Bull. Nov. Comp. Center, Math. Model. in Geoph., 10 (2005), 67–80

The pulse action on saturated porous media*

Yuri Perepechko, Vitaly Dorovsky,
Kholmatzhon Imomnazarov

Abstract. The pulsed action on fluid-saturated nonlinearly deformed porous media is considered. A mathematical model of a nonlinear two-velocity medium was obtained on the basis of the method of conservation laws. The model is thermodynamically consistent and hyperbolic in the reversible approximation. The numerical model is based on Godunov's explicit scheme with the use of a parallel computational algorithm. Numerical modeling of non-equilibrium nonstationary processes in heterophase deformable media for various modes of the pulsed action was made for various values of thermodynamic and kinetic parameters. Dilatancy areas are correlated with a partial density distribution of the saturating fluid. The approach proposed can be used to simulate the operation of oil collectors and in geophysical prospecting.

1. Introduction

Applications of seismic methods for studying the geological structure of the Earth become more complicated, and mathematical modeling of seismic fields is gaining in importance. Mathematical modeling in seismology and seismic prospecting is carried out on the basis of analytical, numerically-analytical, and numerical methods. Analytical methods have limited application areas for solving relatively simple problems. Complex nonlinear problems, arising in the adjacent branches, which need to be solved to develop hi-tech technologies, make it necessary to create and solve combined mathematical models. An effective approach in the theoretical study of wave processes in the upper Earth's layers, is to use the numerical modeling methods based on well-posed physical models. The basic models for solving problems of geophysical prospecting and describing seismic wave processes in layered media consisting of porous oil- and water-saturated rocks are those by Frenkel and Biot [1–3]. On the basis of the Frenkel–Biot model, the basic characteristics of seismic wave propagation in such media have been investigated. Precise and approximate methods have been developed, for instance: numerically-analytical approaches for solving the Biot equations of the dy-

*Supported by the Russian Foundation for Basic Research under Grant 03-05-64403, by Council for Grants of the President of the Russian Federation under the grants NSh-2118.2003.5 and NSh-1573.2003.5, and by the Ministry of Industry, Science and Technology of Russian Federation under the State contract 43.043.1.1.1601.

namics of layered porous media were developed by Philippacopoulos [4] and by Miroshnikov, Fatyanov [5]; finite difference methods for solving the Biot equations were used in papers [6–8]. However, the calculation results using the linear Biot model do not meet the modern practical requirements. Note the ill-posedness of the Cauchy problem for the Biot equations with dissipation of energy [9]. In real processes in a fluid and in an elastically deformed body, there occur complex wave effects accompanied by the manifestation of physically and geometrically nonlinear properties of interacting media. Only the allowance for nonlinear phenomena can give a description of the processes observed in practice.

In this paper, the pulsed action of the point or the spatially distributed type on the fluid-saturated nonlinearly deformed porous media is considered on the basis of the mathematical dynamical model of dynamics of a nonlinear two-velocity medium [10, 11]. This model of the dynamics of saturated deformable porous media earlier obtained within the method of conservation laws is thermodynamically consistent and hyperbolic in the reversible approximation. The thermodynamically consistent two-velocity model, in contrast to the Frenkel–Biot models, is nonlinear, does not contain additional material parameters, and can be considered as a well-posed generalization of the Darcy law. The model yields results that agree with the experimental data and shows the existence of three types of acoustic oscillations. The hyperbolicity of the reversible equations of the model allows us to use, for the numerical modeling, Godunov's difference scheme. This makes it easier to use a parallel computational algorithm. The numerical modeling of non-equilibrium nonstationary processes for various modes of a pulsed action with fluid pumping was carried out for various values of thermodynamic and kinetic parameters. The influence of nonlinear dynamics on the electric phenomena in a heterophase medium can be considered with the Helmholtz–Smolukhovsky approach.

2. Mathematical model

We consider nonstationary phenomena taking place at the pulsed action on an elastically deformed matrix into which the Newtonian fluid is injected. A distinctive feature of the phenomenological theory is the assumption about the local non-additivity of entropy, which ensures the hyperbolicity of the reversible equations of two-velocity hydrodynamics and makes it possible to use for numerical analysis the algorithms based on the hyperbolicity of governing equations.

As independent variables, it is convenient to choose the density ρ, the energy per unit volume of the medium E, the momentum $\boldsymbol{j}$, the velocity of the saturated fluid $\boldsymbol{v}$, and the metric tensor g_{ik}. The system of governing equations is as follows [11]:

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + \operatorname{div} \boldsymbol{j} = 0, \\
&\frac{\partial g_{ik}}{\partial t} + (\boldsymbol{u}, \nabla) g_{ik} + g_{kn}\partial_i u_n + g_{in}\partial_k u_n = 0, \\
&\frac{\partial j_i}{\partial t} + \partial_k \Pi_{ik} = 0, \\
&\frac{\partial v_i}{\partial t} + (\boldsymbol{v}, \nabla) v_i = -\frac{1}{\rho}\,\partial_i p + \frac{\rho_s}{2\rho}\,\partial_i w^2 - \frac{h_{kj}}{2\rho}\,\partial_i g_{jk} + \chi \rho_l w_i, \\
&\frac{\partial E}{\partial t} + \operatorname{div} \boldsymbol{Q} = 0.
\end{aligned} \tag{1}$$

In system of equations (1), ρ_s and ρ_l are partial densities of the porous matrix and the saturating fluid; $\boldsymbol{u}$ and $\boldsymbol{v}$ are velocities of the porous matrix and fluid, respectively; $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$ is a relative velocity of the components; p is pressure. The partial densities and the deformation tensor are related as follows:

$$\rho_s = \mathrm{const}\,\sqrt{\det(g_{ik})}, \quad \rho_l = \rho - \rho_s.$$

The reversible flows are determined in a standard way:

$$\begin{aligned}
\boldsymbol{j} &= \rho_s \boldsymbol{u} + \rho_l \boldsymbol{v}, \\
\Pi_{ik} &= \rho_s u_i u_k + \rho_l v_i v_k + h_{in} g_{nk} + p\,\delta_{ik}, \\
Q_i &= (E + p)\frac{j_i}{\rho} + \frac{1}{\rho}\,u_k (j_k - \rho v_k)(j_i - \rho u_i) + u_k h_{ij} g_{jk}.
\end{aligned}$$

Since the energy dissipation results in entropy production, the equation of entropy takes the following non-divergent form:

$$\frac{\partial(\rho s)}{\partial t} + \operatorname{div}(\boldsymbol{j} s) = \frac{R}{T}.$$

In the model, it is assumed that the energy dissipation takes place only due to the intercomponent friction. Here $R = \chi(\boldsymbol{j} - \rho u)^2$ is a dissipative function, χ is the kinetic coefficient, which takes into account the permeability of the medium and the intercomponent friction, s is the entropy per unit mass of the medium, and T is temperature.

The total energy of the system E can be related to the internal energy ε with the help of the Galilei transform:

$$E = \rho\varepsilon + \frac{1}{2}\rho v^2 + (\boldsymbol{v}, \boldsymbol{j} - \rho \boldsymbol{v}).$$

The equation of state $\varepsilon = \varepsilon(\rho, s, g_{ik}, w^2)$ obtained by the work of Dorovsky, Perepechko, and Romensky [12], allows us to express p, T, and h_{ik} in terms of independent thermodynamic parameters

$$
\begin{aligned}
p &= -\frac{K}{2}\delta g_{ll} + \frac{T\beta}{C_v\alpha}\delta\rho + \left(\frac{C_p}{\rho_0 C_v\alpha} + \frac{K}{\rho_0}\right)\delta s + \frac{\rho_s w^2}{2},\\
T &= T_0 + \frac{T}{C_v}\delta s + \left(\frac{C_p}{\rho_0^3 C_v\alpha} + \frac{K}{\rho_0^3}\right)\delta\rho,\\
h_{ik} &= \frac{\lambda}{2}\delta_{ik}\delta g_{ll} + \mu\delta g_{ik} - \frac{K}{\rho_0}\delta_{ik}\delta\rho - g^{ik}\frac{\rho_s w^2}{2}.
\end{aligned}
$$

Continual thermodynamic parameters (heat capacities C_p and C_v, coefficient of isothermal compression α, and coefficient of volume expansion β) are associated with respective physical parameters by the expressions

$$
\begin{aligned}
\rho C_p &= \rho_s^f(1-x_0)C_{p,s}^f + \rho_l^f x_0 C_{p,l}^f, \qquad & \rho C_v &= \rho_s^f(1-x_0)C_{v,s}^f + \rho_l^f x_0 C_{v,l}^f,\\
\rho\alpha &= \rho_s^f(1-x_0)\alpha_s^f + \rho_l^f x_0\alpha_l^f, & \rho\beta &= \rho_s^f(1-x_0)\,\beta_s^f + \rho_l^f x_0\beta_l^f.
\end{aligned}
$$

The need to satisfy the Darcy relation in one of the limits of equation (1) allows us to relate the parameter χ to the permeability of the medium k and the viscosity of the fluid η being filtered: $\chi = \eta/(\rho\rho_l k)$. It makes it possible to determine the dimensionless number Re_p, an analog to the Reynolds number Re for porous media:

$$
\frac{\rho_0\chi b}{c_{\|,1}} = \frac{\eta b}{c_{\|,1}\rho_0 k} = \mathrm{Re}\,\frac{b^2}{k} = \mathrm{Re}_p,
$$

which determines the type of motion of the two-velocity medium.

3. Difference scheme

Godunov's explicit scheme successfully applied into elasto-plastic calculations was used to numerically solve the problem. A standard method to construct difference equations is well known [13]. Its application allows us to obtain a difference approximation of system of equations (1) [12]. Following [14], a modified version of Godunov's scheme, which allows us to carry out a simple generalization for mobile grids, was used.

The problem of breakdown of a discontinuity, necessary for the determination of flows of independent variables ($\boldsymbol{j}$, Π_{ij}, $\boldsymbol{Q}$, etc.), because of complexity of the system, was numerically solved in the acoustic approximation. The Courant stability condition for the given difference scheme has the following form:

$$
\frac{1}{\Delta t} \ge \frac{\lambda_x}{\Delta x} + \frac{\lambda_y}{\Delta y},
$$

and allows us to determine the time step Δt. Here, λ_x and λ_y are maximum eigenvalues for the corresponding one-dimensional problems, which are taken for all the points, at which the breakdown of discontinuity is calculated; Δx and Δy are the spatial steps for a given time layer.

The dissipative term in equations (1), since it is a nonlinear function, was taken from the upper time layer. This ensures stability of the scheme with arbitrary values of the product $\Delta t\,\rho\chi$.

The computational domain of the problem on the plane $\{x, y\}$ is a finite rectangle: $\Omega = \{x, y \mid x \in [0, a],\ y \in [0, b]\}$. Since the computational domain does not depend on time, a fixed rectangular non-uniform computational grid is used. On the left boundary ($x = 0$) of the computational domain, the pulsed action and the conditions of pumping a fluid with velocity V_0 are given:

$$\sigma_{xx} = f(t, y), \quad \sigma_{xy} = 0, \quad \boldsymbol{v} = (V_0, 0), \quad u_x = 0, \quad T = T_0.$$

The pulse has the following form:

$$f(t, y) = \bigl(1 + \cos(\omega(t - t_0))\bigr) \exp\bigl(-(\omega(t - t_0)/\gamma)^2\bigr) \exp\bigl(-\zeta(y - b/2)^2\bigr).$$

Here $\sigma_{ij} = -p\,\delta_{ij} - h_{ik}g_{kj}$ is a stress tensor.

On the right boundary of the computational domain ($x = a$), "non-reflecting" boundary conditions imitating an infinite half-space are given. This means that the invariants transferred by the characteristics going into the computational domain are equal to zero. On the upper and the lower boundaries, a porous matrix is considered to be non-deformed, and the zero normal velocity condition is

$$u_x = u_y = v_y = 0.$$

At the boundary points of the computational domain, the problem of boundary breakdown of a discontinuity is numerically solved as well.

Parallelization of the program on the basis of an algorithm that realizes the virtual topology "line" was carried out. The algorithm realized the one-dimensional distribution of data between computers in two columns with further realization of the scheme of exchanges between the parallel program branches.

4. Results

The purpose of the numerical experiment was to investigate the character of nonstationary processes in porous elastically deformed bodies under the pulsed action with various values of the medium parameters. As a model system, a medium with the initial physical densities $\rho_s^f = 2.7 \cdot 10^3$ kg/m^3, $\rho_l^f = 0.9 \cdot 10^3$ kg/m^3 and the volume fraction of the fluid component $x_0 = 0.2$ was considered. The physical heat capacities were $C_s^f = 0.48$ kJ/(kg·K) and $C_l^f = 0.21$ kJ/(kg·K). The volume expansion coefficients were $\beta_s =$

$-3 \cdot 10^{-6}$ K^{-1} and $\beta_l = -6 \cdot 10^{-4}$ K^{-1}. The compressibility coefficients were $\alpha_s = 6 \cdot 10^{-11}$ Pa^{-1} and $\alpha_l = 6 \cdot 10^{-10}$ Pa^{-1}. The Lamé coefficients were $\lambda = 2.5 \cdot 10^9$ J/m^3 and $\mu = 9.3 \cdot 10^9$ J/m^3. The dynamic viscosity was taken as $\eta = 0.1$ kg/(m·s). The geometrical dimensions of the domain were $a = 3 \cdot 10^3$ m and $b = 1 \cdot 10^3$ m. At the initial time, the temperature of a fluid-saturated porous medium was $T_0 = 293$ K.

The calculation results are represented in Figures 1–14. In Figures 2–9, examples of distribution of the measured parameters of the model for the dimensionless times, corresponding to (a) $0.8 \cdot 10^5$, (b) $1.6 \cdot 10^5$, (c) $2.4 \cdot 10^5$, and (d) $3.2 \cdot 10^5$ steps, are given. All the figures, except for Figure 1, were obtained for the same amplitude of the pulse f.

Figure 1 demonstrates the dependence of the pulsed action propagation on its amplitude. The distribution of components of the stress tensor σ_{ij} is shown in Figures 2–4.

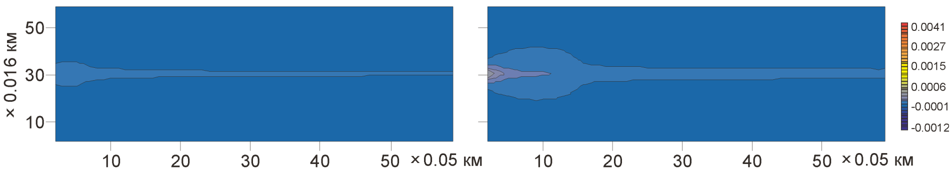

Figure 1. Distribution of $\{xx\}$ component of the stress tensor h_{ij} of the porous matrix versus the power pulse f. The dimensionless amplitudes of pulse are 0.01 (to the left) and 0.03 (to the right)

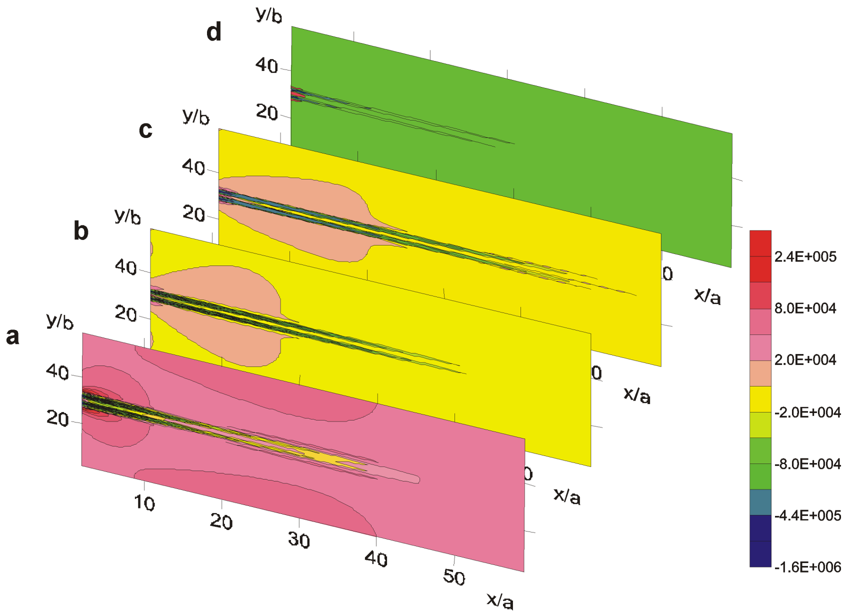

Figure 2. Distribution of stresses in the porous matrix for σ_{xx} component

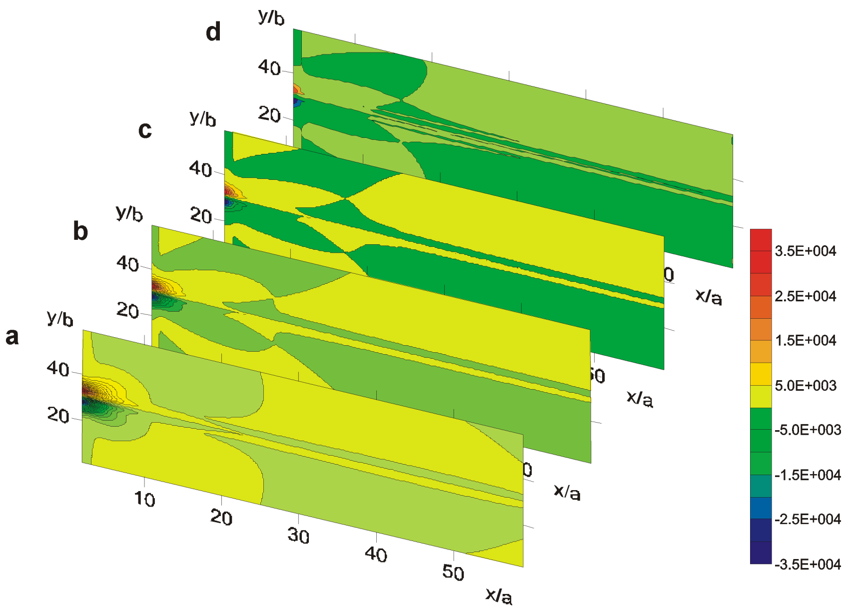

Figure 3. Distribution of stresses in the porous matrix for σ_{xy} component

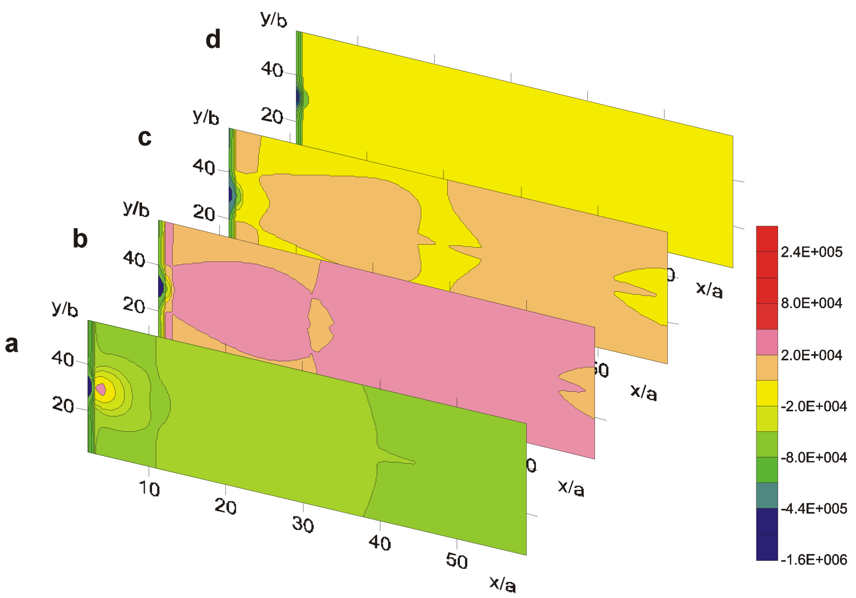

Figure 4. Distribution of stresses in the porous matrix for σ_{yy} component

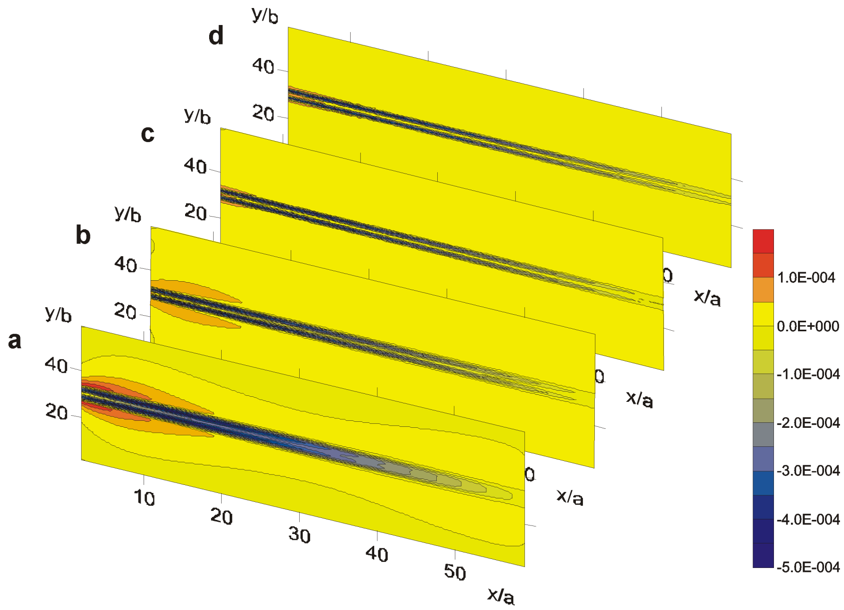

Figure 5. Distribution of deformations in the porous matrix for g_{xx} component

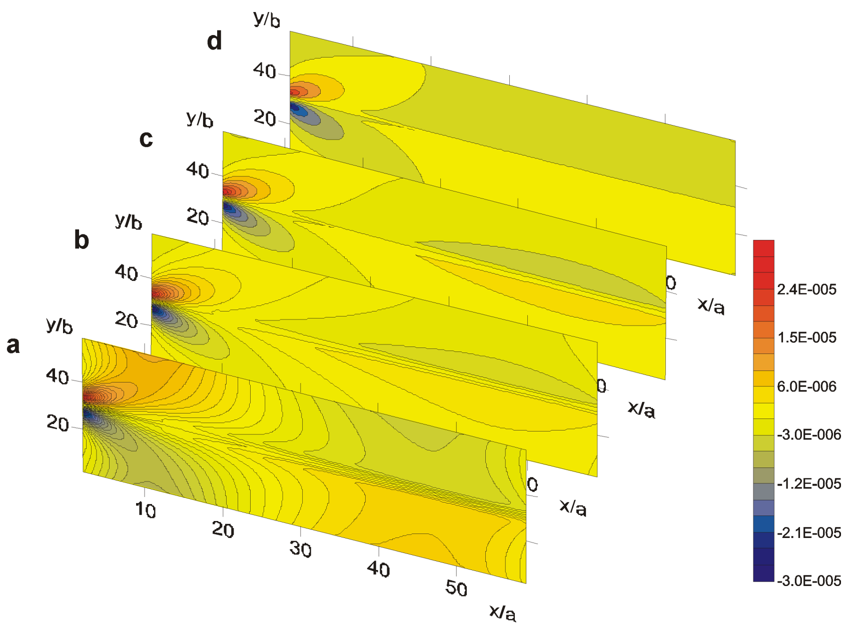

Figure 6. Distribution of deformations in the porous matrix for g_{xy} component

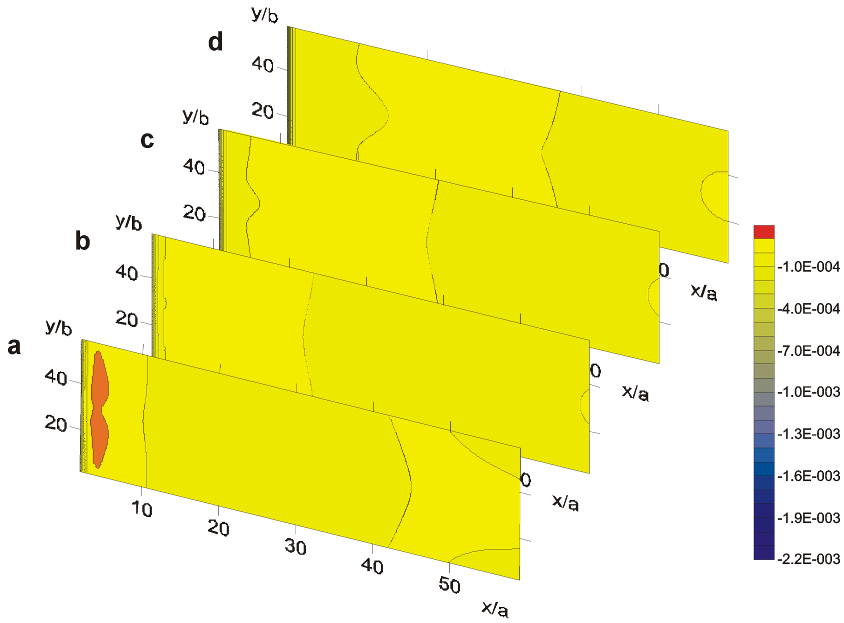

Figure 7. Distribution of deformations in the porous matrix for g_{yy} component

The dilatancy zones were estimated using the following standard formula:

$$D_\tau = \tau - \alpha p_s - Y,$$

where $p_s = -\sum_i \sigma_{ii}/3$ is the hydrodynamic pressure, $\alpha = 0.5$ is the internal friction coefficient, $Y = 3 \cdot 10^6$ Pa is the adhesive capacity of rocks, and τ is intensity of shearing stresses. The dilatancy field is represented in Figure 8, in which one can see its correlation with distribution of the partial density of the fluid component (Figure 9).

The distribution of the velocity field of the saturating fluid is presented in Figure 10.

To calculate the electrokinetic effect associated with propagation of longitudinal waves, one can use the Helmholtz–Smolukhovsky relation for the longitudinal electric field strength $\boldsymbol{E}$ at a stationary flow of the fluid through the pores of the fixed solid matrix

$$\boldsymbol{E} = -\frac{\varepsilon\varsigma}{\mu\sigma}\nabla P.$$

Here $\varepsilon = 7.1 \cdot 10^{-10}$ $\mathrm{C}^2/(\mathrm{N{\cdot}m}^2)$ is the dielectric permeability, $\mu = 10^{-3}$ Pa·s is the dynamic viscosity of the fluid, $\sigma = 0.022$ S/m is the conductivity of the fluid, and $\varsigma = -60$ mV is the electrokinetic potential. It is assumed that the time of formation of the gradient of the electrokinetic potential is small enough in comparison with the period of oscillations, that is, the

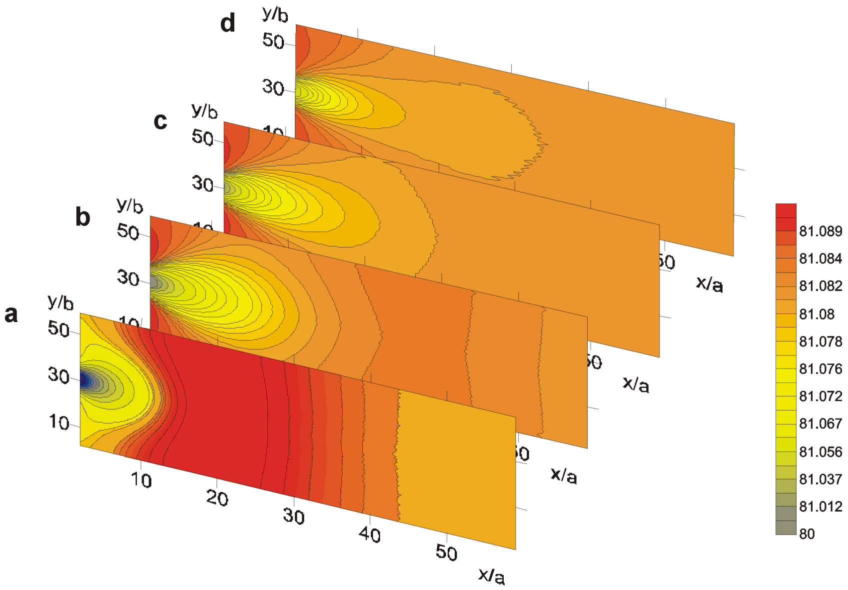

Figure 8. Distribution of dilatancy field

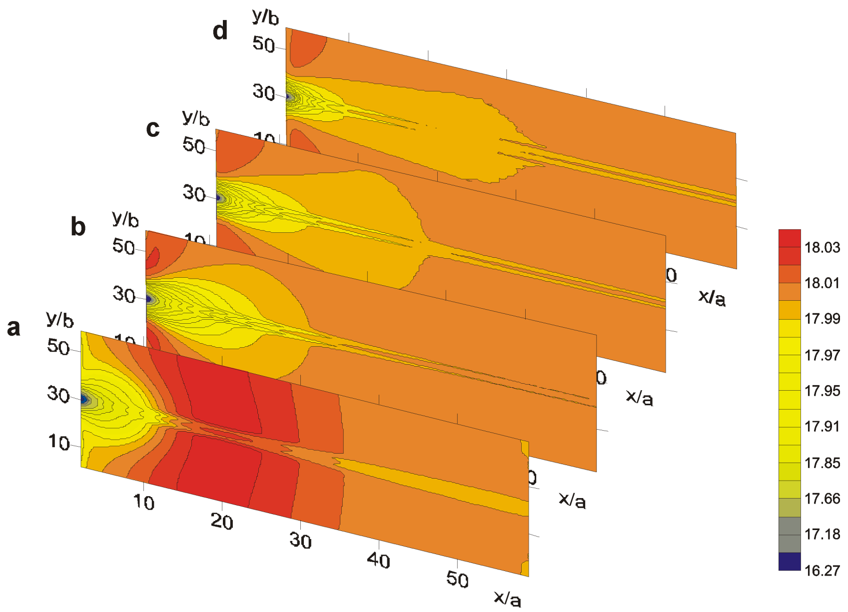

Figure 9. Distribution of the fluid component density

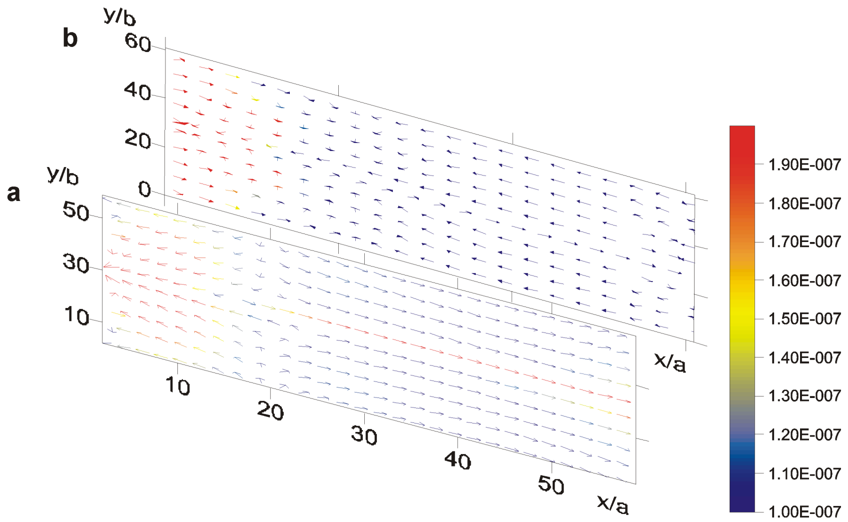

Figure 10. Velocity fields of the saturating fluid $\boldsymbol{v}$ for the dimensionless times (a) 300 and (b) 8000

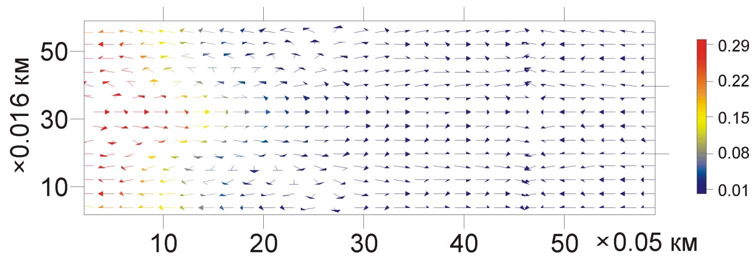

Figure 11. Electric field strength using the Helmholtz–Smolukhovsky formula

value $\boldsymbol{E}$ corresponds to an instantaneous value of the pressure gradient. The corresponding profiles are represented in Figure 11.

The plots of the temperature-time dependence also illustrate the dissipative effects (Figure 12) when a heat conduction is neglected. A high level of stresses and energy dissipation due to the intercomponent friction leads to a noticeable warming of a medium, which increases temperature by 10 K.

The effectiveness of using the multiprocessor cluster is demonstrated in Figure 14, in which one can see a high efficiency of using the "line" topology for Godunov's explicit scheme.

Analysis of the dynamics of pulsed action on saturated porous oil collectors reveals an essentially nonlinear behavior of this system. The dependence on the dissipative parameter χ shows that it is necessary to take into account the visco-plastic phenomena in this medium and use the Maxwell rheology for an adequate description of the propagation of wave processes in real media.

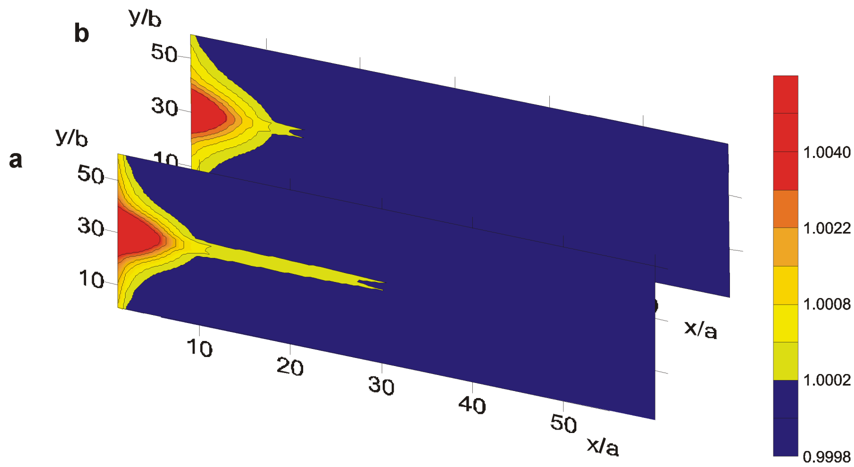

Figure 12. Temperature field. The dimensionless times are (a) 80000 and (b) 300000

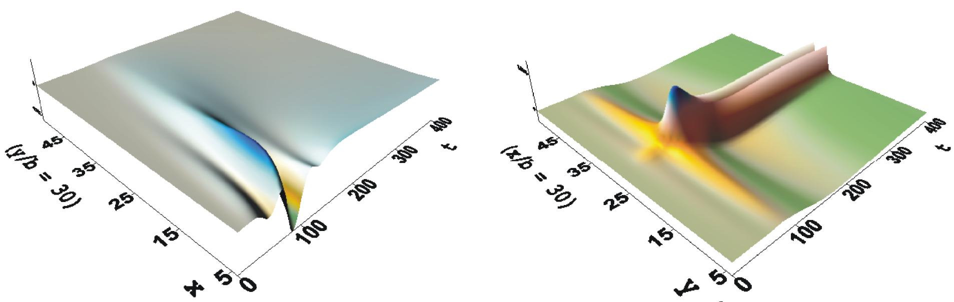

Figure 13. Time distribution of stresses for h_{xx} component at the center of (a) longitudinal and (b) transverse cross-section of computational domain

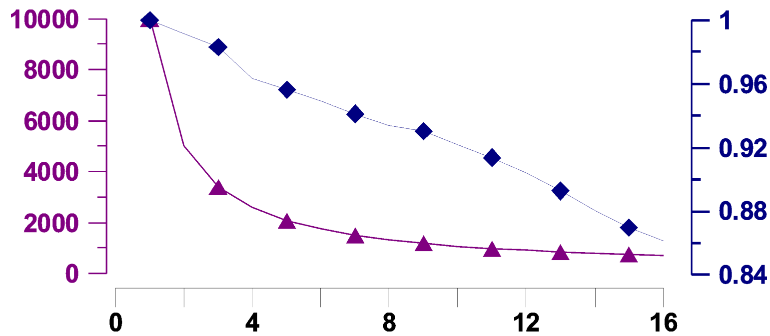

Figure 14. Calculation time and effectiveness versus the number of processors

Thus, the numerical modeling of non-equilibrium nonstationary processes for various modes of the pulsed action of the distributed type at pumping a fluid was carried out for various values of the thermodynamic and the kinetic parameters. An essentially nonlinear behavior of such systems is shown. The dilatancy areas reveal a correlation with distribution of the partial density of the saturated fluid.

References

[1] Frenkel Ya.I. On the theory of seismic and seismoelectric phenomena in a moist soil // J. Physics. USSR. — 1944. — Vol. 8. — P. 230–241.

[2] Biot M.A. Theory of elasticity and consolidation for a porous anisotropic solid // J. Appl. Physics. — 1955. — Vol. 26. — P. 182.

[3] Biot M.A. Theory of propagation of elastic waves in a fluid-saturated porous solid // J. Acoustical Society of America. — 1956. — Vol. 28. — P. 168–191.

[4] Philippacopoulos A.J. Lamb's problem for fluid-saturated porous media // Bull. Seismological Society of America. — 1988. — Vol. 78. — P. 908–923.

[5] Miroshnikov V.V., Fatyanov A.G. Semianalitical method for computational of wave fields in layered porous mediums // Proc. of Computing Center of the Siberian Branch of the Russian Academy of Science. Mathematical modelling in Geophysics. — Novosibirsk, 1993. — Vol. 1. — P. 27–57 (In Russian).

[6] Zhu X., McMechan G.A. Finite-difference modeling of the seismic response of fluid-saturated, porous, elastic media using Biot theory // Geophysics. — 1991. — Vol. 56. — P. 328–339.

[7] Hassanzadeh S. Acoustic modeling in fluid saturated porous media // Geophysics. — 1991. — Vol. 56. — P. 424–435.

[8] Dai N., Vafidis A., Kanasewich E.R. Wave propagation in heterogeneous, porous media: A velocity-stress, finite-difference method // Geophysics. — 1995. — Vol. 60. — P. 327–340.

[9] Imomnazarov Kh.Kh. Some remarks on the Biot system of equations describing wave propagation in a porous medium // Appl. Math. Letters. — 2000. — Vol. 13. — P. 33–35.

[10] Dorovsky V.N. Continual theory of filtration // Soviet Geology and Geophysics. — 1989. — Vol. 30. — P. 34–39.

[11] Dorovsky V.N., Perepechko Yu.V. Phenomenological description of two-velocity media with relaxing shear stresses // J. Appl. Mechanics and Technical Physics. — 1992. — Vol. 33. — P. 403–409.

[12] Dorovsky V.N., Perepechko Yu.V., Romensky E.I. Godunov's scheme for the analysis of the nonlinear wave process in the two-velocity media with elastic interaction // ASME. — 1994. — Vol. FED-196. — P. 355–366.

[13] Godunov S.K., Zabrodin A.V., Ivanov M.Ya., Kraiko A.N., Prokopov G.P. Numerical Solution of Multicomponent Gas Dynamics Problems. — Moscow: Nauka, 1976 (In Russian).

[14] Romensky E.I. Methods of calculation of two-dimensional dynamics equations of non-linear elasto-plastic Maxwell media // Proc. Institute of Mathematics of the Siberian Branch of the Academy of Science of the USSR. — 1990. — Vol. 18. — P. 83–100 (In Russian).