A CA-model of population dynamics of organisms living in Baikal. Verification and investigation of pollution influence

Ivan Afanasyev

Abstract. The CA-model was modified in order to take into account the birthrates seasonal dependency and the influence of water streams on movements of organisms. The model was verified within the production-to-biomass and the relative average quantity criteria. A difference in the verification results and the assessments of physical values is about 20%. The simulation of a possible pollution influence was carried out. The assessment of the critical pollution that leads to the total death of individuals and acceptable pollution with no observable influence are presented.

1. Introduction

Investigation of the population dynamics is an important problem. A realistic result of the investigation is the ability to assess limits of the anthropogenic influence on the ecosystem, which is environmentally acceptable and economically sound.

Up till now, the population dynamics has been investigated with the help of differential equations systems [1, 2]. In the models used, the number of interacting organisms groups is less or equal to 3, and the population parameters are uniformly distributed over the modeling area. The model of eight populations in lake Baikal has been proposed and investigated in [3] using ODE. So, the first restriction was overcome using the numerical simulation.

A CA-model of the population dynamics of eight organisms groups living in lake Baikal is presented in [4]. This model allows us to take into account the spatial distribution of organisms and a possible local pollution. However, the modeling area is proposed to be in the form of a square of the size much less than that of the lake. Also, the seasonal influence and water streams are neglected.

In this paper, a modified version of the CA-model [4] is presented. The area under simulation matches the lake Baikal geometry. Also, the seasonal influence on birthrates and water streams is taken into account. The data about the organisms population used in the model are taken from [3].

The model is verified by comparing the assessments of the physical values and the simulation results. The fact is, it is difficult to find data for local areas in Baikal, so the verification criteria proposed use the global assessments.

*Supported by Presidium of Russian Academy of Sciences, Project 15.9-2014.
averaged for a simulated area. The production-to-biomass and quantity-to-quantity criteria were chosen for the verification. Simulation results differ from assessments of physical values in about 20%.

The model was used to investigate a possible pollution influence. Assessments of a critical and an acceptable pollution are presented.

2. A composite CA-model

Three kinds of organisms are investigated: *macrohectopus, comephorus dybovsky* and *comephorus baikalensis*.

Each kind of organisms is divided into the age groups (letters \(\{m,d,b\}\) designate the kind, and numbers \(\{1,2,3\}\) designate the age group):

- Macrohectopus: immature \(m_1\), puberal \(m_2\).
- Comephorus dybovsky: one-year-old \(d_1\), immature \(d_2\), puberal \(d_3\).
- Comephorus baikalensis: one-year-old \(b_1\), immature \(b_2\), puberal \(b_3\).

Totaly, there are eight groups of organisms. Prey-predator and demographic relations are considered for groups.

Let us define the CA-model of the population dynamics of the Baikal organisms as

\[ \mathcal{N} = (\Sigma, M, f, \rho), \]

where \(\Sigma\) is the alphabet of states, \(M\) is the cells names set, \(f\) is the global transition operator, \(\rho\) is the functional mode.

The model is a parallel composition \([5]\) of eight CA, each being designed to simulate a concrete group of the population dynamics.

Let \(Q\) be a square mesh that covers the surface of Baikal. A set of cell names \(M\) is a union of eight pairwise disjoint subsets \(M^\alpha_i\):

\[ M = M^m_1 \cup M^m_2 \cup M^d_1 \cup M^d_2 \cup M^d_3 \cup M^b_1 \cup M^b_2 \cup M^b_3. \]

Assume that bijective mappings \(\psi^\alpha_i: Q \rightarrow M^\alpha_i\) exist for all \(i\) and \(\alpha\).

Let us define the *cell* as an element of the set \(M \times \Sigma\). Cell states are integers \(n \in \Sigma\), to characterize the density of organisms in this cell. A finite collection of cells

\[ S(c) = \{(\phi_1(c), n_1), \ldots, (\phi_k(c), n_k)\} \]

is a *local configuration*, where \(n_i \in \Sigma\) and \(\phi_i: M \rightarrow M\) are naming functions, which define the names of cells, interacting with the cell named \(c\).

In the general case, *Local transition operator* \(f\) is

\[ f: \{S(c)\} \mapsto \{S(c)\}. \]
The result of applying the operator $f$ to a cell named $c$ is replacing the local configuration $S(c)$ by the local configuration $f(S(c))$. *Iteration* or *application of the transition operator* $F$ is application of the local operator to all the cells.

The operator $F$ is a sequential composition of the two operators [5]:

$$ F = F_1 \circ F_2, $$

where $F_1$ is the movement operator, $F_2$ is the demographic operator.

The movement operator $F_1$ on its part is a sequential composition of a free movement operator $F_d$ and a water-stream operator $F_s$:

$$ F_1 = F_d \circ F_s. $$

### 2.1. Free movement operator

Let $f_z$ and $F_z$ be local and global transition operators of integer diffusion according to [6]:

$$ f_z : \{S_1(c)\} \mapsto \{S_1(c)\}, $$

where $S_1(c)$ is a set of the neighbors of the cell named $c$ including the cell itself. Let us define cells with the names $c, c' \in M_{\alpha i}$ as neighbors if they correspond to the neighboring cells of the square mesh $Q$.

Application of $f_z$ to the cell named $c$ is performed with the following algorithm:

1. Let $(c_1, n_1), \ldots, (c_k, n_k) \in S_1(c)$ be the neighbors of the cell $(c, n)$, $k \leq 4$. Number $i \in \{1, \ldots, k\}$ is randomly chosen with equal probability.

2. The new states $n'$ and $n'_i$ of the cells named $c$ and $c_i$ are obtained by the formula

$$ n' = n - \lfloor \sigma \cdot n \rfloor + \lfloor \sigma \cdot n_i \rfloor, \quad n'_i = n_i + \lfloor \sigma \cdot n \rfloor - \lfloor \sigma \cdot n_i \rfloor, $$

where $\sigma$ is the diffusion ratio, $0 \leq \sigma \leq 1$.

The operator $F_z$ is asynchronously applied.

Let us define the free movement operator $F_d$. Let $l$ be the physical size of a cell from the mesh $Q$. Let $v_{cr}$ be a cruising speed of organisms of the kind $\alpha$ and the age group $i$. Let $\Delta t$ be a physical time step equivalent to the model iteration. The maximum number of cells visited by the individual can be calculated as

$$ K_{i}^\alpha = \frac{v_{cr} \Delta t}{l}. $$

Let us define $F_d|_{M_{\alpha i}}$ as a $K_{i}^\alpha$-wise sequential application of $F_z$:

$$ F_d|_{M_{\alpha i}} = (F_z)^{K_{i}^\alpha}. $$
2.2. The water-stream operator

Let \( \text{stream}_c : M \mapsto \mathbb{R}^2 \) be a water streams map. The two-dimensional vector \( \text{stream}_c(c) = (v_x, v_y) \) is a physical speed of water stream in the cell named \( c \). The direction \( x \) corresponds to the West-to-East direction. The direction \( y \) corresponds to the South-to-North direction.

The local operator \( f_s \) is designed to simulate the organisms movement with water streams. The algorithm of \( f_s \) application to the cell named \( c \) is recursive. The cell named \( c \) is observed at the first level of recursion. The neighbors of the cell named \( c \) are observed at the second level of recursion. Then the neighbors of the neighbors, and so on. The number of cells observed at the \( k \)-th level does not exceed \( 2^k \).

Let \( l \) be the size of a cell from the mesh \( Q \). Let \( \Delta t \) be a time which corresponds to one iteration.

Application of \( f_s \) to a cell named \( c \) is performed with the following algorithm:

1. Find \( c^k_x \) and \( c^k_y \) the neighbors names of the cell named \( c^k \) in the directions \( v^k_x \) and \( v^k_y \), where \( k \) is a level of recursion, \( c^k \) is the name of cell currently being observed, \( (v^k_x, v^k_y) = \text{stream}(c^k) \) is the stream vector in the cell named \( c^k \).

2. Find \( n^k_x \) and \( n^k_y \), i.e., the number of organisms of the cell named \( c^k \) moved by water streams to the cells named \( c^k_x \) and \( c^k_y \), respectively:

\[
\begin{align*}
n^k_x & = \begin{cases} n^k \frac{v^k_x}{|v^k_x| + |v^k_y|}, & s^k_x > 1, \\ n^k \frac{v^k_x}{|v^k_x| + |v^k_y|} s^k_x, & s^k_x \leq 1, \end{cases} \\
n^k_y & = \begin{cases} n^k \frac{v^k_y}{|v^k_x| + |v^k_y|}, & s^k_y > 1, \\ n^k \frac{v^k_y}{|v^k_x| + |v^k_y|} s^k_y, & s^k_y \leq 1, \end{cases}
\end{align*}
\]

where \( n^k \) is a model number of moving organisms of the cell named \( c^k \), \( n^k \) is calculated at \( (k-1) \)-th level of recursion at Step 3 if \( k > 1 \) and \( n^1 \) is equal to the state of the cell named \( c \) if \( k = 1 \), \( s^k_x \) and \( s^k_y \) are a number of cells, covered with a water stream during the time \( \tau^k \):

\[
\begin{align*}
s^k_x & = \frac{\tau^k v^k_x}{l}, \\
s^k_y & = \frac{\tau^k v^k_y}{l},
\end{align*}
\]

\( \tau^k \) is the physical time calculated as follows:

\[
\begin{align*}
\tau^1 & = \Delta t, \\
\tau^k & = \tau^{k-1} - \Delta \tau^{k-1},
\end{align*}
\]

\( \Delta \tau^{k-1} \) being the time spent by the water stream to cover the distance \( l \) at the previous recursion level:

\[
\Delta \tau^{k-1} = \frac{l}{v^k_{x-1}} \quad \text{or} \quad \Delta \tau^{k-1} = \frac{l}{v^k_{y-1}}.
\]
3. If $s^k_x > 1$, then the algorithm is recursively applied for the cell named $c^{k+1} = c^k_x$ and $n^{k+1} = n^k_x$ and $\Delta \tau^k = \frac{1}{v^k_x}$. The same checking is done for y-direction.

2.3. The demographic operator

Let $f_2$ be a local demographic operator:

$$f_2 : \{S_2(c)\} \mapsto \{S_2(c)\},$$

where $S_2(c)$ is a set of twins (Figure 1).

Let us define cells with the names $c^\alpha_i \in M^\alpha_i$ and $c^\beta_j \in M^\beta_j$ as twins if they correspond to the same cell in $Q$, i.e., $(\psi^\alpha_i)^{-1}(c^\alpha_i) = (\psi^\beta_j)^{-1}(c^\beta_j)$: $\exists i \exists \alpha : c \in M^\alpha_i$,

$$q = (\psi^\alpha_i)^{-1}(c), \quad q \in Q,$$

$$S_2(c) = \begin{cases} (\psi^m_1(q), n^m_1), & (\psi^m_2(q), n^m_2), \\ (\psi^d_1(q), n^d_1), & (\psi^d_2(q), n^d_2), \\ (\psi^b_1(q), n^b_1), & (\psi^b_2(q), n^b_2), \\ (\psi^b_3(q), n^b_3), & (\psi^b_3(q), n^b_3) \end{cases}$$

The functional mode of the local operator $f_2$ is synchronous.

The new state of the cell named $c^\alpha_i \in M^\alpha_i$ is given by the formula

$$(n^\alpha_i)' = n^\alpha_i + (\rho^\alpha_i n^\alpha_j - \lambda^\alpha_i n^\alpha_i - \theta^\alpha_i n^\alpha_i) \Delta t,$$

where $j$ is the age group of organisms that generates organisms of the $i$-th age group, $\Delta t$ is the time which corresponds to one iteration, $\rho^\alpha_i n^\alpha_j$ is an income into the group due to the birth or aging, $\lambda^\alpha_i n^\alpha_i$ is an outcome from the group due to the death, $\theta^\alpha_i n^\alpha_i$ is an outcome from the group due to aging.

The birthrates $\rho^\alpha_i$, the deathrates $\lambda^\alpha_i$ and the aging rates $\theta^\alpha_i$ are taken from [3].

1. Aging rates $\theta^\alpha_i$ are constant,
2. Deathrates $\lambda^d_2, \lambda^d_3, \lambda^b_2, \lambda^b_3$ are constant,
3. Deathrates $\lambda^m_2, \lambda^m_3, \lambda^d_1, \lambda^b_1$ are obtained as follows:

$$\lambda^\alpha_i = a^\alpha_i + b^\alpha_i (n^d_2 + n^d_3) + d^\alpha_i (n^b_2 + n^b_3), \quad (1)$$

where $a^\alpha_i$, $b^\alpha_i$, $d^\alpha_i$ are constant, $b^\alpha_i$ and $d^\alpha_i$ define deaths of preys due to eating by predators, $a^\alpha_i$ define deaths due to other reasons,
4. Birthrate $\rho^m_1$ is constant,
5. Birthrates $\rho_1^d$, $\rho_1^b$ are obtained as follows:

$$\rho_1^\alpha = \mu_\alpha (n_1^m + n_2^m) + \nu_\alpha (n_1^d + n_1^b),$$

where constants $\mu_\alpha$ and $\nu_\alpha$ depend on preferences of predators.

In order to take into account the seasonal dependency, the birthrates $\rho_1^d$ and $\rho_1^b$ are multiplied by the periodic functions $s_d(t)$ and $s_b(t)$ (Figure 2). Their period is equal to one year. Zero corresponds to the 1st of January.

Figure 2. Graphs of $s_b(t)$ (grey) and $s_d(t)$ (black)

3. The model verification

The model verification is done for the three kinds of organisms (data are summarized by the age groups).

The following criteria were chosen for the verification:

- $P_\alpha/B_\alpha$ is the ratio of production to biomass for organisms of the kind $\alpha$. The production $P_\alpha$ is the positive change of biomass per year. The production includes the born and aged organisms, but does not include dead organisms, $B_\alpha$ is an averaged biomass per year.

- $N_\alpha/N_\beta$ is the ratio of the number of organisms of the kind $\alpha$ to the number of organisms of the kind $\beta$. $N_\alpha$ is an averaged number of organisms per year.

$$P_m = P_1^1 + P_2^1, \quad B_m = B_1^1 + B_2^1, \quad N_m = N_1^1 + N_2^1,$$

$$P_d = P_1^d + P_2^d + P_3^d, \quad B_d = B_1^d + B_2^d + B_3^d, \quad N_d = N_1^d + N_2^d + N_3^d,$$

$$P_b = P_1^b + P_2^b + P_3^b, \quad B_b = B_1^b + B_2^b + B_3^b, \quad N_b = N_1^b + N_2^b + N_3^b.$$

The assessment of $B_m$ is given in [7] and equals 110,000 ton. The assessment of $P_m$ is also given in [7] and varies from 330,000 to 900,000 ton. Thus, the assessment of $P_m/B_m$ is 3–8.

The assessment of $P_d$ is given in [8] and equals 1.24. Assessments of $N_m/N_d$ and $N_m/N_b$ is given in [3].

The verification results are shown in the table. The difference in the model results from the assessments given in [3, 7, 8] is about 20%.

<table>
<thead>
<tr>
<th>$P_m$</th>
<th>$P_d + P_b$</th>
<th>$N_m$</th>
<th>$N_d$</th>
<th>$N_m/N_d$</th>
<th>$N_m/N_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[3, 7, 8]</td>
<td>3–8</td>
<td>1.24</td>
<td>6.05</td>
<td>21.52</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>5.77</td>
<td>1.49</td>
<td>6.00</td>
<td>20.45</td>
<td></td>
</tr>
</tbody>
</table>
4. The pollution influence

The model allows us to take into account the influence of the possible pollution. Let $\text{poll}(c) : M \mapsto R^+_+$ be a pollution map, i.e., a function of the cell name with a range in the positive numbers characterizing pollution intensity. Let us assume that pollution affects deathrates.

1. The new deathrates of predators $\lambda^d_2, \lambda^d_3, \lambda^b_2, \lambda^b_3$ are given as follows:

$$ (\lambda^o_i(c))^' = \lambda^o_i(1 + \text{poll}(c)). $$

2. The new deathrates of preys $\lambda^m_1, \lambda^m_2, \lambda^d_1, \lambda^b_1$ are given as follows:

$$ (\lambda^o_i(c))^' = \lambda^o_i + \text{poll}(c)a^o_i, $$

where $a^o_i$ is a constant deathrate from (1).

The pollution map $\text{poll}(c)$ used is shown in Figure 3. The function $\text{poll}(c)$ is the normal distribution density with the center in the cell $c_0$ south of lake Baikal multiplied by a constant. Grey color means land. White color means water. Darker color means a greater pollution intensity.

5. Computational experiments

In further computational experiments, the initial state is a uniform distribution of individuals and organisms’ density equal to the stable state $\text{poll}(c_0) = 14^3$.

Some iterations for the simulation of the puberal macrohectopus population dynamics are presented in Figure 4. The pattern formation process is observed outside of the polluted area. Non-uniform distribution is a result of the water streams influence.

Population dynamics in the north area of Baikal (Figure 5) tends to steady annual oscillation process. Annual oscillations are the result of seasonal dependency of birthrates.

The population dynamics in the pollution epicenter $c_0$ is presented in Figure 6; $\text{poll}(c_0) = 14$.

The population dynamics in the cell with $\text{poll}(c) = 2.3$ is given in Figure 7. The number of preys is increased and the number of predators is decreased as compared to non-polluted area.
Figure 4. Some iterations for simulating the puberal macrohectopus population density. A darker color means a higher individuals density

Figure 5. The population dynamics in a cell in the north area of Baikal for immature macrohectopus (left) and immature comephorus dybovsky (right)

Figure 6. The population dynamics in the pollution epicenter for immature macrohectopus (left) and immature comephorus dybovsky (right). A brighter line means the same dynamics in the cell without pollution

Figure 7. The population dynamics if the cell with poll(c) = 2.3 for $m_1$ (left) and $d_2$ (right). A brighter line means the same dynamics in the cell without pollution
In order to investigate the dependency of the average annual density on the pollution intensity, several points along the segment given in Figure 8 were investigated.

The distribution and density of the pollution intensity at the 2,000th iteration along the segment is presented in Figure 9.

The following conclusions might be proposed:

- If \( \text{poll}(c) > 10 \), then the macrohectopus density at the 2,000th iteration is less than 3% of average density in non-polluted area. That means total death of organisms.
- If \( \text{poll}(c) \in (0.15, 5) \), then the number of preys increases and number of predators decreases in comparison to non-polluted case. Concrete values depend on \( \text{poll}(c) \) intensity.
- If \( \text{poll}(c) < 0.03 \), then pollution influence is not observable within natural annual oscillations.

6. Conclusion

The CA-model [4] of the population dynamics of eight organisms in lake Baikal was corrected and extended by taking into account the lake geometry, seasonal dependency of birthrates and water streams.

A difference in the model verification results from the assessments given in [3, 7, 8] is about 20%.

The dependency of the model behavior on the pollution intensity is investigated. The influence of local pollution is localized in the polluted area. The assessments of the critical pollution that leads to the total death of organisms have been obtained, as well as the acceptable pollution whose influence is not observable.
References


