Bull. Nov. Comp. Center, Math. Model. in Geoph., 4 (1998), 23–44

On a multidisciplinary approach to an integral earthquake precursor and near-surface dilatant zones*

A.S. Alekseev, A.S. Belonosov, and V.E. Petrenko

A physical-mathematical model and approaches to the measurement of the integral earthquake precursor proposed in [4] and [5] in the form of a theoretical scheme are developed. This model can probably reduce the acuteness of contradictions which often take place between precursors of different physical nature when a multidisciplinary model of prediction is used.

Reasons for the choice of the space–time density of cracks as an integral earthquake precursor in the area of preparing earthquake source and in the zone of appearance of anomalous geophysical fields are given.

The possibility of formation of near-surface migrating dilatant zones that can cause variations of geophysical fields-precursors is shown. This conception gives new possibilities to the determination of an integral earthquake precursor.

Schemes for quantitative estimation of the integral precursor using seismic and combined (multidisciplinary) inverse problems for fracturous media are proposed.

Introduction

Precursors of various geophysical nature are used in earthquake prediction to reduce the probability of "missing" large earthquakes. Several years prior to such earthquakes there occur anomalies of geophysical fields, such as deformation fields of the Earth's crust, fields of seismicity, electric conductivity, geomagnetism, gravics, gas and fluid permeability. Zones of manifestation of such anomalies migrate in time at a territory at a distance of 200–300 km from the epicenter of a future earthquake. The migration mechanisms of anomalous zones of different geophysical nature and interrelation between them reflect the evolution of the field of seismotectonic stresses and apparently contain useful information about the preparation of earthquakes. Over the past 30 years this multidisciplinary information has been collected in such countries as China, Japan, USA, Greece, Turkey, Russia, and some others. In China, this information is available for a hundred of large earthquakes (of magnitude $M > 6$) from a rather dense observation network. Now it is in the form of computer Data Bases.

*Supported by the Russian Foundation for Basic Research under Grants 96-05-66058, 96-15-98542.

The accumulated information is used to investigate problems of earthquake prediction by analyzing statistical relations of space and time characteristics of geophysical anomalies of various types at different stages of earthquake preparation. Some successful earthquake predictions by Chinese geophysicists were based on synthesized information indices of changes in the behavior of anomalies at medium- and short-range preparation stages of earthquakes which occurred earlier [1–3].

The use of the multidisciplinary concept of prediction and the expected achievements raised, however, new important questions concerning determination of the mechanism of interrelation between seismicity processes, variations of different geophysical fields, and the sources of anomalies of these fields.

The Chinese specialists [2, 3] point out that a large number of anomalies-precursors are contradictory, even if observations are for one and the same tectonic area and the same observation point. Researchers are faced with the following questions: "what theory can be used to give an acceptable explanation" for this phenomenon [3] and what method of decision-making about predictions can be proposed now instead of the method of "making decisions by simple ballot" on the basis of a large number of precursors of different physical nature [2].

These questions are of the same origin as those for multiparametric models and multidisciplinary methods of investigation of complex systems when it is necessary to choose criteria for division of possible variants of behavior of such systems into physical types.

In the present paper, a physical-mathematical model of integral precursor is developed. It was proposed in the form of a theoretical scheme in [4, 5]. It can apparently reduce the acuteness of contradictions that occur when multidisciplinary precursors are used.

Let us consider an artificial precursor which is synthesized with the help of a numerical method. It is not measured directly, but periodically calculated during geophysical monitoring using multidisciplinary quantitative data about measured fields-precursors. This integral precursor is determined by solving combined inverse problems for the corresponding geophysical fields (displacement and deformation fields of the Earth's surface, the electric conductivity field, anomalies of the gravitational field, the ground water level, etc). As is shown in [4], the accuracy of determination of the integral parameter-precursor can exceed the accuracy of determination of the other characteristics of the medium by individual geophysical methods.

An advantage of such a precursor must be its integrating content, which is free from inconsistencies between data of precursors of different geophysical nature.

Researchers have certain freedom in choosing a physical quantity reflecting time variations of the integral precursor. Here, two conditions are necessary. First, this quantity must reflect quantitatively the stage and degree of preparation of the earthquake source for destruction. Second, it must determine the space-time variations of various geophysical fields-precursors.

We propose to use, as the integral precursor, the space-time crack density function in the earthquake preparation area and in zones of anomalous geophysical fields at the Earth's surface.

This proposal is based on:

- the results of the kinetic theory of destruction developed by S.N. Zhurkov et al. (see [6, 7]);
- the experimental and theoretical investigations of earthquake preparation processes by V.I. Myachkin, B.V. Kostrov, G.A. Sobolev, and O.G. Shamina, the researchers of the Institute of the Earth's Physics of the Russian Academy of Sciences [8, 9];
- the results of numerical analysis of "source" and "surface" dilatant zones, which are given below.

The main facts relating to the mechanical properties of the integral precursor are considered.

The phenomenon of formation of "the boundary dilatant layer" detected in computational experiments is described. It is loosening of extensive zones of the upper part of the Earth's crust under the action of tangential and tensile tectonic stresses near the daily surface which is free from stresses. The scheme of vibroseismic sounding of dilatant zones of the "source" and "boundary" types for more reliable determination of the integral precursor is presented. The structure of the precursor model and a method for estimation of the crack density function using multidisciplinary data are described.

1. The main properties of the integral precursor

Earthquakes are realized through destruction of rocks, beginning with destruction of rocks in the source zone. Therefore, study of the *destruction preparation processes* and monitoring of these processes are of major importance for earthquake prediction. An investigation of the destruction processes of samples of various materials in laboratory conditions, as well as large-scale natural objects, in particular, blocks of the Earth's crust of earthquakes, revealed the general characteristics of the destruction process.

Step-by-step development of this process in time is the most general principle. Some kinetic laws and concepts of destruction were established by S.N. Zhurkov and his colleagues from the Physical–Technical Institute of RAS in St. Petersburg [6, 7]. They are close to the concept of the scheme of destruction of large-scale objects of the Earth's crust during earthquakes proposed by researchers from the Institute of the Earth's Physics of RAS.

G.A. Sobolev [10] formulates the following three principles, which are of "major importance for problems of search for precursors and earthquake prediction":

• *The concept of development of a system of cracks* in earthquake preparation areas as a result of increase in the volume density of microcracks, from the stages of increasing crack sizes and decreasing crack number to formation of large fractures.

• The generality of step-by-step transition from smaller cracks to larger cracks, when smaller cracks reach some critical value, *in accordance with the concentration criterion* of S.N. Zhurkov [6]:

$$K^* = \frac{N^{-1/3}}{L}, \tag{1}$$

where N is the number of cracks of size L, K^* is the critical average distance between cracks in the units of average length of cracks, and $N^{-1/3}$ is the volume concentration of cracks. When the average distance becomes smaller than a critical value, there is an abrupt reorganization of the whole system of cracks, with increasing average sizes of cracks in some geometrical proportion and their decreasing average volume concentration. Cracks tend to localize in the area of a future macrofracture. These phenomena are typical for any scale level and any regime of loading.

• Reorganization of the system of cracks manifests itself as a *change of some characteristics of the medium in a developing earthquake source* and as *formation of anomalies of some geophysical fields.* In particular, concentration of the crack formation process can be evident as a change in the space–time seismicity regime for weak earthquakes and in the mechanism of preceding microearthquakes, appearance of anisotropy of rocks in a future earthquake source. The appearance of anisotropy is most conspicuous in formation of anomalies of the propagation velocities of longitudinal and transverse seismic waves prior to large earthquakes [11].

Some geophysical fields can be subjected to the process of opening of microcracks. In particular, it is natural to assume that there is an increase in gas and fluid permeability in those areas of the Earth's crust where this process takes place. Therefore, the ground water level, the intensity of gas flows, and the electrical resistance can change. Where the field of local tectonic stresses is responsible for intensification of crack formation, loosening of rocks due to increasing total volume of cracks must also cause anomalies of the gravitational field.

An analysis of the stress field in areas of earthquake preparation [12, 13] and the results of numerical modeling of conditions for the appearance of dilatant zones presented below make it possible to assume that cracks of some scale level can be formed at distances of 200–300 km from the source of a future earthquake.

Thus, the choice of the crack density function as a measure of preparation for destruction has some advantages, because this function is present in the formulation of all the three principles, which "are of major importance for the problems of earthquake prediction".

One more advantage is that the crack density function can be more accurately and reliably determined from multidisciplinary data due to its presence in quantitative models of anomalous geophysical fields of various nature (the principle of complementability [14]).

2. The boundary dilatant layer of the Earth's crust

Interaction between regional and local tectonic forces in seismically active zones can lead to the appearance of areas of high concentration of tectonic stresses. After a time, destruction of the medium resulting in an earthquake takes place in some of these areas.

Although the earthquake preparation process lasts long (several years), it is an energy-intensive process. Considerable rheological changes in the medium take place, and anomalous zones of geophysical fields of various nature are formed. Opening of cracks in zones with increased values of shearing and tensile stresses is the most universal mechanism of development of changes in the medium. Such zones are formed in the vicinity of the sources of future earthquakes, if here the distribution of forces in space is nonuniform. The majority of seismologists consider that the initial stage of opening of cracks and the subsequent state of the medium, when destruction processes develop, are connected with the dilatancy of the medium described in [15, 16].

Dilatancy is a nonlinear loosening of the medium due to formation of cracks of shear. This takes place, when tangential stresses exceed a certain threshold. A dilatancy area is considered to incorporate a set of points of an elastic medium, for which at a given stress field $\{\sigma_{ij}\}$ the following condition is fulfilled:

$$D_\tau \equiv \tau - \alpha(P + \rho g z) - Y \geq 0. \quad (2)$$

Here ρ is the density of rocks, g is the gravitational acceleration, z is the depth of a point, and P is the hydrodynamic pressure

$$P = -\frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}),$$

where α is the coefficient of internal friction, Y is the cohesion of rocks, τ is the intensity of tangential stresses:

$$\tau = \frac{\sqrt{3}}{2}\Big[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 + 6(\sigma_{12}^2+\sigma_{13}^2+\sigma_{23}^2)\Big]^{1/2}.$$

Condition (2) coincides with the Schleicher–Nadai criterion of material destruction under the action of shearing loads. It describes satisfactorily the beginning of the rock destruction process. It can also be used at the "predestruction" stage (when loading constitutes up to 60–90% of the critical value) for qualitative description of the shape of areas with intensification of crack opening.

Let us demonstrate the complex character of dilatant zones using the simplest model of the Earth's crust, which is initially a uniform, isotropically elastic half-space. This complexity manifests itself even when a unit concentrated force is a source of tectonic stresses.

Exact solutions for elastic displacements and stresses from a point source satisfying the conditions of absence of stresses at the boundary $z = 0$ were used to model the stress field in an elastic half-space [17].

The domain boundary $D_\tau = 0$ from (2) for the source, one simple force at a depth of 15 km, is shown in Figure 1. Here the parameters of the elastic half-space are as follows: $v_P = 6000\,\mathrm{m/s}$, $v_S = v_P/\sqrt{3}$, $\lambda = \mu = \rho v_S^2 = 3.48 \cdot 10^{10}\,\mathrm{Pa}$, $\rho = 2900\,\mathrm{kg/m^3}$, $g = 9.8\,\mathrm{m/s^2}$, $Y = 3 \cdot 10^6\,\mathrm{Pa}$, $\alpha = 0.5$, $F = 10^{16}$ N.

Tangential stresses inside the domain $D_\tau \geq 0$ dominate over compressional stresses. The resistance of the medium to shearing forces is overcome due to cohesion. Conditions favorable to the increase in the number of cracks are created. The mechanisms of crack opening and the rheological changes of the medium in the zone $D_\tau \geq 0$ are not described by the solutions used. The solutions are valid only for determination of the boundary of transition from the elastic state to the state of nonlinear loosening with formation of a system of cracks.

Formation of two dilatant zones, such as the "source" zone in the vicinity of the force application point and the "boundary" zone in the layer near the free surface, is of interest. Here the stress field from the source can have a large contribution of tangential stresses on the background of a weak influence of compressional stresses and the hydrostatic pressure due to the proximity of the surface which is free from external stresses.

The behavior of the boundary dilatant zone is variable depending on the following parameters: h (the depth of the source), F (the intensity of the source), the angle ϑ (the force orientation in the source), Y (the cohesion of the medium's elements). It can vanish with increasing depth of the source or merge with the "source" zone as the intensity of the source increases (Figures 2–4). In some cases, the horizontal size of the "boundary" zone is 200 and more kilometers, and it is of complex shape when projected onto the Earth's surface. It is easily seen that the picture of displacement of dilatant

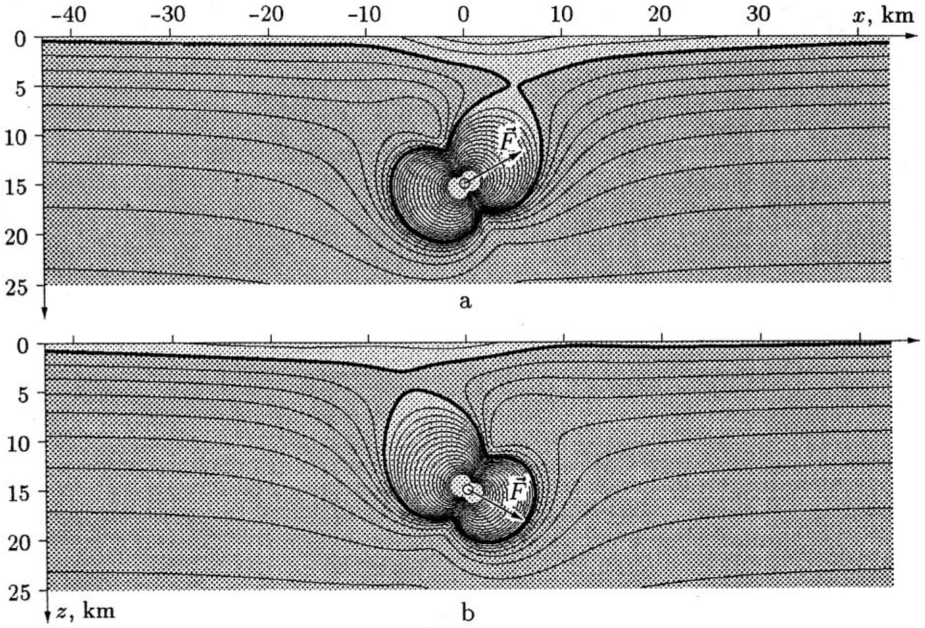


Figure 1. The shapes of the "source" and "surface" dilatant zones in the plane $y = 0$. Source depth $h = 15\,\text{km}$, force $\vec{F} = F\,\vec{n}$, $F = 5 \cdot 10^{16}\text{N}$, $\vec{n} = (\cos\vartheta, 0, \sin\vartheta)$: a) $\vartheta = -30°$, b) $\vartheta = +30°$.

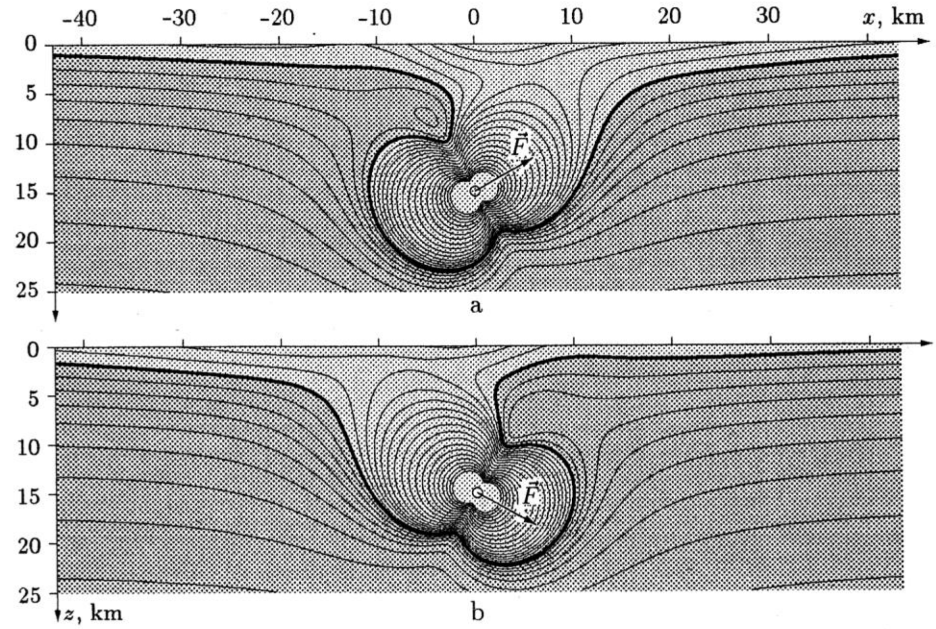


Figure 2. The shapes of dilatant zones versus the magnitude F of the force. In comparison to Figure 1, here $F = 10^{17}$ N.

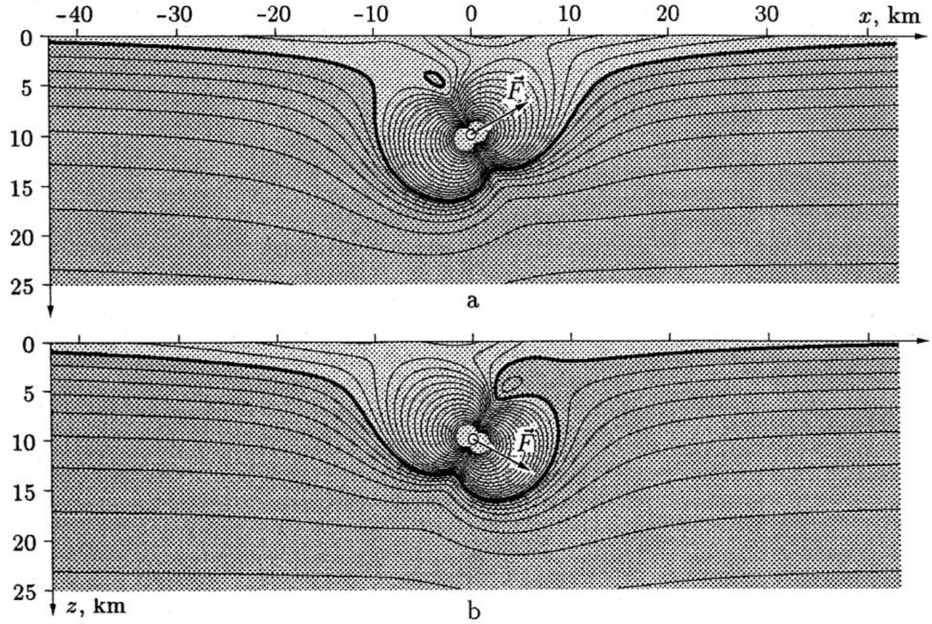


Figure 3. The shapes of dilatant zones versus the depth h of the source. In comparison to Figure 1, here $h = 10\,\mathrm{km}$

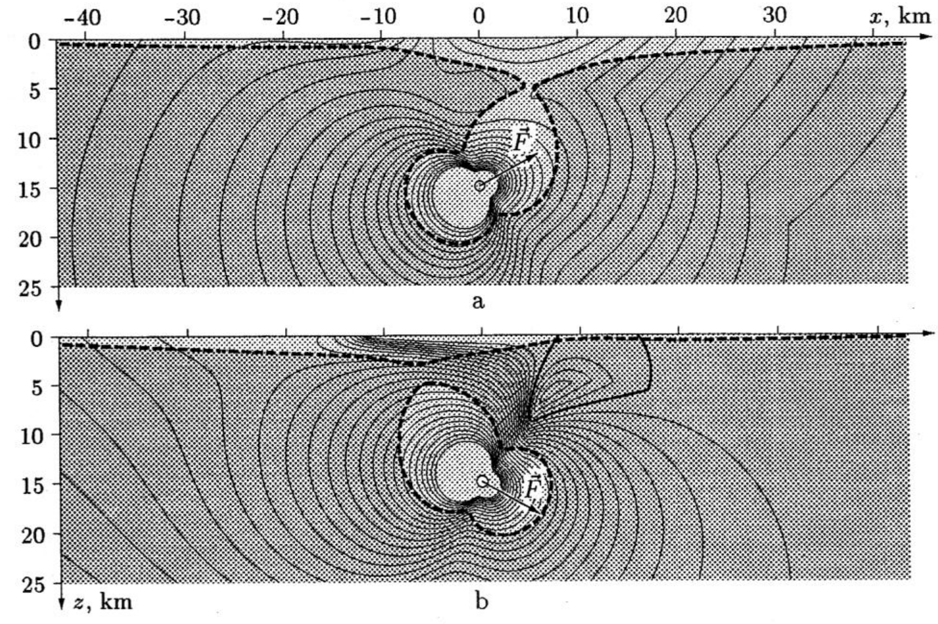


Figure 4. The "source" and "surface" dilatant zones in the field of main normal stresses σ_1. The force and the depth are the same as in Figure 1

zones along the surface can be complex under the action of several sources distributed in space, the intensity of which varies with time.

Since the zones of anomalous values of geophysical fields are naturally related to the surface dilatant zones, the location of the dilatant zones must be determined more reliably to investigate the sources of these anomalies.

The condition of loosening of rocks of the Earth's crust taking into account the mechanisms of crack formation by tension of the medium can be written in the form of a new criterion:

$$0 \le D_\sigma \equiv \begin{cases} \frac{1}{2}\sigma_1(1-\sin\varphi) - \frac{1}{2}\sigma_3(1+\sin\varphi) - Y\cos\varphi, & \text{if } -\sigma_3 \ge \sigma_s, \\ -\sigma_3 - \sigma_s, & \text{if } -\sigma_3 < \sigma_s. \end{cases}$$

Here σ_1 and σ_3 are the largest and the smallest main stresses, respectively; φ is the angle of internal friction; Y is the cohesion, σ_s is the strength of the medium under tension. This criterion determines dilatant zones in conditions, when the medium withstands large shearing stresses, but offers less resistance to tensile forces.

Figures 5 and 6 show the isolines of the functions D_τ and D_σ for the same medium as in Figure 1. The values of the new parameters for the criterion D_σ are as follows: the angle of friction $\varphi = 30°$; the ultimate resistance to tension $\sigma_s = 8 \cdot 10^6$ Pa. All parameters are for the granite type of rocks. It is seen from these figures that for this case the forms of the dilatancy areas are similar in accordance with the two criteria. Calculation of the dilatancy areas for a large set of realistic variants of the medium's structure and loading conditions shows a good agreement for the two criteria considered. Taking into account that these criteria are alternative (one criterion takes into account the mechanism of displacement, and the other the mechanism of tension), it can be supposed that the boundary dilatant zone can exist in a wide range of parameter values of the medium. It is pertinent to note that so far the existence of the boundary dilatant zones in real conditions should be considered a hypothesis. The use of this hypothesis for practical analysis of accumulated extensive data on monitoring of anomalies of various geophysical fields is an attempt to find the reasons for space-time mobility of anomalies of these fields and the mechanisms of their interrelation.

Control of variation in the crack density in the source zone is of special importance for short-range earthquake prediction. This control should not be separated from observations of the boundary dilatant zone and the anomalous fields associated with it. First, the location of the future source is not yet known exactly, and its determination is closely related to the behavior of surface anomalous zones. Second, the reliability and accuracy of estimation of the crack density function in the "source" zone depends on the information about field anomalies in "boundary" zones.

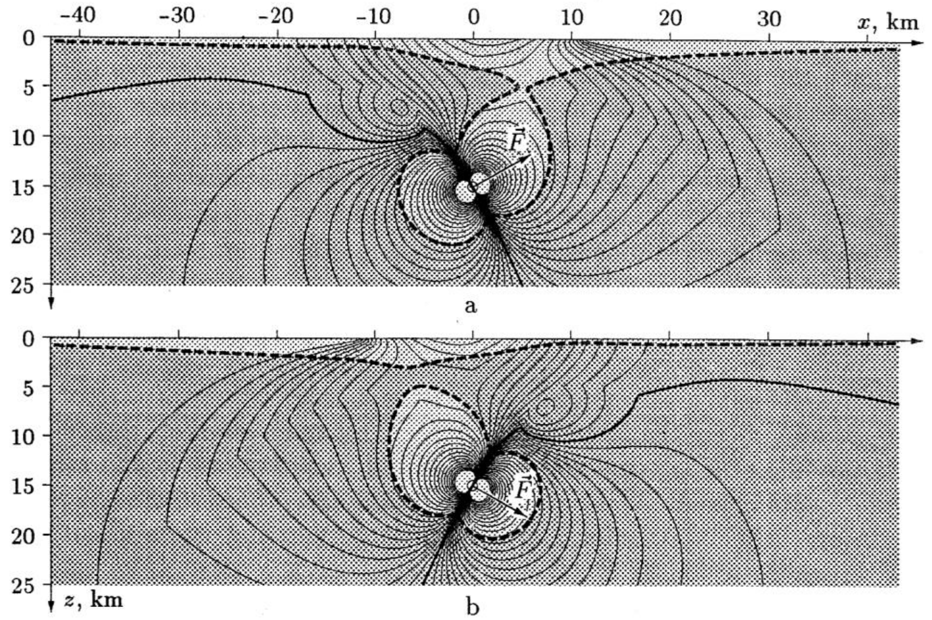


Figure 5. The "source" and "surface" dilatant zones in the field of main normal stresses σ_2. The force and the depth are the same as in Figure 1

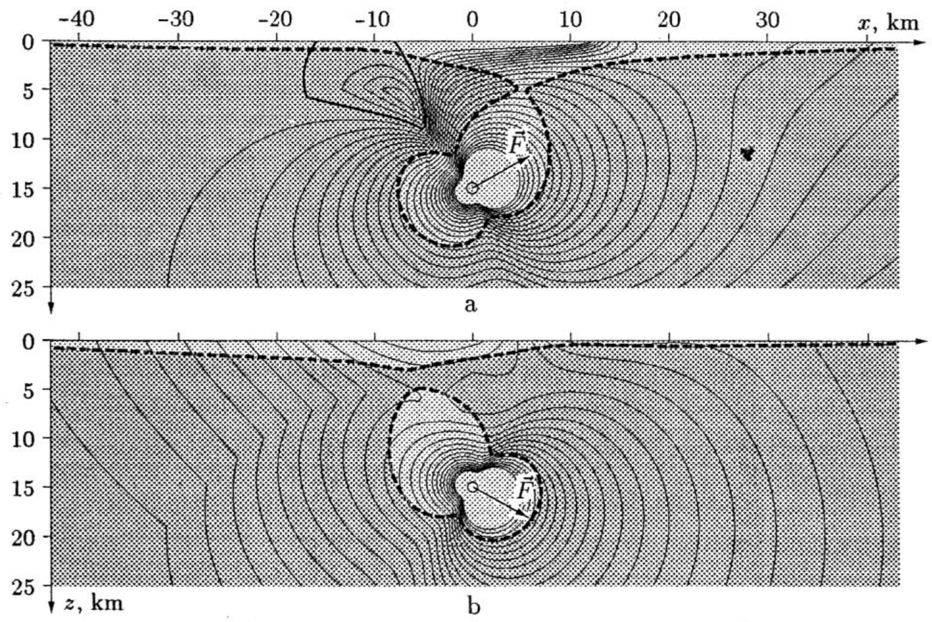


Figure 6. The "source" and "surface" dilatant zones in the field of main normal stresses σ_3. The force and the depth are the same as in Figure 1

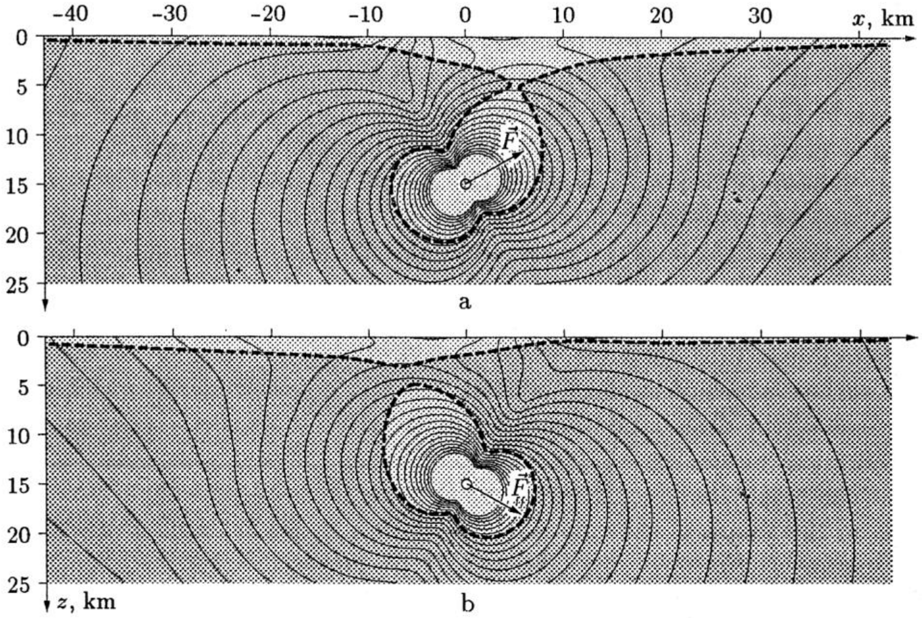


Figure 7. The "source" and "surface" dilatant zones in the field of tangential stresses $\tau_2 = (\sigma_3 - \sigma_1)/2$. The force and depth are the same as in Figure 1

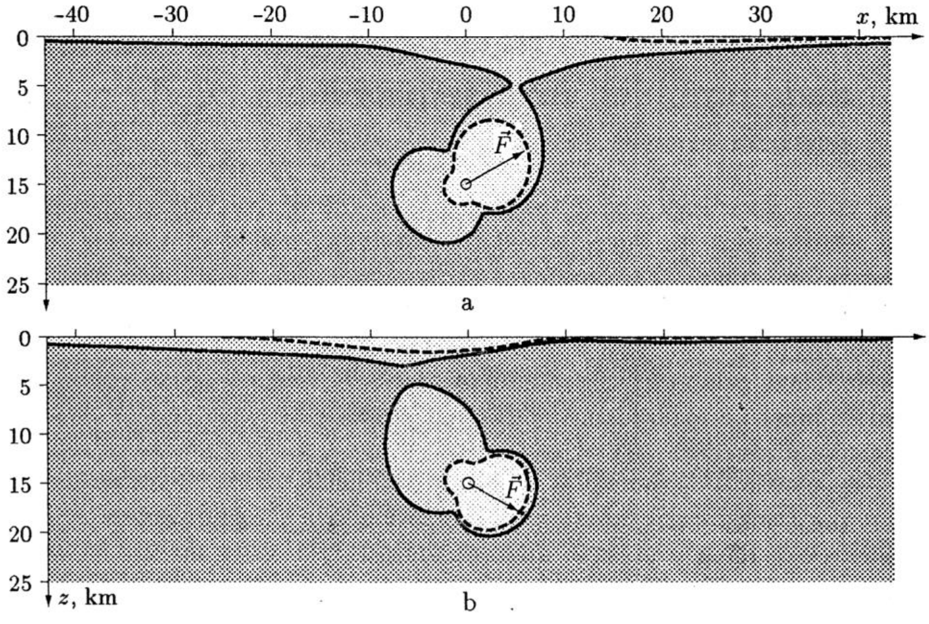


Figure 8. A comparison of dilatant zones for two criteria of the beginning of crack formation: destruction due to shear (solid line); destruction due to tension (dashed line). The force and the depth are the same as in Figure 1

Preparatory to considering the scheme of estimation of the integral precursor in the source using multidisciplinary monitoring, it is reasonable to consider the mechanism of interrelation between the crack formation processes in the "source" and "boundary" dilatancy areas. A common stress field is an energetic medium, in which the interrelation between the "source" and "boundary" zones is realized. This field is generated by forces in the source of a preparing earthquake. The lines of the largest tangential stresses (or the lines of the largest tensile stresses) determine the estimate of the predominant orientation of cracks, which occur in the zones of the "source" and "boundary" types (Figures 7 and 8). When dilatancy zones of the two types are combined into one zone, there is a joint area of cracking. It combines the surface zones of anomalous fields with the source zone, in which development of the destruction process can directly influence the change in the geophysical anomalies. When dilatancy areas are separated, the "source" zone can retain its direct influence on the anomalies of some fields (for example, on the values of the velocities v_P and v_S) by a joint area of introduced anisotropy of the medium. The anisotropy coefficients can be expected to vary in a special way during the process of crack growth, because the orientation of the axes of symmetry remains the same. This can simplify the problem of estimation of the average number of cracks by a high-resolution vibroseismic method.

3. Multidisciplinary model of integral precursor and combined inverse problems for its monitoring

The main assertions about the key role of the destruction processes and the relationships for development of systems of cracks at the earthquake preparation stages are considered in Section 1. In accordance with these assertions, the problem of prediction must include the solution to the problem of total and accurate control over the space-time characteristics of the development processes of systems of cracks in seismic prone zones. This control must be realized in the dilatant zones, where systems of cracks are formed and develop.

Observations of anomalies of geophysical fields make it possible to determine the crack density function. It was assumed in the previous sections that the crack opening process in dilatant zones is related to the mechanisms of formation of anomalous fields. Qualitatively, formation of anomalies of the gravitational, electric conductivity, ground water level, gas and fluid permeability fields can be explained by crack opening. The quantitative approach to determination of the characteristics of cracking (the integral precursor) using the data on anomalies of geophysical fields also gives an idea of the boundary dilatant zone.

Let us introduce the function $\Theta(x, y, z, t)$ and try with its help to describe approximately the density of cracks in the medium. In the process of the medium's deformation prior to break of continuity, loosening is characterized by the volume expansion $\Theta = \operatorname{div} \vec{U}$, where the divergence is calculated from the displacement vector of the medium's points. It is assumed in this case that the components of this vector are sufficiently smooth (differentiable) functions. If we consider a small volume v_0, which is v_1 after deformation, then $v_1 = v_0(1 + \Theta)$. If the medium's density is ρ_0, after deformation it is $\rho_1 = \rho_0/(1 + \Theta)$. For large volumes of deformed medium this loosening is considerable. It generates an anomaly of the gravitational field $V(x, y, 0) = V^0(x, y)$, using which we solve the inverse problem

$$\Delta V = -4\pi\rho_1(\Theta), \quad V\Big|_{z=0} = V^0(x, y) \tag{3}$$

to determine the density $\rho_1(x, y, z) = \rho_0/\left(1 + \Theta(x, y, z)\right)$, and can find the loosening Θ (if this inverse problem can be solved uniquely, and the initial density is known). The main difficulty which leads one to consider multidisciplinary (combined) statements of inverse problems is that (3) does not have a unique solution. It is ill-posed; an attempt is made to find the three-dimensional function $\rho_1(x, y, z)$ using the known two-dimensional function $V^0(x, y)$. This is impossible without additional information.

The significance of combined statements of inverse problems is in the use of additional information from the solution to a new inverse problem for the same physical quantity.

In further statements of the problems we shall try to use as much additional information about $\Theta(x, y, z, t)$ (the medium's volume expansion) as possible. The function $\Theta(x, y, z, t)$ in the inverse problem (3) for determination of the density $\rho_1 = \rho_0/(1 + \Theta)$ can be called *the function of the medium's loosening.* This function can be considered piecewise continuous; its determination is not associated with the function for elastic volume expansion ($\Theta = \operatorname{div} \vec{U}$), and it is assumed to be equal to the total relative volume of cracks in the medium's unit volume. The number of cracks in the unit volume can be determined by the formula $N = \Theta/\Theta_L$, where Θ_L is the relative average volume of a crack of length L. In the problems of crack number monitoring, this quantity is to be determined from the estimate of the medium's mechanical parameters, the average crack sizes at the previous scale level of the process development in the source taking into account the concentration criterion (1).

Let us consider the combined inverse problems for the gravitational and electric prospecting methods, for the problem of description of the ground water level, and the seismic method for measurement of the effective anisotropy coefficients for cracked rocks on the basis of the *principle of complementability* of geophysical methods [4, 15] to obtain reliable estimates of the function $\Theta(x, y, z, t)$.

Measurements at the points of the surface $z = 0$ of the corresponding geophysical field

$$U_\nu(x, y, 0, t_k) = U_\nu^0(x, y, t_k)$$

are made for each of these methods.

Here $t_k = kT_\nu$; T_ν is a time interval between the moments of recording of the field values during its monitoring.

Methods for solving direct and inverse problems exist for all geophysical fields which are used in the problem of earthquake prediction. In direct problems, the equation for the field

$$L_\nu(U_\nu, \alpha_\nu, \beta_\nu) = f_\nu(x, y, z, t),$$

the boundary conditions

$$l_\nu(U_\nu, \alpha_\nu, \beta_\nu)\Big|_s = h_\nu(s, t),$$

and the initial data

$$U_\nu(x, y, z, t)\Big|_{t=0} = U_\nu^\circ(x, y, z)$$

are assumed to be given.

Here $\alpha_\nu(x, y, z)$, $\beta_\nu(x, y, z)$ are the physical and geometrical characteristics of the medium; $f_\nu(x, y, z, t)$ are the external volume sources of the field; $h_\nu(s, t)$ are the sources at the surface S (Figure 9).

Now there exist effective numerical methods to solve direct problems of any above-mentioned types on computers. For this, the medium's characteristics $\alpha_\nu(x, y, z)$ and $\beta_\nu(x, y, z)$, the field sources and the boundary surface S must be given.

In inverse problems, the following quantities are given: the field $U_\nu^\circ(s_i, t)$ at a series of the points s_i at the surface S, and the sought-for characteristics of the medium α_ν and β_ν, or other elements of the problem (the shape of the surface S, some sources f_ν or h_ν). All other elements of the problem must be given as in direct problems.

In formulating inverse problems, the following variants are possible:

- Individual inverse problems for each method are formulated separately.
- One combined inverse problem is formulated for a group of "m" methods giving information which is sufficient to determine the necessary characteristics of the medium.

The general scheme of the formulation is given in Figure 9.

Individual inverse problems have, as a rule, the following main drawback: they have deficit of information. The above-mentioned inverse problem for

The system of equations:

$$L_\nu(U_\nu, \alpha_\nu, \beta_\nu) = f_\nu(x,t), \quad \nu = 1, 2, \ldots, m.$$

Conditions:

$$l_\nu(U_\nu, \alpha_\nu, \beta_\nu)\big|_S = h_\nu(s,t), \quad U_\nu\big|_{t<0} \equiv 0, \quad \nu = 1, 2, \ldots, m.$$

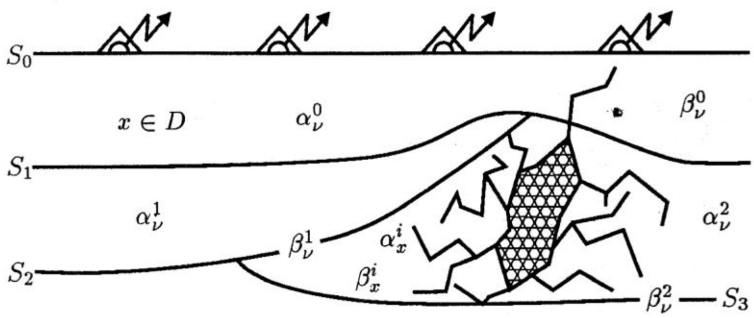
Initial information:

$$U_\nu\big|_S = U_\nu^0(s,t), \quad \nu = 1, 2, \ldots, m.$$

It is necessary to find

$$\alpha_\nu(x), \ \beta_\nu(x), \quad \nu = 1, 2, \ldots, m.$$

One or several fields $U_\nu(x,t)$ are monitored:



α_ν – physical parameters of the medium
β_ν – geometrical parameters of the same body

It is necessary to find the parameters

$$(\alpha_x^i, \beta_x^i), \quad i = 1, 2, \ldots, m_x$$

and make their "computational monitoring".

Figure 9. The general scheme of statement of the multidisciplinary inverse problem

the gravitational field, in which the three-dimensional density function of the medium cannot be determined, is a good example of this.

The model of multidisciplinary (combined) inverse problem is used to determine the integral precursor $\Theta(x,y,z,t)$, i. e., the relative crack density function. In this case, all the geometrical and physical parameters of the medium are considered to be known, with the exception of the function $\Theta(x,y,z,t)$, and the function $\Theta(x,y,z,t)$ is considered not to depend on time during each field measurement session $t_k = kT_\nu$.

The optimization method is used to solve the combined inverse problem.

Let $B_\nu(x, y, z, t, \Theta)$ represent the operator to calculate the field $U_\nu(x, y, z, t)$ in the direct problem for the method with the number ν. The problem consists in determining $\Theta(x, y, z)$ from the condition of minimum of the functional:

$$I(\Theta) = \min_{\Theta \in M_\Theta} \sum_{\nu=1}^{m} \gamma_\nu \left[U_\nu^\circ(x, y) - B_\nu(x, y, 0, t_k, \Theta)\right]^2,$$

where γ_ν are the weight coefficients for individual methods, M_Θ is an *a priori* set of possible solutions Θ, $U_\nu^\circ(x, y)$ is the measured field, $B_\nu(x, y, 0, t_k, \Theta)$ is the calculated test field.

There arise considerable computational difficulties in the optimization method. They are associated with the solution to a large number of direct problems for several methods simultaneously. Besides, the functional that is

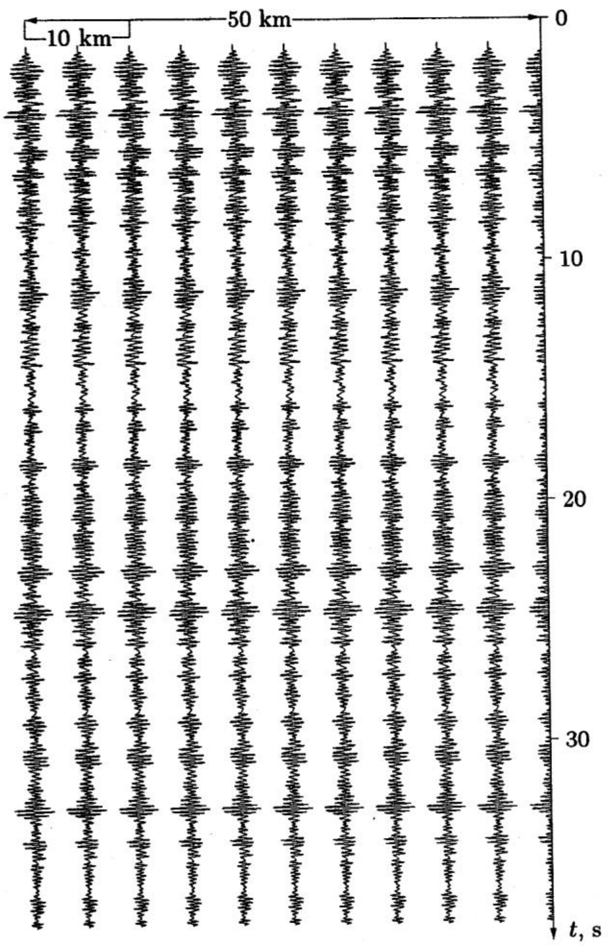


Figure 10. A fragment of CDP time cross-section in the area of the Bystrovka vibrational test site for a profile of 0–95 km (the distance between stations is 5 km). One-dimensional structure of the Earth's crust is assumed

being minimized often has many "ravines". This makes search for the global minimum difficult. To solve such problems successfully, one should use high performance computers and good initial approximations to the sought-for functions.

The seismic method for observation of longitudinal and transverse waves from powerful controllable vibroseismic sources can give more detailed data about the structure of the medium's areas with cracking variable with time. In the case of powerful mobile sources and the use of observation systems with multiple overlaps, this method gives the results in seismology, the resolution of which is similar to that of the well-known results of seismics in seismic oil prospecting (Figure 10).

Here we will not dwell on some capabilities of active seismology [18] using powerful vibrators, signals from which can be recorded at distances of up to 500–1000 km, but only consider the scheme of vibroseismic sounding of dilatant zones of the "source" and "boundary" types to increase the reliability and accuracy in determination of the integral precursor.

4. Scheme of vibroseismic monitoring of seismic prone zones

Powerful vibroseismic sources with precision control systems have been developed in some institutes of the Siberian Branch of RAS since 1977 [19]. Seismograms are obtained at distances of up to 400 km and records of harmonic signals at distances of up to 1000 km using the vibrators with the force of action of 50; 100; 250 tons and recording systems with long accumulation of signals (Figures 11 and 12). The methods of "active seismology" are being developed on this basis. A controllable mechanical source whose epicenter coordinates and the time of beginning of work are known exactly is used in these methods instead of an earthquake, which is an uncontrollable source of seismic signals.

The observation schemes of seismic prospecting, the algorithms and program packages of data processing can be used for the methods of "active seismology".

Among important problems of "active seismology" is creation of a method for vibroseismic monitoring of seismic prone zones and, in particular, a method for determination of the function $\Theta(x, y, z, t_k)$. This function characterizes the development of systems of cracks in the earthquake source and in zones of formation of anomalies of geophysical fields. To determine $\Theta(x, y, z, t_k)$, the scheme of Deep Seismic Sounding (DSS) can be used together with the scheme of the Common Depth Point (CDP) on profiles 150–200 km in length over the source of a preparing earthquake (Figure 13). At the stage of long-range prediction, the period between soundings can

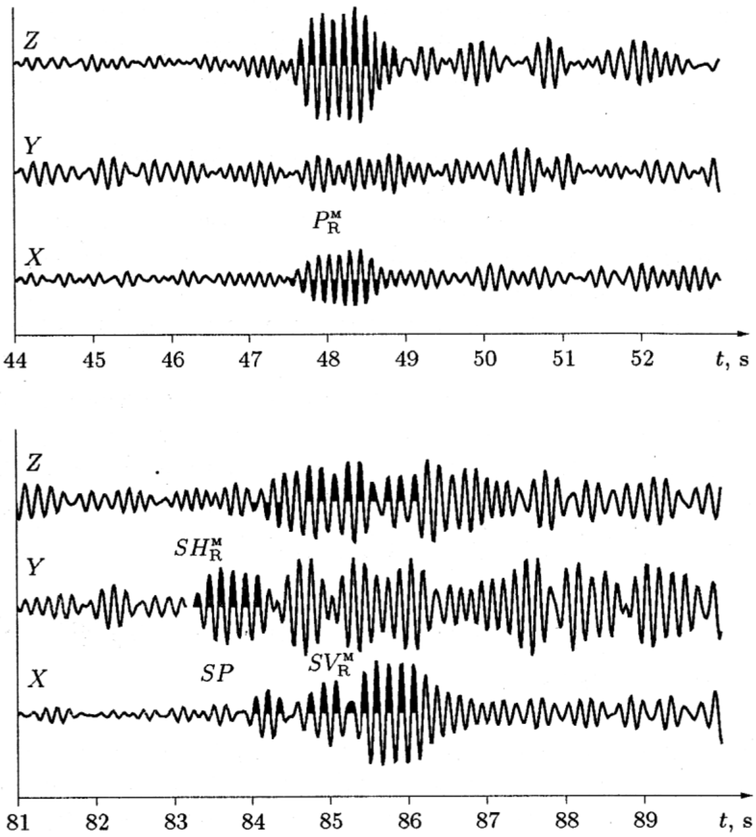


Figure 11. Seismograms at a distance of 320 km (CV-100 vibrator)

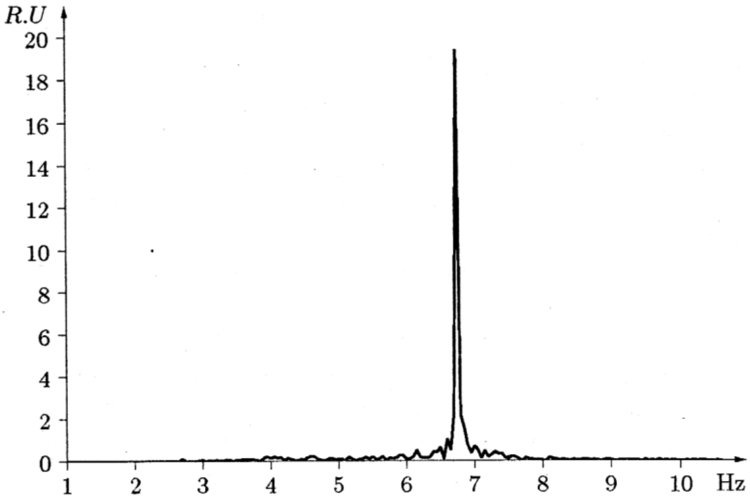


Figure 12. Harmonic signal at a distance of 520 km

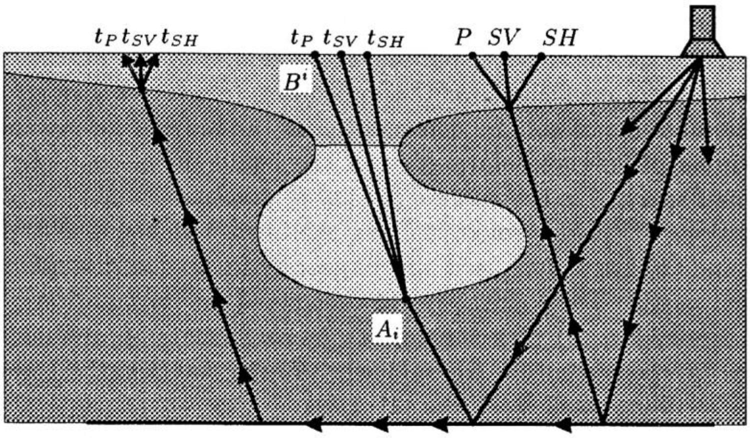


Figure 13. Scheme of the profile of vibroseismic observations of the P, SV, and SH waves for monitoring of the function $\Theta(x, y, z)$ in the dilatant zone

be equal to 6 months or a year. At the stage of short-range prediction, soundings must be more frequent, and observation systems must be more detailed.

Periodic soundings of the medium make it possible to increase the accuracy of determination of the medium's parameters that characterize cracking. It can be assumed that the medium's characteristics vary only slightly in time periods between soundings (these periods must be chosen in this way). These small variations of the medium's properties can be made the major elements of variability of seismic cross-sections with the help of the "interframe correlation" method (i. e., by subtraction of sequential images of the medium and analysis of increments).

An analysis of the experiments on resolution shows that the variability in crack sizes is larger than the variability of the dominant orientation of cracks [20]. Sometimes crack sizes vary abruptly during the transition to the next scale level of destruction [6].

This peculiarity of the monitoring problem of the growth of systems of cracks makes it possible to simplify and refine the algorithms for processing of vibroseismic observations.

The general scheme of monitoring is shown in Figure 13. Automatic processing of observations using this scheme presupposes development of migration methods and solution to the inverse dynamic problems for the total system of equations of the elasticity theory in an anisotropic medium

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho \frac{\partial V}{\partial x_i} = \rho \frac{\partial^2 U_i}{\partial t^2}, \tag{4}$$

with the generalized Hooke law

$$\sigma_{ij} = C_{ik}(\nu_s, K_s, K_f, e)\varepsilon_{kj}. \tag{5}$$

Here σ_{ij} are the stresses, ε_{kj} are the deformations, V is the gravitational potential, C_{ik} are the effective parameters of anisotropy for cracked medium, ν_s is the Poisson coefficient for the imbedding (elastic, isotropic) medium, K_s is the modulus of volume deformation of the imbedding medium, K_f is the modulus of volume deformation of the liquid or gaseous phase in the porous half-space, e is the volume density of cracks.

Equations (4) (often not containing the term with the gravitational potential) and (5) are widely used now in geophysics to describe seismic waves in cracked media. There exist several versions of the formulation of the generalized Hooke law (5) with the anisotropy coefficients approximating wave processes in cracked media for sufficiently large wavelengths (much larger than the medium crack size) (see [21–23]).

The quantity e (the density of cracks) is present explicitly in (5). It depends on the assumed shape of cracks. A. Hoenig [24] gives the following formula:

$$e = \frac{2NA^2}{\pi P}. \tag{6}$$

It is obtained for densely packed parallel plane elliptic cracks of the area A with the perimeter P. He holds that it is valid for any plane cracks with the convex shape of boundary.

Equations (4) and (5) form the basis for the methods of processing of observations which are being developed in oil seismic prospecting and seismology. In practice, simplified kinematic approaches (including the migration methods of wave fields in CDP and DSS) are used.

The scheme of profile observations of waves reflected and refracted from the Moho boundary in the Earth's crust at vibroseismic sounding of the "source" and "surface" dilatant zones is shown in Figure 13. The presence of cracks in these zones and change in their volume density in the periods between monitoring sessions can be determined from the change in the anisotropy coefficients and wave propagation velocities. The transverse wave S splits into SV and SH waves at the boundaries of the dilatant zones. The depth and the shape of the boundaries, as well as the wave velocities, can be determined by the well-known methods from the lags Δt_{SV}, Δt_{SH} of the moments of arrivals of the corresponding waves at the points B^i_{SV}, B^i_{SH} with respect to the generating waves at the point A_i.

For the model of cracking of the type (6) the velocities of all the three wave types v_P, v_{SV}, and v_{SH} are determined approximately by the formula

$$v = \frac{v_0}{\sqrt{1 + ef(\gamma)}},$$

where v_0 is the wave velocity in the medium before the appearance of cracks, $f(\gamma)$ is one function for all types of waves (H.G. Garbin, L. Knopoff [25]), γ is the angle between the direction of wave propagation and the direction which is normal to the orientation of plane cracks. The quantity e from (5) and (6) is the sought-for function $\Theta(x, y, z, t_k)$.

This quantity can be determined not only from observations by the scheme of vibrational monitoring, but also from routine seismologic observations of the velocities $v_P(t_k)$, $v_{SV}(t_k)$, and $v_{SH}(t_k)$ at seismic stations. In this case its determination is less detailed and accurate due to smaller density of observation points and lower accuracy of determination of wave lag times. It should not be used as a "good" initial approximation for multidisciplinary monitoring of the crack density function.

Acknowledgements. The authors would like to thank the Chinese colleagues Ai Runbiao, ChenShaoxu, ZhouQingliang, PingJianjun, and LiWenying from the Seismological Bureau of the Hebei Province for cooperation and useful discussions.

References

[1] Shiron Mei. Progress in earthquake prediction in China during the 80-ies // J. of Earthquake Prediction Research. – 1992. – Vol. 1, № 1. – P. 43–57.

[2] Guomin Zhang, Zhaocheng Zhang. The study of multidisciplinary earthquake prediction in China // Ibid. – P. 71–85.

[3] Features of precursor fields before and after the Datong–Yanggao earthquake swarm / Ma Li, Chen Jianmin, Chen Qifu, Liu Guiping // J. of Earthquake Prediction Research. – 1995. – Vol. 4, № 1. – P. 1–30.

[4] Alekseev A.S. A multidisciplinary mathematical model of combined foreshock for earthquake prediction research // J. of Earthquake Prediction Research. – 1993. – Vol. 2, № 2. – P. 137–150.

[5] A multidisciplinary mathematical model for earthquake prediction studies and vibroseismic monitoring of seismic prone zones / A.S. Alekseev, B.M. Glinsky, V.V. Kovalevsky, B.G. Mikhailenko. Proc. 2nd Int. Conf. on Seismology and Earthquake Engineering, May 15–17. – Teheran, JJEES, 1995. – Vol. 1. – P. 97–104.

[6] Zhurkov S.N. The kinetic concept of strength of solid bodies // Vestnik Akad. Nauk SSSR. – 1968. – № 3. – P. 46–52. – (in Russian).

[7] Zhurkov S.N., Kuksenko V.S., Petrov V.A. On the question of prediction of rock destruction // Izv. Akad. Nauk SSSR. Ser. Fiz. Zemli. – 1977. – № 8. – P. 11–18 (in Russian).

[8] Laboratory and theoretical investigations of earthquake preparation processes / V.I. Myachkin, B.V. Kostrov, G.A. Sobolev, O.G. Shamina // Izv. Akad. Nauk SSSR. Ser. Fiz. Zemli. – 1974. – № 10. – P. 107–112 (in Russian).

[9] The principles of the source physics and earthquake precursors / V.I. Myachkin, B.V. Kostrov, G.A. Sobolev, O.G. Shamina // Physics of Earthquake Source. – Moscow: Nauka, 1975. – P. 6–29 (in Russian).

[10] Sobolev G.A. Study of formation of earthquakes and precursors of shear-type fracture in laboratory conditions // Search for Earthquake Precursors. – Moscow: Nauka, 1978. – P. 86–99 (in Russian).

[11] Nersesov I.L., Semyonov A.N., Simbiryova I.G. Space-time distribution of travel times of transverse and longitudinal waves in the Garmsk region // Experimental Seismology. – Moscow: Nauka, 1971. – P. 334–345 (in Russian).

[12] Liangtian Miao. Monitoring and prediction of the Datong earthquake // J. of Earthquake Prediction Research. – 1993. – Vol. 2, № 2. – P. 299–310.

[13] Chunhua Wang, Sugiong Liao. Experimental study on the preparation and occurrence of strong earthquakes // J. of Earthquake Prediction Research. – 1996. – Vol. 4, № 5. – P. 525.

[14] Alekseev A.S. Complementary features of geophysical methods and the computational aspect of joint data inversion // Proc. 54th Meeting of European Association of Exploration Geophysicists. – Paris, 1992. – P. 750–751.

[15] Brace W.F., Paulding B.W., Scholz C. Dilatancy in the fracture of crystalline rocks // J. of Geophys. Research. – 1966. – Vol. 71, № 16. – P. 3939–3952.

[16] Nikolaevskii V.N. A review: the Earth's crust, dilatancy and earthquakes // Adv. in Science and Engineering. – Moscow: Mir, 1982. – P. 133–215 (in Russian).

[17] Mindlin R., Cheng D. The unit force in elastic half-space // J. of Applied Physics. – 1950. – Vol. 21, № 9. – P. 118–133.

[18] The problems of active seismology / A.S. Alekseev, B.M. Glinsky, V.V. Kovalevsky, and B.G. Mikhailenko // Trans. of 2-nd Intern. Conference "The Structure of Upper Mantle". – Moscow, 1997. – P. 1–8.

[19] Alekseev A.S., V.V. Kovalevsky. Powerful vibrator for deep Earth interior investigations // LX Annu. Intern. Meeting Soc. of Exploration Geophysicists. Sept. 23–27. – San-Francisco, California, 1990. – P. 956–957.

[20] Nur A. Effects of stress on velocity anisotropy in rocks with cracks // J. of Geophys. Research. – 1971. – Vol. 78, № 8. – P. 2022–2034.

[21] Crampin S. Seismic wave propagation through a cracked solid: polarization as a possible dilatancy diagnostics // Geophys. J. of Roy. Astr. Soc. – 1978. – Vol. 53. – P. 467–496.

[22] Crampin S. Effective anisotropic elastic constants for wave propagation through cracked solids // Geophys. J. of Roy. Astr. Soc. – 1984. – Vol. 76. – P. 135–145.

[23] Budiansky B., O'Connell R.J. Elastic moduli of a cracked solid // Int. J. Solids Structures. – 1976. – Vol. 12. – P. 81–97.

[24] Hoenig A. Elastic moduli of a nonrandomly cracked body // Int. J. Solids Structures. – 1979. – Vol. 25. – P. 137–154.

[25] Garbin H., Knopoff L. Elastic moduli of a medium with liquid-filled cracks // Quart. Appl. Math. – 1975. – Vol. 33 – P. 301–303.

[26] Destruction. The Mathematical Foundations of the Destruction Theory / G. Libovits, ed. – Moscow: Mir, 1975. – Vol. 2 (in Russian).