, & (m-1)Q < r \le n, \ n \ge mQ, \\ 1 + [(N-r)/Q], & n < r \le N. \end{cases} \quad (34)$$

To make sure that formula (33) has similar symmetry, it is sufficient to show that the function

$$r \to \bar{j} - \underline{j} \equiv \nu_r, \qquad r = 1, 2, \ldots, N, \tag{35}$$

has a property of the type (29):

$$\nu_r = \nu_{N-r+1}, \qquad r = 1, 2, \ldots, N. \tag{36}$$

From the definitions of the quantities $\bar{j}$ and $\underline{j}$ (see (27)), it is easy to see that ν_r is expressed by the formula

$$\nu_r = \begin{cases} [n/Q], & (r-1)' \le n' - 1, \\ [n/Q] - 1, & (r-1)' > n' - 1, \end{cases} \tag{37}$$

where $(r-1)'$ and n' are the remainders from the division of $r-1$ and n, respectively, by Q. Property (36) will be proved if we show that the inequalities $(r-1)' \le n' - 1$, $(r-1)' > n' - 1$ from (37) are invariant under the replacement of r by $N - r + 1$.

Let $(r-1)' \le n' - 1$. Then

$$\begin{aligned} ((N - r + 1) - 1)'((7)) &= (n - 1 - (r-1))' = (n' - 1 - (r-1)')' \\ &= (n - 1 - (r-1))' = (n' - 1) - (r-1)' \le n' - 1, \end{aligned}$$

since $(r-1)' \ge 0$.

Now, let $(r-1)' > n' - 1$. Then, first we have, similarly to the previous case

$$\begin{aligned} ((N - r + 1) - 1)'((7)) &= \ldots = \ldots = (n - 1 - (r-1))' \\ &= (n' - 1) - (r-1)' + Q > n' - 1, \end{aligned}$$

since $(r-1)' < Q$.

Thus, the above invariance really takes place. Thereby, (36) and, hence, (29) hold also when κ_r is expressed by formula (33).

Note that the right-hand sides of (33) and (34) can be written in a nonstructured form with the help of the "min" operator. This leads to the following formula for κ_r, $r = 1, 2, \ldots, N$:

$$\kappa_r = \begin{cases} 1 + \min\left\{ \left[\dfrac{r-1}{Q}\right], \ \nu_r, \ \left[\dfrac{N-r}{Q}\right] \right\}, & n < mQ, \\ 1 + \min\left\{ \left[\dfrac{r-1}{Q}\right], \ m - 1, \ \left[\dfrac{N-r}{Q}\right] \right\}, & n \ge mQ. \end{cases} \tag{38}$$

Finally, we notice that the inequality $\nu_r \le m - 1$ holds at $n < mQ$, and the inequality $\nu_r \ge m - 1$ holds at $n \ge mQ$, and obtain a unified expression for κ_r, $r = 1, 2, \ldots, N$:

$$\kappa_r = 1 + \min\left\{ \left[\frac{r-1}{Q}\right], \ \nu_r, \ m - 1, \ \left[\frac{N-r}{Q}\right] \right\}.$$

Let us formulate

Result 1. *Let n be the RP number in the CBP arrangement, let m be the BP number, and let Q be natural, $1 \le Q \le n$, and denote the multiplicity of the BP spacing by a half-RP spacing. Then the multiplicity κ_r of the CDP arrangement over an arbitrary r-th picket, $r = 1, 2, \dots, N \equiv (m-1)Q + n$, is expressed by the formula*

$$\kappa_r = 1 + \min\left\{ \left[\frac{r-1}{Q}\right],\ \nu_r,\ m-1,\ \left[\frac{N-r}{Q}\right] \right\}, \tag{39}$$

where

$$\nu_r = \begin{cases} \left[\dfrac{n}{Q}\right], & (r-1)' \le n' - 1, \\ \left[\dfrac{n}{Q}\right] - 1, & (r-1)' > n' - 1; \end{cases}$$

$(r-1)'$, n' are the remainders from the division of the numbers $r-1$ and n, respectively, by Q.

Remark 1. Maybe there exists a shorter way to formula (39). It is clear that in this case the property of invariance of the quantity κ_r with respect to the inversion of the numeration order of pickets must form the basis of the conclusion (and must not be obtained as a consequence of this conclusion). It should be noted that initially this invariance is not evident, because it is masked by the side character of the initial CBP arrangements.

Remark 2. In fact, we obtained all possible forms of formula (39):

- Non-structured forms subdivided into the cases $n < mQ$ and $n \ge mQ$ (formula (38));
- Structured formulas (33) and (34) for the cases $n < mQ$ and $n \ge mQ$, respectively.

Remark 3. In practice, mainly the case $n \ll mQ$ is realized. Then the work formula for κ_r is (33) or the upper line of (38). In (33), the quantity $\bar{j} - \underline{j}$ is ν_r (see (37)), which can take only two values differing from each other by unity. Therefore, in all "internal" pickets the multiplicity κ_r of the CDP groups can take maximum two values differing from each other by unity. It also follows from (33) and (34) that the multiplicity of the CDP groups over "internal" pickets is the same if and only if n is divided evenly by Q, or $n \ge mQ$. Then, at $n \ge mQ$,

$$\kappa_r = m, \qquad (m-1)Q < r \le n,$$

and at $n < mQ$,

$$\kappa_r = \left[\frac{n}{Q}\right], \qquad (m-1)Q < r < mQ. \tag{40}$$

Remark 4. Formula (30) shows that the numeration of pickets beginning from zero would be more "natural". In any case, in this case the "exotic" quantity $\underline{j}$ would not appear.

The least distance between the CDP group and the blast. The least distance, ℓ_r, between the CDP group over the r-th picket and the blast is the least of the distances $\ell_{ik} \equiv \ell + (k-1)\Delta x$ between the i-th BP and k-th RP in the i-th CBP arrangement, where the pairs (i,k) are the solution to problem (10), (11) investigated above.

At $1 \le r < mQ$, in accordance with (12), (14), and (21), we have

$$k = r' + Qj, \qquad j = \underline{j}, \underline{j}+1, \dots,$$
$$r' = r - \left[\frac{r}{Q}\right]Q, \qquad \underline{j} = \begin{cases} 1 & \text{at} \quad r' = 0, \\ 0 & \text{at} \quad r' \ne 0. \end{cases}$$

Therefore,

$$\ell_r = \ell_{i\underline{j}} = \ell + (r' + Q\underline{j} - 1)\Delta x.$$

It can be easily checked by direct calculations that $r' + Q\underline{j} - 1 = (r-1)'$, where $(r-1)'$ is the residual of division of $r-1$ by Q. Hence,

$$\ell_r = \ell + (r-1)'\Delta x, \qquad 1 \le r < mQ. \tag{41}$$

Now, let $r \ge mQ$, $r = mQ + s$, $s = 0, 1, 2, \dots, n-Q$. Then, in accordance with (12), (14), (24),

$$k = r' + Qj, \qquad j = \left[\frac{s}{Q}\right] + 1, \left[\frac{s}{Q}\right] + 2, \dots .$$

Therefore

$$\ell_r = \ell_{ik}|_{k=[s/Q]+1} = \ell + \left(r' + Q\left(\left[\frac{s}{Q}\right] + 1\right) - 1\right)\Delta x.$$

Since $r' = s'$, we have

$$r' + Q\left(\left[\frac{s}{Q}\right] + 1\right) - 1 = \left(s' + Q\left[\frac{s}{Q}\right]\right) + Q - 1 = s + Q - 1.$$

Thus, at $r = mQ + s$, $s = 0, \dots, n-Q$, we have

$$\ell_r = \ell + (s + Q - 1)\Delta x, \qquad s = 0, 1, 2, \dots, n - Q. \tag{42}$$

Let us formulate

Result 2. *At any admissible values of the quantities n, m, and Q, the least distance, ℓ_r, between the CDP group over the r-th picket and the blast, $r = 1, \dots, N = (m-1)Q + n$, is given by the formula*

$$\ell_r = \begin{cases} \ell + (r-1)'\Delta x, & \text{if } 1 \le r < mQ, \\ \ell + (s + Q - 1)\Delta x, & \text{if } r = mQ + s, \quad s = 0, 1, \dots, n - Q, \end{cases} \tag{43}$$

where ℓ is the least distance, Δx is the RP spacing in the CDP arrangement, and $(r-1)'$ is the residual from the division of $r - 1$ by Q.

3. The case of the two-sided (not necessarily centered) CBP arrangements can be considered on the basis of formulas obtained with the use of the superposition principle widely used in [1]. Here, to calculate the parameters of the CDP groups we should make use of the fact that any two-sided arrangement can be considered as two side arrangements. One of them is the right side arrangement (the RP are to the right of the BP), and the other one is the left side arrangement (the RP are to the left of the BP). Here, we shall not dwell on the derivation of relevant formulas.

References

[1] Petrashen' G.I., Rudakov A.G. Main problems of the method of multiple CDP overlaps and technological data processing methods // Geofizika. – 1996. – № 1. – P. 3–13.

[2] Kondratyev O.K. On imaginary and actual contradictions of seismic prospecting technologies // Geofizika. – 1996. – № 1. – P. 14–18.

[3] Encyclopedia of Elementary Mathematics. – Vol. I: Arithmetic. – Moscow: Gostekhizdat, 1951.