

Increase in the accuracy of determination of the parameters of seismic sources using the vibroseismic method for Earth's sounding

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It is proposed to increase the accuracy in the determination of the seismic sources' coordinates and power, taking into account quantitatively the kinematic and dynamic anomalies of the seismic field, associated with a considerable spatial non-homogeneity of the Earth along the traces of the waves' passage and in the places of location of the seismic stations. This is realized on the basis of the calibration sounding of seismic traces from the expected sources to the seismic stations using powerful seismic vibrators with the calibrated characteristics. This method has a number of physical, technological and ecological advantages in comparison to the method of large-calibrational chemical explosions. Together with the reciprocity principle, it can be used in the variant of the accumulative wave recording in the points of probable explosions, when weak, long and well-synchronized signals are radiated from the vibrators located at a small distance from the recording stations.

Introduction

The effectiveness of the systems of control over the nuclear tests depends to a large extent on the accuracy in the determination of the explosions' coordinates (time, geographical coordinates and depth of explosions), and of their power in accordance with the data of recording of the excited seismic waves by the control stations. This accuracy influences the reliability of identification of seismic events as nuclear explosions, and the choice of methods and means of the control systems in the places of probable explosions.

The main difficulty in the accurate determination of the seismic source's parameters is associated with a substantial horizontal and vertical non-homogeneity of the Earth's crust and upper mantle, which causes large variations of the parameters of seismic waves recorded at different distances of the stations from the source and at different azimuths of their location relative to the source. The values of deviations of the waves' times of arrivals and amplitudes from the standard (global or regional) hodographs (averaged by the azimuths) and 1D calibration curves recorded by a group of stations

determine the errors in the calculation of the sources' parameters.

The dependence of these errors on such factors as mutual location of the source and of the seismic array, locations of the stations of the array in different geological provinces or in the presence of strongly varying conditions in the upper part of the medium is complex in its character. The general regularities of the errors' dependencies on these factors can be described only if the detailed structure of the Earth in the areas of the source, the seismic traces "source-station", and the locations of all the stations used are known. In this case, the so-called "influence function" – the Green tensor – can be computed. With the help of the Green tensor, the inverse dynamic problem of seismology could, in principle, be solved numerically. However, as it will be shown below, the present-day data on the 3D structure of the Earth's crust and upper mantle are still not sufficient for this purpose, and it is necessary to consider each specific situation separately, orienting both to the information available about the character of the Earth's structure in the given region, and the empirical estimates of the Green tensor obtained with the help of the calibration measurements for a specific network: source – a network of seismic stations.

In the approach proposed, the problem on the determination of the source's parameters is solved in two stages. At the first stage, some integral characteristics of the medium where the seismic waves propagate from the place of a possible explosion to the fixed observation points are determined empirically by means of the imitation sounding using the scheme "mobile vibrator-recorders". For this purpose, the method and technology of deep vibroseismic sounding of the Earth, developed in the Siberian Branch of the Russian Academy of Sciences is employed. In this case, the vibrator is located at the recording points, and the recorders are in the places of possible explosions, or vice versa. Owing to the correlation principle (kinematic and dynamic), there is equivalence of the obtained data. At the second stage, the problem of determination of the coordinates and parameters of the explosion using the data of recording from a network of seismic stations is solved, on the basis of the numerical determination of the seismic moment tensor for the equivalent source. The data on the calibration of the specific seismic traces and the conditions at the stations obtained at the first stage by means of the active vibromonitoring of the medium are taken into account.

The method proposed will be considered in the following sequence:

- quantitative characteristics of the Earth's non-homogeneity;
- a method and a technology for the vibroseismic sounding and determination of trace and station corrections;
- a numerical algorithm for the solution of the inverse problem of determining the source's coordinates and parameters with the introduction of trace and station corrections.

1. Basic characteristics of non-homogeneity of the Earth's crust and mantle and data on the lateral variations of times and amplitudes of a seismic field

Beginning in the 50-ies in connection with the problems of oil geological prospecting, and in the 60-ies in connection with the problem of control of nuclear tests, a large number of works has been carried out using the Method of Deep Seismic Sounding (DSS) [1, 2, 4, 5, 6] and the method of profile seismologic observations [3] in the USSR and USA [7, 8, 9].

In accordance with the data of Russian and American geophysicists [5, 6, 9], the variations of hodographs of the first waves in the crust and mantle can reach 4-5 seconds for the distances of more than 1200 km from the source, depending on the type of the geological province and the azimuth to the epicenter [6] (Figure 1).

Variations of the amplitudes with the distance and depending on the azimuth to the epicenter are even more significant. Figure 2 presents the empirical laws of attenuation of the waves' amplitudes in the first arrivals up to the distances of 1200-1800 km for different geological regions. If these curves are brought to one level near the source, then the variations of amplitudes at equal distances will be estimated in 1-2 orders. This value can increase to 2-3 orders, if the source and the recording station are located in different geological regions and in different surface conditions.

The well-known global hodographs of Jeffreys-Boolean, Gutenberg and Herrin (averaged over the regions and azimuths) and a common calibration curve of amplitudes (Figure 3) are most commonly used in seismology. It is often difficult to use, in addition to these averaged laws, more accurate information on the influence of the regional peculiarities of the structure of the Earth's crust and mantle on the waves' times and amplitudes in the practical estimation of coordinates and energetic characteristics of explosions. In this case, errors of 1-2 seconds can be in the determination using only seismic data from the moment of explosion, errors of 50-80 km in the estimate of the epicenter's coordinates, and 50-100% errors in the estimates of the explosion's power. Thus, it is necessary to use 4D hodographs $\tau(x_o, y_o, x, y)$ and amplitude curves $A(x_o, y_o, x, y)$ and, later, 5D hodographs taking into account the depth of explosion h (x_o, y_o are the coordinates of the epicenter, x, y are the coordinates of the stations). Even if only the corrections for the waves' times and amplitudes for real traces and the stations' kinematic and dynamic corrections are taken into account, the accuracy in the determination of the times, coordinates and power of explosions can increase by 30-40% in the most unfavourable combinations of the group of points "source-receivers" relative to the considerable horizontal non-homo-

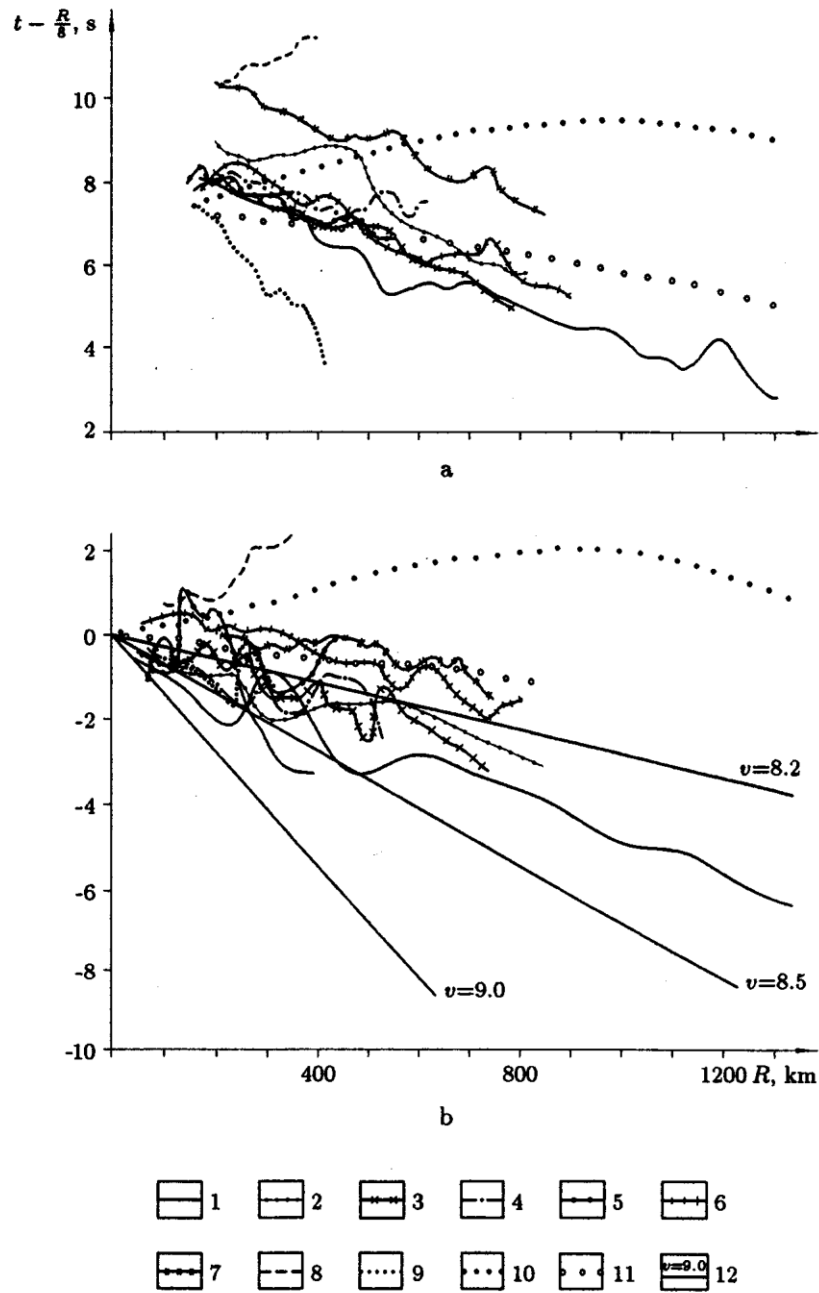


Figure 1. Observed (a) and recalculated to the levelled base of the Earth's crust (b) hodographs of waves $P^M(P_n)$, recorded in the first arrivals by the method of deep seismic sounding (DSS) [6]

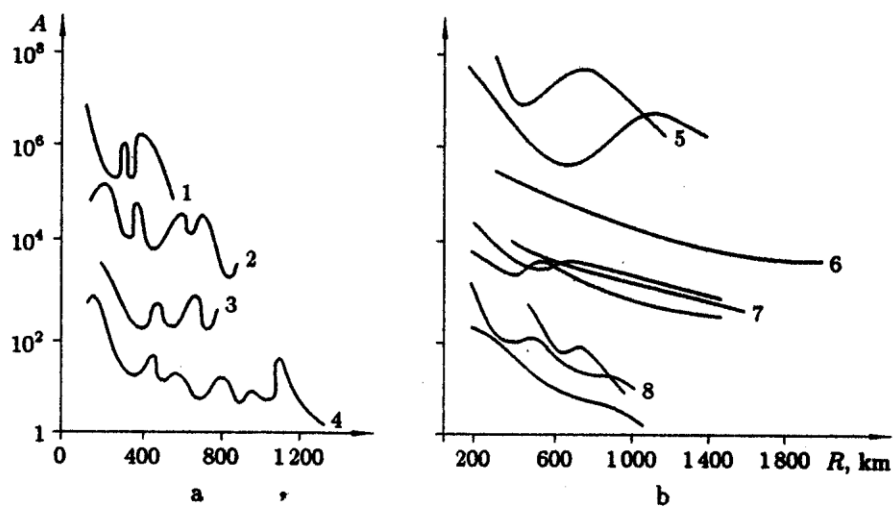


Figure 2. Graphs of variation with distance of amplitudes of waves $P^M(P_n)$ in the first arrivals for different areas of the Earth in accordance with the data of DSS (a) and seismology (b) [6]

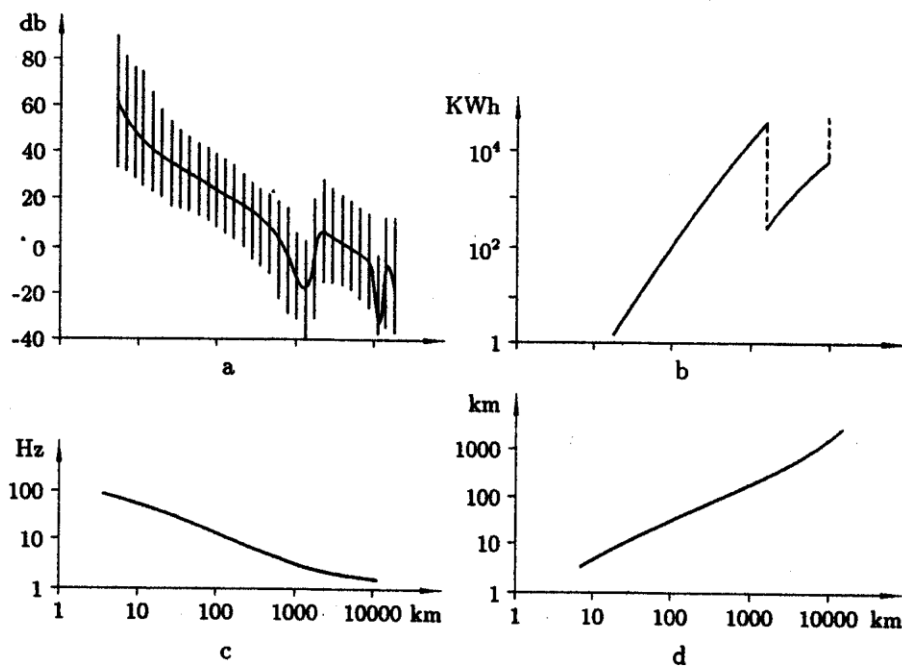


Figure 3. Characteristics of the seismic wave P and the parameter of sounding in the function of distance from the source [6]

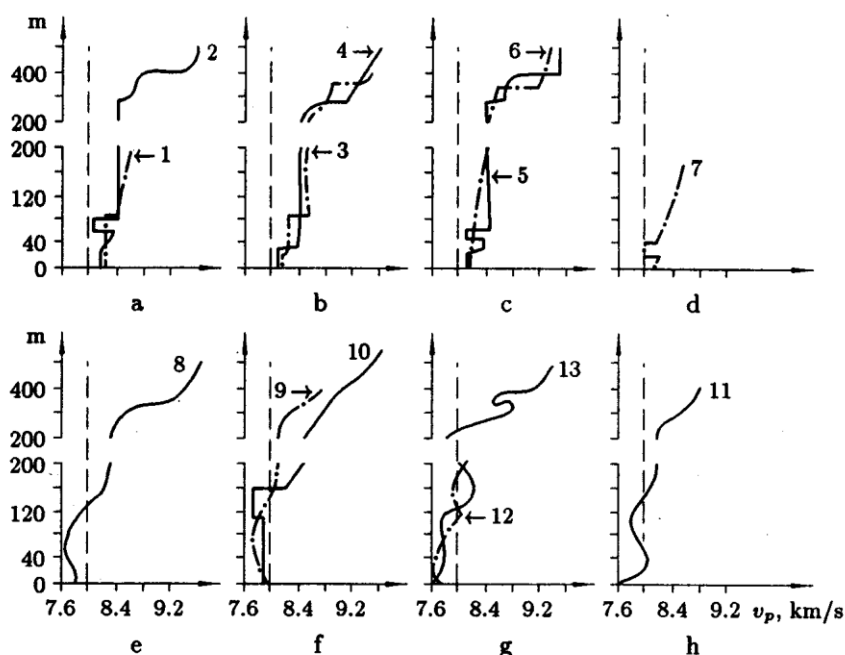


Figure 4. Typical velocity cross-sections of the upper mantle of some platform and tectonically active areas in accordance with [6]

geneities of the Earth's structure. The method proposed is designed for the improvements of such scale.

Well-known results of the horizontal differences in the depth structure of the Earth's crust and upper mantle will be cited (Figure 4) for the better orientation of the readers in the questions of allowing for geological-geophysical factors of the Earth's structure for the estimation of the accuracy in the determination of the seismic fields' kinematic and dynamic characteristics using the observation systems in different geological regions. These differences were systematized, estimated and explained from the geological viewpoint in [4] and [5]. An important factor of differences is the layer of "seismic asthenosphere" at the depths of 100–200 km which is not present globally, but can be referred to the zones of active present-day tectonism (the zones where the mantle is "not depleted yet", but is partly melted).

2. A method and a technology for vibroseismic sounding and determination of trace and station corrections

As noted above, one of the ways to increase the accuracy in the determination of the coordinates and power of the sources of seismic events is taking

into account the corrections for the waves' passage times and amplitudes caused by the geological non-homogeneity of the medium structure along the traces of wave propagation and in the place of location of the seismic array. The development of an instrumental method for the introduction of corrections at seismic stations is a topical problem of seismology. In the given paper, a solution to this problem on the basis of the calibration of seismic traces and seismic stations using sounding signals with high metrological characteristics is proposed. Such signals can be obtained from the vibrators controlled by the amplitude, frequency and time. By locating such sources in the areas of seismic stations and recording signals from them at the points of tests, it is possible to determine the corrections of the waves' passage time and amplitudes with high accuracy. This can be done by taking into account the differences in the phases and amplitudes of the seismic vibrations measured by seismic sensors in the harmonical regime of sounding, or the waves' passage times during the emission of broad-band signals. For the latter, signals with frequency modulation (sweep signals), phase-modulated or noise-type signals, i.e., signals described by a narrow correlation function can be used. In this case, the vibrational seismogram – the analog of the impulse seismogram – is reconstructed at the place of recording as a result of continuous convolution of long seismic signals with the reference signals reconstructed at the recording points. A seismogram obtained in this way reflects the main types of seismic waves with measurable characteristics.

The first regime of sounding is applied for teleseismic distances, and the second regime is used within the first hundreds of kilometers. The first one is associated with a higher power output in a narrow frequency band near the resonance in the system "vibrator-medium", and the selective properties of the seismic traces themselves depending on the distance of sounding.

A considerable scientific-technical potential for the solution of this problem has been accumulated in the Siberian Branch of RAS. During last 15 years, a number of powerful low-frequency vibrators with the acting force of up to 100 tons-force and the frequency range of 3–15 Hz have been developed and created. Let us present brief characteristics of some of them.

The eccentric vibromodule CV-100 (Figure 5) creates a vertically oriented perturbing force with the help of synchronously rotating unbalances mounted on the platform. The use of the radiation regime near the resonant frequency of the system "vibrator-ground" in the area of 7 Hz makes possible a multiple increase of the radiation power. The distances of sounding of more than 1000 km were achieved in the experiments with this source.

The eccentric vibromodule CV-40 is a transportable variant of the previous source. An action of 40–50 tons-force can be developed. Its active frequency range is 5–10 Hz.

The hydroresonant vibrator HRV-50 (Figure 6) creates a perturbing force with the help of a vertically vibrating water volume having a mass



Figure 5. Hydroresonant vibrator HRV 50

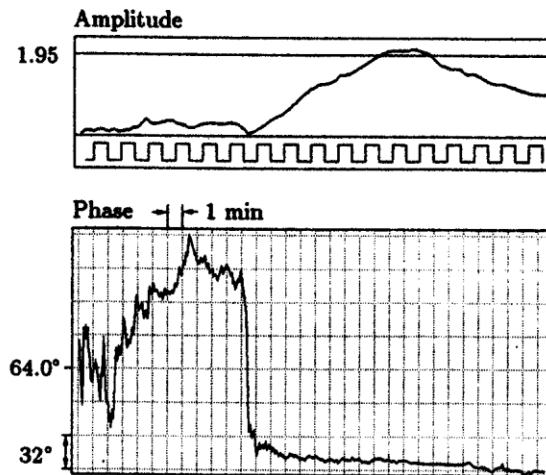


Figure 6. Results of amplitude and phase determination of the vibroseismic signal. The distance is 430 km. The band width is 0.001 Hz

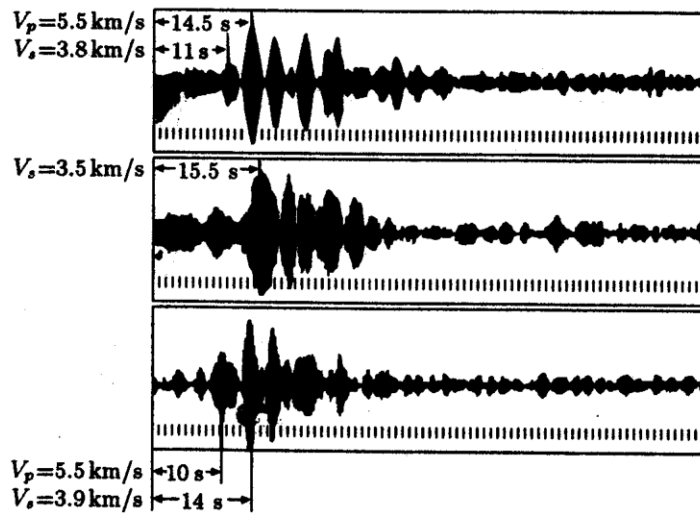


Figure 7. An example of a 3-component vibrational seismogram. The distance is 60 km. The frequency range is 4.98–7.03 Hz

of 60 tons in the frequency range of 2–15 Hz. The source is characterized by small inertness, high stability of the parameters and relative simplicity of its manufacturing. The other advantage of the source is a possibility to increase the force of action owing to the increased sizes of the working chamber. At present, a variant of such vibrator for the force of action of 1000–10000 tons-force using water-filled mines has been developed.

High accuracy in the measurement of the parameters of vibroseismic signals is achieved by high metrological characteristics of the sources and the use of statistic methods of signal processing. As an example, Figure 6 gives the results of a determination of an amplitude and a phase of the vibroseismic signal recorded at the distance of 430 km with the sounding frequency of 6.5 Hz.

In the upper graph, the section of the presence of the sounding signal is characterized by the increase of some statistics proportional to the amplitude of the seismic signal. The lower graph shows the behaviour of the signal's phase, reaching its stable value in the signal section in a jumpwise manner. The accuracy of phase measurements is approximately 1.5 degrees, and that of the amplitude measurements – within 5%. Figure 7 gives an example of a 3-component vibrational seismogram obtained from the vibrator CV-100 in the frequency range 4.98–7.03 Hz at the distance of 60 km from the source.

Numerous experiments on the deep vibroseismic sounding of the Earth which have been carried out in the Siberian Branch of RAS during last 15 years justify the efficiency of works on the calibration of seismic traces and seismic stations.

3. A numerical method for the solution to the inverse problem of determination of the explosions' parameters

As a result of the calibration of the propagation channels of seismic information on the basis of vibrosounding we obtain, in particular, two types of data:

- arrival times of the signal from the place of possible nuclear tests to the receiving points;
- amplitude-frequency characteristics of the propagation channels of seismic information;

The information of the first type allows us to synchronize the processing of information from the recorders in time (i.e., automatically identify the pieces of records of the same event from different stations), and thereby it gives a possibility to organize continuous monitoring of coordinates of the events occurring in the given region of the probable explosions.

The information on the spectral properties of propagation channels makes it possible to take into account the amplitude-phase distortions which can occur both due to the block non-homogeneity of the Earth and as a result of the frequency dependence of absorption in the medium. In this case, the correction of data of the real events' records can be done, for example,

within the framework of the convolution model – on the basis of the solution to the integral equation of the form

$$v_r(t; x_k, y) = \int v(\tau; x_k, y) H(t - \tau; x_k, y) d\tau.$$

Here v_r are initial data, v are the data after correction, H is the impulse characteristics of the seismic channel, t is time, x_k are the coordinates of the k -th recorder, y are the coordinates of the explosion's hypocenter.

The procedure of finding the function $v(t; x_k, y)$ can be considered as the fitting of data into the correction domain, i.e., the area where the inverse problem has a stable solution within the framework of simpler *a priori* assumptions on the mathematical model of the processes of seismic waves' propagation in the medium. In our case, we shall consider the model of homogeneous absolutely elastic medium with the known velocities of propagation of pressure and shift waves.

3.1. Statement of the problem of inverse seismic moment tensor

A system of the Lamé equations is considered, which describes the motion of the homogeneous ideally elastic medium:

$$\frac{\partial \sigma_{ij}}{\partial x_j} - \rho \frac{\partial^2 u_i}{\partial t^2} = M_{ij}(t) \frac{\partial}{\partial x_j} \delta(x - y). \quad (1)$$

Here $i, j = \overline{1, 3}$, $x, y \in R^3$, $t \in R^1$; ρ is the medium density, σ_{ij} is the stress tensor related to the displacement vector $u(x, t) = (u_1, u_2, u_3)$ in the form:

$$\sigma_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \lambda \delta_{ij} \frac{\partial u_k}{\partial x_k},$$

where μ, λ are the Lamé constants and the repetition of indices means summation.

The symmetric tensor of the rank 2 – $M_{ij}(t)$ is called the seismic moment tensor which is in energy units ($\text{g} \cdot \text{cm}^2 \cdot \text{s}^{-1}$) and, in terms of equivalent forces, it describes shift discontinuity, shift destruction of the medium, transformational change of the volume, crack of separation, etc. [10].

Let the parameter t_o characterize the time of process beginning in the source ($M_{ij}(t) \equiv 0$, $t < t_o$). Let us also suppose that λ, μ and ρ are the known constants.

The inverse problem is in the determination of the parameters t_o, y and the tensor $M_{ij}(t)$ from the data of the form:

$$v_k(t) = u(x_k, t) + \varepsilon_k(t), \quad k \geq 4. \quad (2)$$

Here $x_k \in R^3$, ε_k is the noise of normal distribution, the zero mean value and some known covariance matrix $G_\varepsilon(x_k, x_{k'})$.

3.2. Description of the solution algorithm

Let us assume that there exists a formal procedure \mathcal{A} with the help of which we may determine, using the data of form (2), either the arrival times of p-waves and s-waves separately, or only the difference of the arrival times of p-waves and s-waves

$$\mathcal{A}(v_k(t)) = t_p^k, t_s^k, t_s^k - t_p^k. \quad (3)$$

Then it is reasonable to implement the solution of inverse problem (1)–(3) in two steps: at the **first step** we determine the parameters t_o, y from data (3); at the **second step** we determine, from data (2), the dynamic characteristics of the source, that is the tensor $M_{ij}(t)$, when its coordinate y and the time of the beginning process t_o are known.

The problems are solved at the both steps by minimizing the functional of the form:

$$J(\eta) = (w - F(\eta))^T V (w - F(\eta)) + \Omega(\eta) + Sh(\eta). \quad (4)$$

Here $w = (w_1, \dots, w_N)$ is the vector of measurements determined from (2) and (3), $\eta = (\eta_1, \dots, \eta_n)$ is the vector of the desired parameters; $F(\eta)$ is the vector of functions for the solution determined from equation (1) of the same dimension as the data w ; $(\cdot)^T$ denotes the vector line; V is a positively determined symmetric $N \times N$ matrix which can completely coincide with G_ϵ^{-1} , and $Sh(\eta)$ is some external penalty functional. The functional Ω in (4), in accordance with A.N. Tikhonov [11], is a stabilizing functional:

$$\Omega(\eta) = (\eta - \eta_0)^T G (\eta - \eta_0). \quad (5)$$

Here G is some positively defined symmetric matrix of the regularity coefficients of $n \times n$ dimension, η_0 is the vector of *a priori* values of the stabilizing functional. It should be noted that if we know the covariance matrix of the parameters $G_\eta = \langle \eta \eta' \rangle$, then we may take $G = G_\eta^{-1}$.

3.3. The method of minimization

The minimum of functional (3) is determined by the method which is the combination of the Marquardt technique and the regularized quasi-Newtonian method using only the first derivatives, and the controlled iterative step is given as follows:

$$\eta^{k+1} = \eta^k + \alpha_k p^k, \quad (6)$$

$$p^k = -A^{-1}(\eta^k, \zeta^k) Y(\eta^k, \zeta^k), \quad (7)$$

Here α_k is the step of regulation of the parameter, $1 \leq k \leq k_{\max}$,

$$Y = \left(\frac{\partial F}{\partial \eta} \right)^T V(F - w) + G(\eta - \eta_0) + \frac{\partial Sh}{\partial \eta}, \quad (8)$$

$$A = \left(\frac{\partial F}{\partial \eta} \right)^T V \left(\frac{\partial F}{\partial \eta} \right) + \Lambda + \frac{\partial^2 Sh}{\partial^2 \eta}. \quad (9)$$

The vector ζ denotes the combination of all free parameters of functionals (4)–(5) and the iterative processes (6)–(9) adjusted at each step with respect to k :

$$\zeta^{k+1} = \zeta^k + \Delta \zeta(\eta^1, \dots, \eta^k).$$

Thus, in solving inverse problem (1)–(3), an adaptive approach is used when the penalty functional parameters and the Tikhonov stabilizer are formed at each iterative step. The methods are combined through the introduction of the independent matrices Λ and G . In fact, if $\Lambda = G$, we obtain the regularization of the quasi-Newtonian method. When $G = 0$, algorithm (6)–(9) is the generalization of the Marquardt method (Λ is an arbitrary matrix, in contrast to the conventional technique when $\Lambda = \lambda I$).

The parameters α_k in (6) at each step of the iterative process are selected as the maximal parameters from the set $\{\alpha_k | 0 < \alpha_k < \alpha_0\}$, so that the inequality

$$J(\eta^k + \alpha_k p^k) - J(\eta^k) \leq \varepsilon \alpha_k (Y, p^k),$$

where $0 < \varepsilon \leq 1/2$, is fulfilled.

When $\alpha_0 = 1$, the method with the step regularization coincides with the conventional method of the first. In the case of $\alpha_0 > 1$, the search for the global minimum in the direction of the antigradient is carried out. If this global minimum is not found, the standard technique of subdividing α is used.

Iterative process (6)–(9) is completed if $k = k_{\max}$, or if the differences between the values of several parameters become smaller in the absolute values than some values δ_i , $i = 1, \dots, n$.

3.4. Discrepancy method of the regularization and questions of stability

One of the most complicated problems in solving the unstable inverse problems such as (1)–(3) is the problem of regularization of the solution, that is, concordance of the values of regularization parameters of the matrix G with the existing noise level in the data $w = (w_1, \dots, w_N)$. In this paper, the discrepancy method is applied for the selection of the regularity parameters.

Let us illustrate this by the solution of a test problem. Let $\Lambda = \alpha I$, where I is a unit $n \times n$ matrix. Based on the solution of equation (1) [12], the data of the form (2)–(3) were simulated for four observational points (with the average distance between them of about 5 kilometers). The noise was

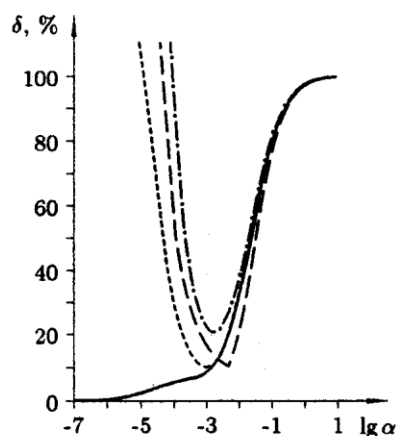


Figure 8. Dependencies of relative errors δ on the regularization parameter α for different noise levels in the data

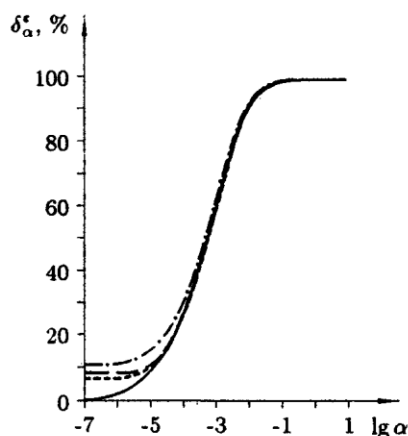


Figure 9. Dependencies of relative discrepancies δ_α^ϵ on the regularization parameter α for different noise levels in the data

simulated by the random numbers generator with the normal distribution and the zero mathematical expectation. Tests calculations were made for the cases when the relative noise level ($\varepsilon = \|v_\varepsilon - u\|/\|u\|$, where $\|\cdot\|$ is a mean quadratic norm) was 0, 10, 20, 40 per cent. The parameters of the source y , t_o and the seismic moment tensor were selected arbitrarily.

The procedure of coordination of the regularity parameter α is in this case in determining the optimal value of the parameter $\alpha - \alpha_{opt}$ at which, for the given noise level ε , the relative error of determining the sought-for parameter $\delta = \|\eta - \eta_\varepsilon\|/\|\eta\|$ is minimal.

Figure 8 shows the graphs of the δ dependence on α for different noise levels, when the vector η consists of the components of the Fourier transform tensor $\hat{M}_{ij}(f)$ for the frequency value $f = 20$ Hz. It was assumed that the coordinates of the source and t_o are accurately determined at the first stage. The problem at the second stage is solved in the spectral domain, that is the value $\hat{M}_{ij}(f)$ is determined at each of 256 frequencies.

Analysis of test calculations showed that the value α_{opt} essentially depends on the mutual location of the earthquake source and observation stations, amount of noise and the value of frequency f . So, it should be noted that, with the increase of frequency and noise, α_{opt} also increases and may take the value 0.1.

It is interesting that, in the presence of even small noise in the data (less than 10 per cent) it is impossible to obtain a stable solution without the regularization procedure, i.e., for $\alpha = 0$ (Figure 8).

Let u_α denote the solution of equation (1) with the parameters η_α determined on the basis of the minimization of functional (4) for some fixed

value of the regularity parameter α . Then $\delta_\alpha^\varepsilon$ denotes the value of relative discrepancy u_α and the experimental data v_ε for the given noise level ε : $\delta_\alpha^\varepsilon = \|u_\varepsilon - v_\varepsilon\|/\|v_\varepsilon\|$.

Figure 9 shows the dependencies of the discrepancy $\delta_\alpha^\varepsilon$ on the regularity parameter for different noise levels: $\varepsilon = 0 \div 40\%$. The determination of the optimal value of regularity parameter for our problem, in accordance with the *discrepancy method*, consists in the determination of $\alpha_{\text{opt}}^\varepsilon$ such that $\alpha_{\text{opt}}^\varepsilon = \sup\{\alpha | \delta_\alpha^\varepsilon \leq \varepsilon\}$. Due to the monotonicity of the curves $\delta_\alpha^\varepsilon$, the value $\alpha_{\text{opt}}^\varepsilon$ is unique at each noise level (see Figure 9).

3.5. Results of numerical experiments

The method proposed for solving the inverse problem was tested by the processing of seismological records of an earthquake of the energy class $k = 7$ which occurred in Altai in 1985. The event was recorded by four three-component stations "Region" (Figure 10). The coordinates of the source were determined based on the data of the difference of arrival times of p and s-waves. The depth of the source was $y_3 = 14.5 \pm 0.5 \text{ km}$. Parameters of the medium had the following values: $\rho = 2.7 \text{ g/sm}^3$, $c_p = 6.1$, $c_s = 3.5 \text{ km/s}$.

Figure 11 shows one of the components of a digitized record of station 4 and its spectrum.

Based on the method described above, the optimal values of the regularity parameters were calculated on 256 frequencies assuming that the noise level in the data did not exceed 30 per cent. In accordance with the results of the test calculations, the accuracy of determination of kinematic parameters of the source and the components of seismic moment tensor in this case must not be worse than 30 per cent.

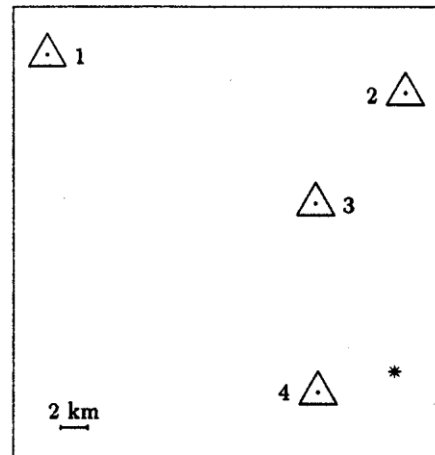


Figure 10. The scheme of location of seismological stations and the earthquake epicenter - *

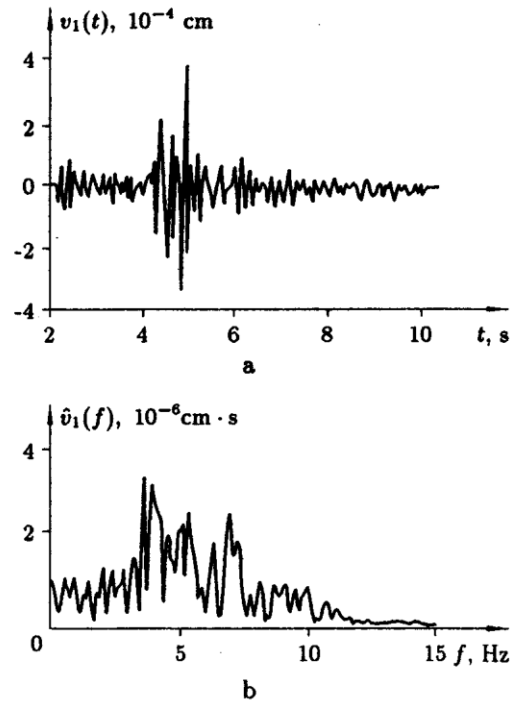


Figure 11. A record of one of the components of vector displacement at station 4 and its amplitude spectrum

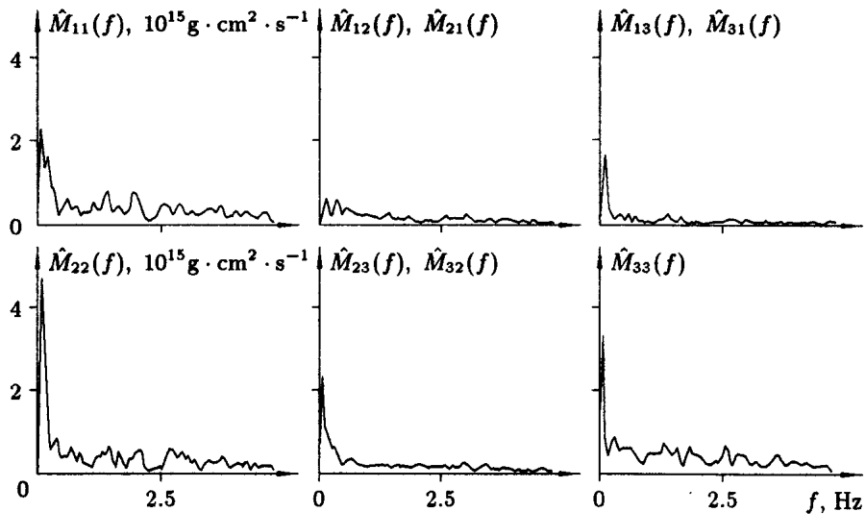


Figure 12. Spectral densities of components of the determined seismic moment tensor

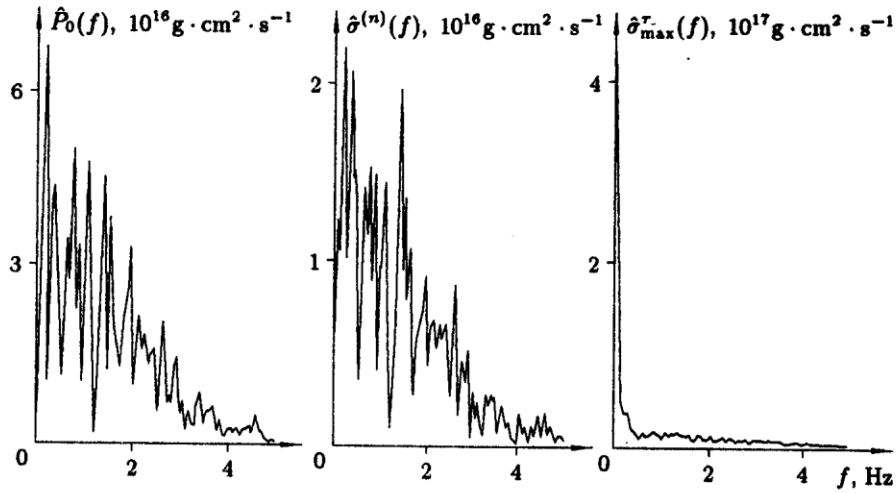


Figure 13. Spectral densities in the source: a – the hydrostatic tensor $p_o(t)$; b – the separation stresses $\sigma^n(t)$; c – the shift stresses $\sigma_{\max}^{\tau}(t)$

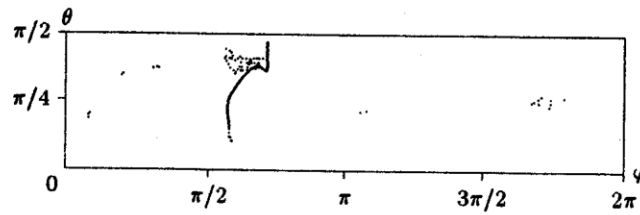


Figure 14. Projection of the normal vector of area elements of the maximal tangential stresses in the earthquake source on the unit semi-sphere. $\Delta t = 0.015$ sec

Figure 12 shows spectral densities of components of the seismic moment tensor $\hat{M}_{ij}(f)$. It is seen from the figures that, along with the shift stresses, hydrostatic stresses are also present in the source. It is possible to make a distinction between them by reducing them, at each time moment, to the main axes of stresses in the source, which, in their turn, are determined by the eigen-vectors $M_{ij}(t)$ (c.f. [13]). The further interpretation is possible on the basis of the separation of the spherical tensor $p(t)$ and the deviator of stresses from the obtained tensor. From the latter procedure based on the problem solution for the extremum of tangential stresses, it is possible to determine the normal vector $n(t)$ in the areas of maximal tangential stresses, the shift stresses – $\sigma_{\max}^{\tau}(t)$ and the separation stresses – $\sigma_n(t)$ (Figures 13–14).

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