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On reliability of wireless ad hoc networks with imperfect nodes

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Abstract. In this paper, we present a new network reliability measure that is useful to evaluate performance of ad hoc networks with imperfect nodes and perfectly reliable links. An ad hoc network is modeled as undirected probabilistic graph. A specific feature of our model is that a network contains initially excessive amount of nodes to properly provide the functioning of the network. Thus, we consider the networks which carry on operating acceptably even if some of the nodes fail. The problem of calculation of the new reliability measure is NP-hard, just like problems of computing other reliability parameters. The method for calculating a new reliability measure has been obtained. It is shown that this method can be used for the optimal sink nodes placement in networks in order to obtain the most reliable version.

Introduction

The area of wireless ad hoc has received a lot of attention in the research community over the past several years. Research into generic wireless ad hoc networking is also ramified to special types of networking like wireless mesh networks, wireless sensor networks, vehicular networks, radio frequency identification networks, etc. In this paper, we study at the problem of reliability in these networks.

We consider ad hoc networks with imperfect nodes and perfectly reliable links. An operational probability is associated with every node. It is assumed that the node failures are statistically independent.

Probabilistic graph models have been extensively used in the literature for studying the network reliability problems, especially in the case of unreliable edges [1–12]. Case of unreliable nodes was also a subject for studying [13, 14]. But as a rule, these studies do not take into account specific features of ad hoc network. The reliability of sensors in wireless sensor network was the subject of studying in [15,16]. In [17], the problem of reliability of wireless distributed sensor networks is considered. Reliability is defined as probability of existence of operating communication path between the sink node (command node), and, at least, one operational sensor in a target cluster.

A special feature of our model is that the network contains initially excessive amount of nodes to provide a proper functioning of the network. The failure of one or more nodes can cause the operational data sources to be disconnected from the data sink nodes (command nodes). However, operational nodes in the faulty nodes neighborhood may still be able to communicate with end-users, although, through a larger number of hops resulting in a larger delay of receiving information. For example, a wireless sensor network may work acceptably even if a certain amount of nodes fails. In other words, it works until there is a sufficient number of workable nodes which are connected to any sink node. Another requirement for the network operation is the connectivity of sink nodes through workable nodes. We define the reliability of such a network as probability of the proper functioning in the above meaning.

The rest of the paper is organized as follows: in Section 1, the basic notations and definitions are presented. Sections 2, 3 describe the method of reliability calculation, Section 4 describes the numerical experiments which demonstrate an optimal sink nodes placement in the network.

1. The basic definitions and notations

We represent the ad hoc network with an undirected probabilistic graph $G = (V, E)$, whose vertices are the nodes and whose edges represent the links. We assume each node to succeed or to fail independently with an associated probability. Further on, we refer to this probability as the node reliability. We suppose the links to be perfectly reliable. We use the following notations for the number of network elements: $|V| = N$, $|E| = M$. K is a specific set of nodes that correspond to the sink nodes of the ad hoc network. We call terminals the elements of this set. It is assumed that K contains at least one element. As a rule, sink nodes are perfectly reliable. We have also an integer T such that $1 \leq T \leq N - |K|$. It is assumed that the ad hoc network is functioning properly if sink nodes are connected with each other and, at least, T nodes are workable and connected to any sink node.

Let us introduce some definitions.

An elementary event Q is a special realization of the graph defined by the existence or absence of each node. By V_Q we denote a set of all existing nodes in Q that are not sink nodes.

The probability of an elementary event is equal to the product of probabilities of the existence of operational nodes multiplied by the product of probabilities of the absence of faulty nodes.

An elementary event Q is called successful if:

- all sink nodes are connected with each other by nodes from V_Q;
- at least T nodes from V_Q are connected to any sink node.

Otherwise, it is called unsuccessful.

An arbitrary event (an event is a union of elementary events) is called successful if it consists of only successful elementary events.

An event is called unsuccessful if it consists of only unsuccessful elementary events.

We define the reliability of the ad hoc network as the probability of the event consisting of all successful events and of them only. We denote it by $R_{K,T}(G)$. Further on, under the network reliability we assume this index, unless otherwise stated. In other words, the introduced reliability index is the probability that sink nodes are connected with each other and at least T nodes are workable and connected to any sink node.

2. The factoring method for network reliability calculation

The calculation of $R_{K,T}(G)$ may be done by the well-known factoring method [1, 5, 7] which for this purpose has been modified. This technique partitions the probability space into two sets, based on the success or failure of one of the network particular elements (a node or a link). The chosen element is called a factored element. Thus, we obtain two subgraphs, in one of them factored element is absolutely reliable (branch of contraction) and in second one, a factored element is absolutely unreliable, that is, is absent (a branch of removal). The probability of the first event is equal to the reliability of a factored element; the probability of the second event is equal to the failure probability of the factored element. Thereafter, the subgraphs obtained are subject to the same procedure. The total probability law gives an expression for the network reliability. In the general case, for the system S with unreliable elements it takes the following form:

$$R(S) = r_e R(S \mid e_{\text{works}}) + (1 - r_e) R(S \mid e_{\text{fails}}),$$

where $R(S)$ is the reliability of S, $R(S \mid e_{\text{works}})$ is the reliability of the system S when the element e is in operation, and $R(S \mid e_{\text{fails}})$ is the reliability of the system S when the element e is not in operation, r_e is the reliability of the element e.

Figure 1 illustrates the factoring method for the all-terminal reliability of the graph G with unreliable edges. The corresponding formula takes the following form:

$$R(G) = p_e R(G_e^*) + (1 - p_e) R(G \setminus e),$$

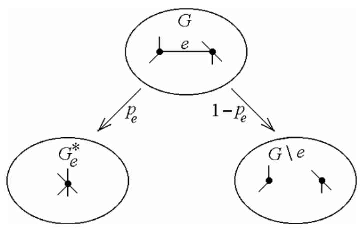


Figure 1

where p_e is the reliability of the edge e, G_e^* is the graph obtained by contracting the edge e from G, $G \setminus e$ is the graph obtained by deleting e from G. Recursions continue until either a disconnected graph is obtained, or until a graph for which the probabilistic connectivity can be calculated directly is obtained: this can be a graph of a special type or a small-dimensional graph [9].

The calculation of $R_{K,T}(G)$ is performed in the same way, but it is more complicated because of the need to fulfil the two conditions: the connectivity of terminals and the availability of a sufficient number of other nodes attached to them. It is convenient to choose as a factoring element one of the nodes adjacent to any terminal or the node adjacent to any already passed one with reliability 1. Thus, we can accumulate the number of nodes connected to a chosen terminal. We keep the name "branch of contraction" and "branch of removal" despite the fact that the process of contraction in a graph is optional, as well as the process of removal. Let us separately consider the branch of contraction and the branch of removal of such a process.

Branch of contraction. In this branch, the number of nodes attached to the terminal is increasing. If its number reaches T, it is necessary to check the connectivity of all terminals via absolutely reliable nodes. If the check is successful, then the final subgraph is obtained which corresponds to a successful event. If the check is unsuccessful, then further factoring procedure continues only in order to ensure the connectivity of the terminals. In other words, we calculate the probability of terminals connectivity in a graph with unreliable nodes. To this end, it is convenient to use the method [13].

Branch of removal. In this branch, the number of nodes which in the process of further factoring could potentially be absolutely reliable is decreasing. Therefore, the event corresponding to the graph obtained by this branch can be authentically unsuccessful due to the disconnection of terminals or impossibility of reaching the required number of nodes attached to the terminals. It is suitable to initially check the first condition, that is, to check whether the terminals are connected via non-zero reliability nodes. If the check is successful, then we should check the second condition: the number of pending nodes (with the reliability of 0 up to 1) should be sufficient to achieve the required number of nodes attached to the terminals, that is, the number T. If the number of pending nodes is just sufficient to ensure this condition, they all become absolutely reliable. Thus, a successful sub-event fully stands out from the considered event. It remains only to take into account the fact that in order to obtain the probability of this event it is required to multiply the value obtained by the reliability of pending nodes.

3. Algorithm

We use the following notations for the algorithm description. All the graphs arising in the factoring processes are presented as the array of probabilities P, where $P[i]$ is the probability that the node i is operational. The reliability of such a graph is denoted as $R(P)$. The array of the nodes reliabilities corresponds to the initial graph G. Let us denote it as P_0. Let us introduce for each graph arised when factoring the value x, that is, the amount of the considered nodes and the value y, that is, the amount of nodes reliably connected to any terminal. We need the function $R_K(P)$ for calculating the probability of connectivity of K terminals (the calculation method proposed is in [13]). Also, we need, a boolean function Connectivity(P) for checking the connectivity of terminals via nodes with nonzero probability. Let $S = |V/K|$.

We assume that the terminals are absolutely reliable. Otherwise, at a preliminary step the terminals become absolutely reliable, and the finally obtained value of the network reliability should be multiplied by the initial values of the reliability of the terminals.

A recursive procedure Factoring(P, x, y) is a factoring algorithm for calculating the reliability of the graph corresponding to the array P. The values of x and y depend on P, however, to avoid recovering each time with use of P, they are present as individual parameters. *Rel* is a local variable for storing an intermediate outcome of the Factoring(P, x, y) procedure, that is a private variable *Rel* created for every start of this procedure. *Result* variable is used to store the final result of the procedure.

The Factoring(P, x, y) algorithm:

1. Choose a node v, such that $P[v] > 0$ and v has the adjacent node j, such that $P[j] = 1$. Let $p = P[v]$;
2. Branch of contraction. Assign the values: $P[v] = 1$, $x = x + 1$, $y = y + 1$.

   If $y = T$ then $Rel = p * R_K(P)$ else $Rel = p *$ Factoring(P, x, y);
3. Branch of removal. Assign the values: $P[v] = 0$, $y = y - 1$.

   If Connectivity(P) then

   if $S - x + y = T$ then $Rel = Rel + (1 - p) * \prod_{P[i]>0} P[i]$

   else $Rel = Rel + (1 - p) *$ Factoring(P, x, y);
4. $Result = Rel$.

Finally, $R_{K,T}(G) =$ Factoring$(P_0, 0, 0)$.

4. The results of numerical experiments

Let us show how the proposed algorithm works. The test problem was the problem of the optimal sink nodes placement in an ad hoc network with unreliable nodes in order to obtain the most reliable version. Figure 2 shows a 5×5 grid network topology that we have analyzed, supposing that all sink nodes are perfectly reliable and reliabilities of the other nodes are equal to

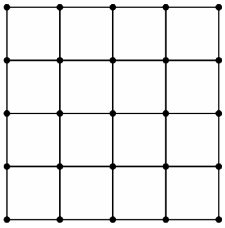
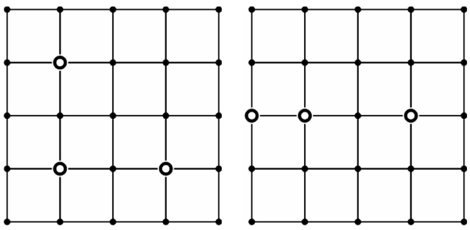


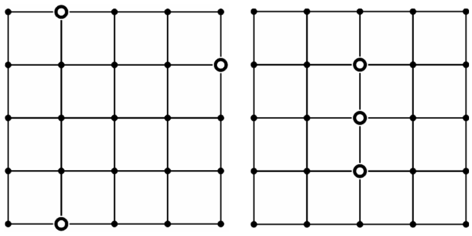
Figure 2

each other. Our objective is to place in the nodes of this grid three sink nodes to maximize the probability of access to sinks for at least T of other nodes in the network provided the sinks are connected to each other. Three values for the indicator T were considered: 10, 15, and 20. For each value of T, the solution was sought for three values of node reliability: 0.1, 0.5, and 0.9.



a b

Figure 3. Optimal sink nodes placements for $T = 15$ for $p = 0.1$, 0.5 (a), and 0.9 (b)



a b

Figure 4. Optimal sink nodes placements for $T = 20$ (a) and 10 (b)

If $T = 15$, we have obtained different optimal sink nodes placements for different values of node reliability (Figure 3). If $T = 10$, 20, optimal sink nodes placements were not different for different values of node reliability (Figure 4).

It is obvious that any sink nodes placement, which is isomorphic to optimal, is optimal, too.

Conclusion

This study proposes a new reliability index, which is applicable to the analysis of ad hoc networks, in particular wireless sensor networks. The classical reliability index (k-terminal network reliability) was taken as a basis of the proposed reliability index. At the same time, important features typical of ad hoc networks were taken into account: unreliable nodes and their excess amount for the network operation.

The index proposed could be used as the basis for other reliability indices, which describe the subject areas more completely. For example, the restriction on diameter of a network makes such a reliability index be more interesting from the point of view of practical applications, but also makes it more difficult to calculate. We can remove the requirement for the sink nodes connectivity. The lack of this requirement is typical of the networks in which sink nodes are able to connect with the base station directly and independently. The methods for calculating the reliability with mentioned changes will not be radically different from the method proposed in this paper. The problem of precise computing these characteristics will be NP-hard. However, with the help of the method from [4,9] it is possible within a reasonable time to obtain the lower and the upper network reliability bounds and to make a decision about the reliability (or unreliability) of the network with respect to a given threshold. This allows us to solve some optimizational problems, such as the sink nodes location in ad hoc networks.

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