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Numerical simulation of currents and admixture transport in a multi-arm river channel*

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Abstract. The paper presents a numerical model of planimetric currents in channel systems of a complicated spatial configuration. The model is based on the two-dimensional Saint–Venant equations. The influence of a non-homogeneous flow structure on the admixture transport processes in the water is studied. Calculation of current and passive substance redistribution is presented for a 16-km stretch of the Ob river near the city of Novosibirsk.

Introduction

The study of models of substance redistribution in river channels is important when analyzing and interpreting various physical processes. They include the following: transport of water pollutants, ice and shuga field formation during the ice drifting, suspended solids transport, channel deformations, and some other processes. Specific features of concentration transport in a river flow are determined by local dynamic conditions, which to a large extent, are related to the river bed morphometry.

The river systems of Western Siberia are notable for a complicated multi-arm structure of water flows with numerous branches, islands, and flow division nodes. Detection of quantitative relations within a multicoupling channel system is a serious theoretical problem, and computer-aided realization of numerical models for calculation of flows of a complicated topology is based on application of nontrivial computational approaches. As a rule, one-dimensional Saint–Venant equations are used with formulation of flow conjugate conditions at branching points.

Solution methods for one-dimensional systems at cell complexes (graphs) are described in [1], where a general edge task is studied for large systems of differential equations, with a definition area not being a single reach but a set of reaches forming a graph. Solution to the problem about the water motion in a system of channels is complicated by the fact that when water flows interact with a current within one reach influences that within the other one. Therefore it is not always possible to assign independent boundary conditions at internal grid nodes. Modelling of multicoupling domains in some cases can result in occurrence of false solutions leading to accumulation of fictitious circulations around internal loops of a graph [2].

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A special attention in the numerical model development for water stream systems was given to the admixture transport description [3]. In statement of the problem an emphasis was made on a correct formulation of the internal boundary conditions at node points, a possible application of effective integration schemes, and development of optimization algorithms. Methods for the assessment of diffusion and dispersion coefficients are discussed in [4] and are devoted to the admixture transport in multi-arm river deltas.

However, the one-dimensional/graph schematization for the description of topologically complex water bodies is not quite universal because it does not account for the effects of winding and meandering of channels, water stream conjugate angles, the influence of floodplain massifs intermitting with river arms and channels. These circumstances necessitate the account of a planimetric current structure. An obvious advantage of a two-dimensional model is a refusal from building a graph of branching nodes and from a complicated system of runs along edges with iteration agreement at nodes.

The objective of this study is to investigate the influence of branching channel morphology on the passive substance migration processes in the water by means of a numerical model for flat (planimetric) currents.

1. Statement of the problem

Hydraulic parameters of a water stream are calculated with a two-dimensional system of the Saint–Venant equations. Let us introduce the Cartesian coordinates with the axes x,y into a horizontal plane. The surface of a channel bed is assigned by the equation, where is a function describing a bottom topography. Equations for planimetric currents are written down as follows [5]

$$\begin{aligned}
\frac{\partial hu}{\partial t}+\frac{\partial huu}{\partial x}+\frac{\partial huv}{\partial y}&=-gh\frac{\partial (h+z_b)}{\partial x}-\frac{g}{C_s^2}|\boldsymbol{u}|u,\\
\frac{\partial hv}{\partial t}+\frac{\partial huv}{\partial x}+\frac{\partial hvv}{\partial y}&=-gh\frac{\partial (h+z_b)}{\partial y}-\frac{g}{C_s^2}|\boldsymbol{u}|v,\\
\frac{\partial h}{\partial t}+\frac{\partial uh}{\partial x}+\frac{\partial vh}{\partial y}&=0,
\end{aligned} \tag{1}$$

where t is time, h is a flow depth, u, v are components of the horizontal velocity vector averaged for the depth, g is acceleration of gravity, C_s is the Chezy coefficient, $|\boldsymbol{u}|=\sqrt{u^2+v^2}$ is a current velocity module.

Let us formulate the boundary conditions. The total river discharge Q_1 will be calculated at the entrance point positioned at some cross-section of the channel. The free surface level converted into the depth terms h is assigned at the exit cross-section.

The hydrodynamic problem is set with the initial conditions for the velocity components $u = v = 0$ and spatial distribution of depths h at the moment $t = 0$. The required parameters of the current were derived by temporal integration of equations (1) till reaching the steady-state condition.

The basic algebraic relations were derived by the finite volume method. Curvilinear grids with nodes spaced apart to the edges of an elementary spatial box were used [6]. System of equations (1) is projected onto a non-rectangular elementary box with sides close to the direction of the curvilinear channel boundaries. Such an approach ensures detalization of a transverse structure of a stream at a relatively small number of nodes located at longitudinal generators approximately retracing channel bends. For the time domain the finite differences method was used with application of implicit algorithms. The spatial discretization of differential operators is based on the concept of Total Variation Diminishing schemes (TVD), ensuring the monotonicity of solution through application of a rearranging pattern and adequate approximation of derivatives at different sites of the numerical solution. The scheme monotonization was performed with the Courant–Isaacson–Rees method [7]. The implicit part of TVD operators was algorithmized according to the method described in [8].

2. Calculation result

The formulated planimetric model for water streams was approved at 16-km Ob river tail-water reach near the city of Novosibirsk. This reach of the river is navigable, depths in some places exceed 11 m. Planimetric shape of the channel with curvilinear grid elements and depths distribution is shown in Figure 1 (arrows indicate to a current direction). The channel has rather a complicated morphological structure: a large Medvezhy island (indicated with "O" in Figure 1) divides the stream into two main arms with smaller channels branching out of them. The arm width does not exceed 800–900 m, a navigable pass following a more full-flowing left arm. The city recreation zone (Zaeltsovsky Park) is located on the right bank of the river, and numerous urban gardens - on banks and islands downstream.

The digital elevation model was developed on the basis of the field survey results obtained by the expedition from Moscow University to this river reach in 2003–2004. Field data serve as a basis for the digital elevation model $z_{i,j} = z_b(x_i, y_j)$ at a curvilinear grid feasible for the use in a planimetric numerical model.

An important stage in the calculation procedure is an algorithm of calculation domain partition into a set of elementary volumes (boxes), at which discrete correlations are built up. Grids of an arbitrary configuration are effectively adapted for the planimetric geometry of a calculation domain and

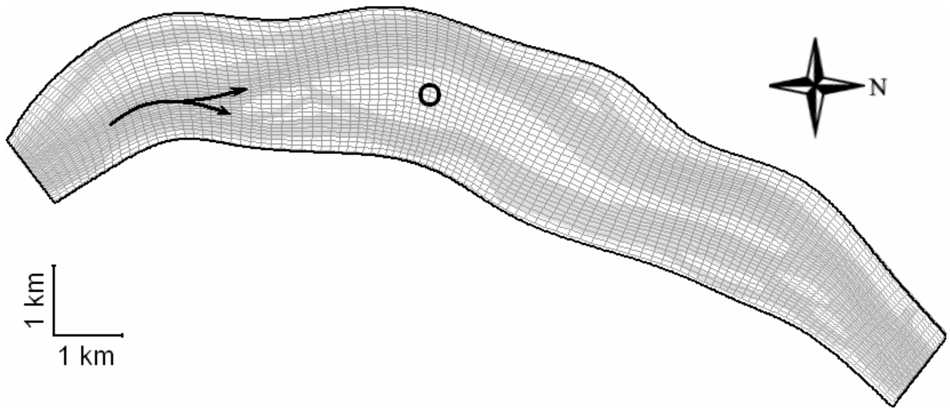

Figure 1. Grid structure of calculation area and channel contours of the Ob river (the grey tint)

morphometry specificy features. At the present time, algebraic, geometric, and differential methods for grid generation in arbitrarily shaped areas are being developed and widely used. Algebraic methods have a variety of advantages including realization simplicity, possibility to build coordinate lines of an assigned configuration, a high velocity of automatic node processing, etc. A hybrid approach combining algebraic and differential/variational methods, having appropriate flexibility, and allowing regulation of the projected grid properties with the help of governing functions and parameters appeared to be most effective in this case. The problem of the grid node system formation for an arbitrary area is posed for a system of quasi-linear elliptic-type equations with boundary conditions in terms of the correspondence of an area contour and a canonical rectangle as pre-image of the initial area. The grid shown in Figure 1 has 100×500 nodes in the transverse and the axial directions, respectively, and provides a mean spatial resolution of 20×30 m^2. A smaller step in the transverse direction is assigned in connection with a need for a more detailed description of the flow variability in the process of its redistribution in arms.

The required parameters of a current were obtained for a given water discharge $Q = 1800$ m^3/s, which corresponds to the spring-summer period, by integration of the dynamic problem with respect to time till reaching the steady-state condition. A stationary solution to equation of continuity (1) suggests the existence of a current function, which is a characteristic of spatial distribution of discharges. The calculated configuration of current lines is shown in Figure 2. The current lines describe the particle trajectories, and moreover, form current tubes, where a water discharge value is constant (Figure 2 shows the current tubes ejecting jets with the same discharge of 180 m^3/s). Note the crowding of isolines in front of the island, which reflects non-homogeneity of a current at the stream partition point. According to

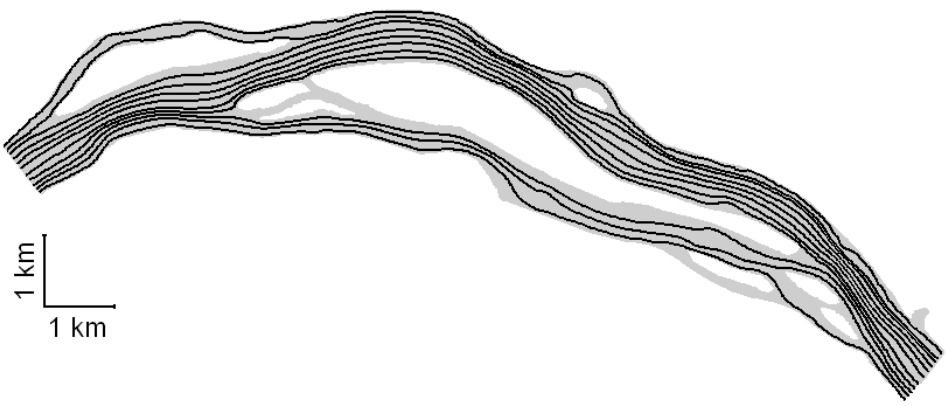

Figure 2. Calculated current lines

Figure 2, the main water stream is going through the left arm, while the right one transits no more than 20–30 % of discharge.

The problem of modeling the passive admixture propagation in its two-dimensional representation reduces to solving the transport and diffusion/dispersion equation

$$\frac{\partial hc}{\partial t} + \frac{\partial huc}{\partial x} + \frac{\partial hvc}{\partial y} = \frac{\partial}{\partial x} hE_x \frac{\partial c}{\partial x} + \frac{\partial}{\partial y} hE_y \frac{\partial c}{\partial y}, \qquad (2)$$

where c is dimensionless concentration (m^{-3}), E_x, E_y are dispersion coefficients. In view of consequences of hazardous and catastrophic events in rivers, it should be noted that abeam Kudryashi village (approximately corresponding to the entrance range of the calculation domain), there is a city purification plant constantly withdrawing wastewaters into the Ob river. The incident suggests a possibility of high concentration unsafe water disposal. This necessities an assessment of pollution ingress into the park zone of the city through the right arm water passage. For this purpose, assign a constant and uniformly distributed over the cross-section value of $c = 1$ at the entrance boundary and consider the concentration field formation along the channel length.

A steady-state spatial pattern of admixture distribution is shown in Figure 3. It is seen that there are a number of high concentration zones within the calculation domain, where c exceeds zero level by 50 % (dark areas in Figure 3). As a rule, these zones form in the areas of flow recompression, where export and dispersion of substance are impeded. This implies, in case of emergency, a discharge with prolonged effect, a possibility of pollutant accumulation in the right arm passage with concentration exceeding that in the source of emission. Values of c in the right arm do not exceed 1 and decrease down the stream.

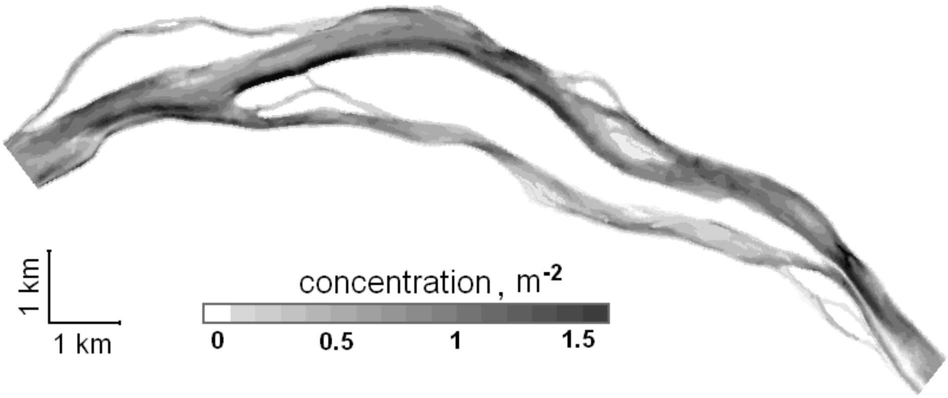

Figure 3. Spatial distribution of pollutant concentration over the channel length

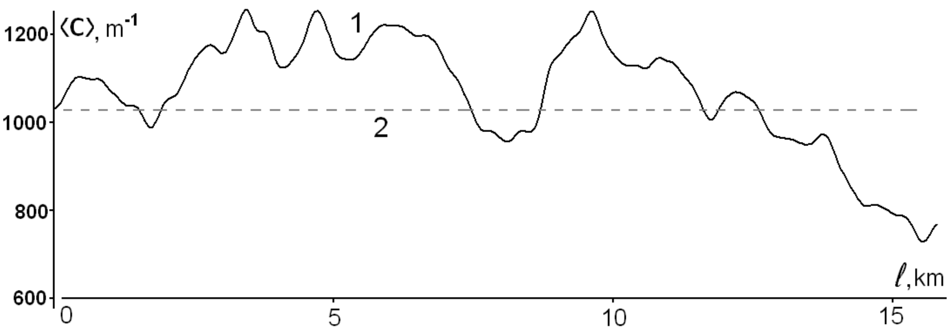

Figure 4. Spatial distribution of total pollutant concentration

Figure 4 presents distribution of the total quantity of admixture in the cross-section $\langle c \rangle$ along the marching coordinate of the channel ℓ. Non-homogeneity of the concentration distribution along the flow is also well seen — values vary within 20–25 % from "non-disturbed" mean value assigned at the entrance range (dotted line 2). Comparison of curves 1 and 2 in Figure 4 shows that the observed non-homogeneity of concentration in the channel is caused by irregularity of current velocity distribution — in a homogeneous flow with a velocity not changing along the channel the concentration remains constant over the whole length, as curve 2 shows.

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