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The impact of the initial density profile on protoplanetary disc evolution simulation*

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Abstract. In the simulation of a protoplanetary disc with a power law density profile, a disc instability is detected. The instability arises only with a power law profile and is affected by the power index. Thus, the impact of initial density profile is large within the employed numerical model.

1. Introduction

The density profile is the dependence of disc surface density on radius. As it is pointed in [1], at present, the true density profiles in protoplanetary discs are unknown. Nevertheless, in many works the density profile is thought to be a power law with different index. The index should be -1.0 to correspond the observation data [2]. In simulation, various indices in the range from -0.5 to -2.5 are being used [1, 3, 4].

The MMSN model (Minimal Mass Solar Nebula) was proposed in [5]. The density of solid particles in this model was obtained by imagining grinding up the planets, distributing their mass smoothly with radius and adding up enough gas to make thee Solar composition. The resulting density profile is the following:

$$\sigma(r) = \sigma_1 \left(\frac{r}{1 \text{ a.u.}} \right)^{-1.5}.$$

Here $\sigma_1 = 1700$ g/cm^2, 1 a.u. $= 1.5 \cdot 10^{11}$ m. The ratio of gas and solid particles mass in the MMSN model is the same as in the Solar System (100 : 1). Unfortunately, this model is scarcely applicable to extrasolar planetary systems as follows from observation of T Tauri stars [2]. Therefore, the simulation of protoplanetary discs with different initial density profiles is conducted [1, 3].

The aim of the present work is to simulate the evolution of the protoplanetary disc with different initial density profiles and to compare the results with the known simulations of the planet system formation.

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2. Simulation

Two density profiles were taken as initial profiles for computational experiments: the solid body profile (σ_S) and the power law profile (σ_P):

$$\sigma_S = \sigma_1\sqrt{1-\Big(\frac{r}{R_D}\Big)^2}, \qquad r < R_D,$$
$$\sigma_P = \sigma_1 r^{\alpha}, \qquad \alpha \in [-1.5, -0.5].$$

Here R_D is a disc radius and the value of σ_1 is set for the disc mass to be equal to a given value.

The computational experiments were conducted in sizeless variables in order to decrease round-off errors. The following quantities were chosen as basic characteristic parameters for transition to sizeless variables:

- the distance from the Sun to the Earth $R_0 = 1.5\cdot 10^{11}$ m;
- the mass of the Sun $M_\odot = 2\cdot 10^{30}$ kg;
- the gravitational constant $G = 6.672\cdot 10^{-11}$ N $\cdot$ m^2/kg^2.

The corresponding characteristic values of the particle velocity (V_0), the time (t_0), the potential (Φ_0) and the surface density (σ_0) are written down as:

$$V_0 = \sqrt{\frac{GM_\odot}{R_0}} = 30 \text{ km/s}, \qquad t_0 = \frac{R_0}{V_0} = 5\cdot 10^6 \text{ s} = 1/6 \text{ year},$$
$$\Phi_0 = V_0^2 = \frac{GM_\odot}{R_0}, \qquad \sigma_0 = \frac{M_\odot}{R_0^2}.$$

In the subsequent text all the parameters are given in sizeless units.

The ratio of central body mass, gas mass and dustsolid particles mass was set according to the MMSN model: the central body mass $M_\odot = 10.0$, the gas mass was $M_G = 1.0$ and solid particles mass $M_P = 0.01$. Both solid particles and gas were given the Keplerian velocity v_K (at any point of the disc the centrifugal force is equal to gravitational one):

$$\frac{\sigma v_K^2}{r} = -\frac{\partial\Phi}{\partial r}$$

The other parameters are the initial disc radius $R_D = 2.0$ and a radius of computational domain $R_M = 9.0$.

The dynamics of a solid particle component of a protoplanetary disc is described by the Vlasov–Liouville kinetic equation. In the sequel, dustsolid particles will be called simply particles. To consider the motion of gas component, the equations of gas dynamics are employed. The gravitational field is determined by the Poisson equation.

If we employ the collisionless approximation of the mean self-consistent field, then the Vlasov–Liouville kinetic equation is written in the following form:

$$\frac{\partial f}{\partial t} + \vec{v}\nabla f + \vec{a}\frac{\partial f}{\partial \vec{v}} = 0,$$

where $f(t, \vec{r}, \vec{v})$ is the time-dependent one-particle distribution function along coordinates and velocities, $\vec{a} = -\nabla\Phi + \vec{F}_{\mathrm{fr}}/m$ is the acceleration of a unit mass particle, $\vec{F}_{\mathrm{fr}}$ is the friction force between gas and particle components of the medium. The gravitational potential Φ could be divided into two parts, $\Phi = \Phi_1 + \Phi_2$, where Φ_1 presents the protostar potential. The second part of the potential Φ_2 is determined by the additive distribution of the moving particles and gas and satisfies the Poisson equation

$$\Delta\Phi_2 = 4\pi G\Sigma\rho.$$

In the case of a flat disc, the bulk density of the mobile media $\Sigma\rho = \rho_{\mathrm{part}} + \rho_{\mathrm{gas}}$ is equal to zero (ρ_{part} is the particle density, ρ_{gas} is the gas density). At the disc with the surface density σ, there is a shear of a normal derivative of potential. This shear gives a boundary condition for the normal derivative of the potential Φ_2:

$$\frac{\partial \Phi_2}{\partial z} = 2\pi G\sigma.$$

The equations of gas dynamics take the following form:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla(\rho\vec{v}) &= 0,\\
\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v}\right] &= -\nabla p + \vec{F},\\
\frac{\partial E}{\partial t} + (\vec{v}\nabla)E &= -\nabla(p\vec{v}) + Q + (\vec{F}, \vec{v}) - \nabla W,
\end{aligned}$$

where $E = \varepsilon + v^2/2$ is the density of the full gas energy, $\varepsilon = \varepsilon(\rho, T)$ is the internal energy of gas, $p = p(\rho, T)$ is the pressure of gas, $\vec{W} = \nabla T$ is the heat flux, Q is an increase in energy due to chemical reactions and radiation, $\vec{F}$ is the external force, which is defined by the following expression:

$$\vec{F} = \rho\nabla\Phi - k_{fr}(\vec{u} - \vec{v}).$$

Here k_{fr} is the coefficient of friction between gas and particle components of the disc, $\vec{u}$ is the particle velocity, $\vec{v}$ is the gas velocity. In the case of a flat disc, the form of equation remains the same with the only exclusion: the bulk density ρ is replaced with the surface density σ. In this paper, we consider only the flat disc model.

The Vlasov–Liouville equation is solved by the Particles-in-Cells method. To solve the equations of gas dynamics, the Fluids-in-Cells method is employed. The Poisson equation is solved by a combination of the FFT and the SOR methods. A detailed description of the code could be found in [6].

In the computational experiments the cylindrical coordinate system was used, grid size is $N_R \times N_\varphi \times N_Z = 300 \times 256 \times 100$. The experiments were conducted with the MVS-1000M multicomputer of the Siberian Supercomputer Centre (32 Alpha21264 processors, 833 MHz).

3. Results

The most interesting result is that a disc with a massive central body is unstable when the initial density profile satisfies the power law. Instability here is the loss of axial symmetry in the central arc of the disc, as is shown in Figure 1: a group of dense gaseous clumps is formed around the central body.

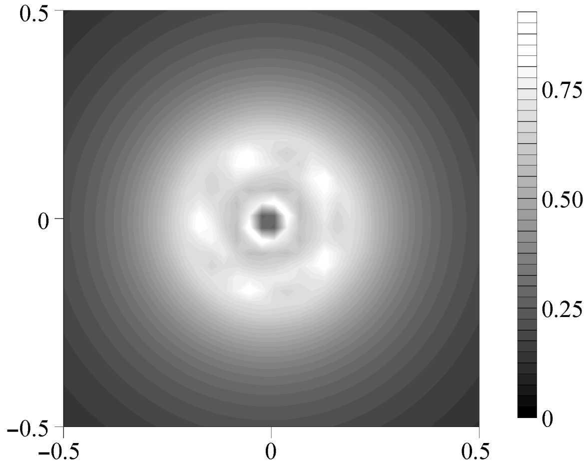

Figure 1. Gas density in the central arc of the disc

It is important, because usually a massive central body suppresses all the angular instabilities [7]. Furthermore, this instability arises for a disc with a power law profile and does not arise for a disc with a solid body profile.

In order to show the development of instability, the Fourier analysis of gas density was conducted along the angular direction:

$$\sigma(r,\varphi,t) = \sum_{k=1}^{N_\varphi-1} S_k(r,t)\cos\frac{2\pi k\varphi}{N_\varphi},$$

$$S_k(r,t) = \frac{1}{N_\varphi}\sum_{n=1}^{N_\varphi-1} \sigma(r,\varphi_n,t)\cos\frac{2\pi k\varphi_n}{N_\varphi}, \qquad \varphi_n = \frac{2\pi(n-1/2)}{N_\varphi},$$

$$S_{\max}(t) = \max\{S_k(r,t):\ 0<r<R_M,\ 1\le k\le N_\varphi-1\}.$$

Figure 2 shows a maximal harmonic amplitude $S_{\max}$ depending on time with various power indices α.

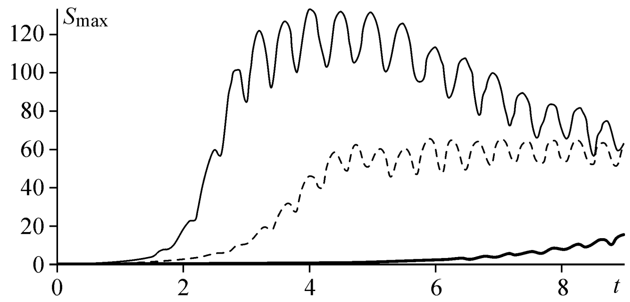

Figure 2. Evolution of instability in the disc with a power law profile for $\alpha = -0.5$ (thick line), $\alpha = -1$ (dash line), and $\alpha = -1.5$ (thin line)

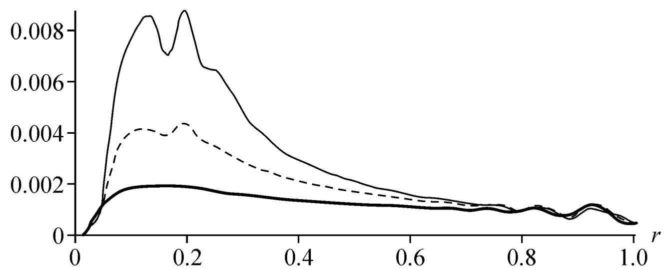

Figure 3. Gas density at $t = 9$ in the central arc of the disc for $\alpha = -0.5$ (thick line), $\alpha = -1$ (dash line), and $\alpha = -1.5$ (thin line)

It should be noted that zero harmonic ($k = 0$) is not displayed in Figure 2 because this harmonic shows instabilities that do not break axial symmetry. One can see from Figure 2 that the harmonic amplitude sufficiently increases with time.

Now let us consider the behavior of a disc with various indices α. Figure 2 shows that with lesser values of α, the instability evolves faster and harmonics have a larger amplitude. This fact corresponds to the results of [1]: in their simulations, discs with lower α formed planets earlier and the planets had a greater mass.

It is also stated in [1, 3] that for steeper profiles (lower values of α), the terrestrial planets are more massive. This result is supported by Figure 3 that shows an average grain particle density depending on a radius.

4. Conclusion

It follows from the computational experiment that within the employed model of the protoplanetary disc, the impact of the initial density profile is essential. The power law profile leads to the development of the angular instability, while the disc with a solid body profile remains stable. Moreover, the instability evolves faster with lower values of a power index. A decrease in the power index also leads to an increase of amplitude of unstable harmonics and to a higher mass of gas and grain clumps.

References

[1] Raymond S.N., Quinn T., Lunine J.I. Terrestrial planet formation in disks with varying surface density profiles // Astrophysical Journal. — 2005. — Vol. 632. — P. 670–676.

[2] D'Alessio P., Calvet N., Hartmann L., Lizano S., Canto J. Accretion disks around young objects. II. Tests of well-mixed models with ISM dust // Astrophysical Journal. — 1999. — Vol. 527. — P. 893–909.

[3] Kokubo E., Ida Sh. Formation of protoplanet system and diversity of planetary systems // Astrophysical Journal. — 2002. — Vol. 581. — P. 666–680.

[4] Rice W.K.M., Wood K., Whitney B.A., Bjorkman J.E. Constraints on a planetary origin for the gap in the protoplanetary disc of GM Aurigae // Mon. Not. R. Astron. Soc. — 2003. — Vol. 342. — P. 79–85.

[5] Hayashi C. // Prog. Theor. Phys. Suppl. — 1981. — Vol. 70, No. 35.

[6] Kuksheva E.A., Malyshkin V.E., Nikitin S.A., Snytnikov A.V., Snytnikov V.N., Vshivkov V.A. Numerical simulation of self-organization in gravitationally unstable media on supercomputers // PaCT-2003 Proceedings. — 2003. — P. 354–368. — (Lect. Notes in Comp. Sci.; 2763).

[7] Bertin G. Dynamics of Galaxies. — Cambridge: University Press, 2000.