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Sound wave propagation simulation in an inhomogeneous medium using Lattice Gas Automata*

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Abstract. The Lattice Gas Automata (LGA) models are based on a microscopic model of physical process being simulated and can be considered as an adjunct to the traditional numerical methods to the spatial dynamics simulation. Here we consider two simple LGA models: HPP and HPPRP. They are based on a square lattice whose nodes can be occupied by the moving particles in the HPP-model, and the moving and the rest particles in the HPPRP-model. In this paper, the possibility of a simple LGA models to simulate sound wave process in inhomogeneous medium formed from two solid materials (aluminium and ebonite) are investigated.

1. Introduction

In the Lattice Gag Automata (LGA) models, the dynamics of an event is described by a set of hypothetical particles, which have moved through space and collided with each other and with obstacles. Space is represented as a regular lattice whose nodes can contain a quantity of hypothetical particles. Each lattice node is assigned to a LGA cell. As opposed to the classical cellular automaton, an initial state of the LGA cell is determined by a set of some particles, locating in the cell at this time instant. There are two types of particles: the moving particles and the rest particles. The moving particles have the same mass (equal to one unit) and equal absolute velocity (equal to one unit). The rest particles have the same velocity (equal to zero) and a different mass. Interactions between particles are simple. Each interaction consists of two successive steps: collision and propagation. The collision rules are chosen in such a way that the mass and momentum conservation laws are satisfied. The collision rules determine the LGA cell transition table. All cells update their own states simultaneously and synchronously. An iterative change of the LGA global state (evolution of the LGA) describes the dynamics of an event on microscopic level.

In order that a modeling process be observed in the usual fashion of a physical event, averaged values of particles density and velocities for each LGA cell are calculated in a certain averaging area. Automata noise arises for small values of the averaging radix. This is the main disadvantage of the

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LGA models. Automata noise cannot be eliminated, but its effect can be reduced by increasing the averaging radix.

In this paper, the ability of a simple LGA models to simulate the sound wave propagation in inhomogeneous media on an example of media of two solid materials: aluminium and ebonite is demonstrated. As the models, two LGA models (HPP and HPPRP) on a square lattice with four neighbors are used. The HPP cells contain only the moving particles. The HPPRP cells can contain the moving particles and the rest particles. As opposed to the HPP model, the HPPRP collision rules can be deterministic or non-deterministic. In [3], it is shown that the HPPRP model corresponds to the wave equation. One can also specify that certain regions of a square lattice have different rest particles numbers. The energy exchanges between the moving and the rest particles in the regions are thus different, and media with different sound speeds can be realized.

This paper is organized as follows. The second section describes the main concepts of the LGA models. In the third section, the influence of the LGA model parameters on the sound wave propagation velocity is experimentally investigated. The sound wave propagation process in an inhomogeneous medium formed of two solid materials (aluminium and ebonite) is discussed in the fourth section.

2. The main concepts of LGCA models

2.1. The HPP model. In the HPP model, each cell contains only the moving particles. The state of the HPP cell is determined by the velocity vector $\boldsymbol{v} = (v_1, v_2, v_3, v_4)$ at this moment. The l-th digit value of the vector, $l = 1, 2, 3, 4$, shows the presence ($v_l = 1$) or the absence ($v_l = 0$) of particles in the direction to the l-th neighbor. The HPP cell with the velocity vector $\boldsymbol{v} = (0, 1, 1, 0)$ is shown in Figure 1. (An arrow in the cell shows the direction of the velocity vector particle.) In the HPP model, the particles collided according to the following rule:

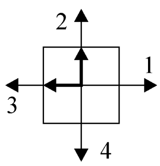

Figure 1

Rule 1 (Head-on collision). Two moving particles, arrive at a cell with an opposite direction of their velocity vector (head-on collision), escape from the cell, changing the direction of their velocity vector by 90 degrees (Figure 2).

Figure 2. Collision rules for the HPP model

2.2. The HPPRP model. As opposed to the HPP cell, each HPPRP cell can contain the moving particles and the rest particles. The state of HPPRP cell is defined by the two vectors: velocity vector $\boldsymbol{v}$ and mass vector $\boldsymbol{m}$. The rest particles have the zero velocity and a different mass. Here we will consider the rest particles with masses equal to 2, 4, 8, and 16. It is evident that two mass 2 rest particles equal to one mass 4 rest particle, four mass 2 rest particles equal to one mass 8 rest particle. The length of the mass vector $\boldsymbol{m} = (m_r, m_{r-1}, \dots, m_1)$ does not depend on the space structure and is equal to the number of the rest particles. The k-th digit value of the vector, $k = 1, 2, \dots, r$, determines the presence ($m_k = 1$) or the absence ($m_k = 0$) of a rest particle with mass 2^k in a cell. The HPP cell with the velocity vector $\boldsymbol{v} = (0, 1, 1, 0)$ and the mass velocity $\boldsymbol{m} = (0, 1, 0)$ is shown in Figure 3. So, the state of a HPPRP cell is represented by an $(r + 4)$ long Boolean vector. (In what follows, the HPPRP model with one rest particle will be indicated by HPP1rp, the HPPRP model with two rest particles will be indicated by HPP2rp and so on.)

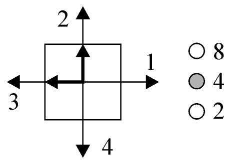

Figure 3

In the HPPRP model, the collision rules differ from the HPP collision rules. In addition to the head-on collision of the moving particles, the energy exchange between the moving and the rest particles occurs. In response to this exchange, either a rest particle is created and moving particles are annihilated or a rest particle is annihilated and moving particles are created. In the general case, the collision rules are deterministic or non-deterministic. They can be divided into the three groups.

Group 1 (the head-on collision). The moving particles collide with each other according to the head-on collision rule (Figure 2) independent of the presence or the absence of the rest particles.

Group 2 (the rest particle creation). If in a cell, the collision rule holds for two (four) moving particles and there is initially no mass 2 (4) rest particle, then moving particles will be annihilated and a mass 2 (4) rest particle will be created, respectively (Figure 4a). If in a cell, the collision rules hold for two moving particles and there are initially no mass 4 rest particle and mass 2 rest particle, then moving particles and mass 2 rest particle will be annihilated and a mass 4 rest particle will be created (Figure 4b).

Group 3 (the rest particle annihilation). If a mass 2 (4) rest particle already exists in a cell, and there are no two (four) moving particles for which the collision rule is hold, then two (four) moving particles will be created after the collision step, respectively, and the rest particle will be annihilated (Figure 5a). If a mass 4 rest particle already exists in the cell,

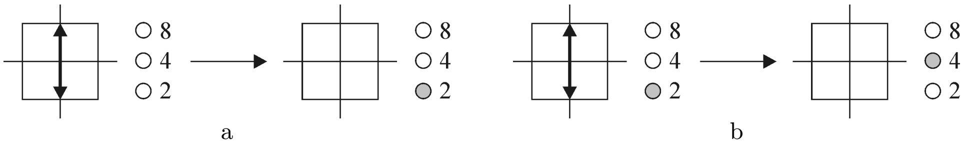

Figure 4. The rest particle creation rules

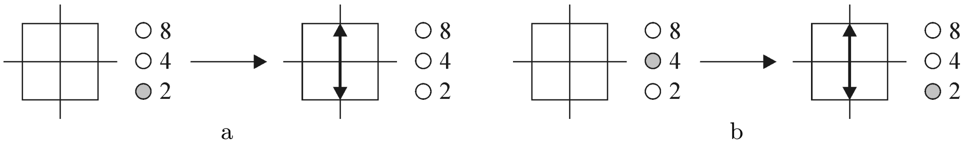

Figure 5. The rest particle annihilation rules

and there are no a mass 2 (4) rest particle and no four (or two) moving particles for which the collision rules hold, then two moving particles and a mass 2 rest particle will be created after the collision step, and a mass 4 rest particle will be annihilated (Figure 5b).

The rest particles are created (annihilated) with a certain probability $\mathcal{P}_k$, $k = 1, 2, \ldots, r$, in so doing, the following limitations should be met:

$$\mathcal{P}_{k+1} \geq \mathcal{P}_k, \qquad \sum_{k=1}^{r} \mathcal{P}_k \leq 1. \tag{1}$$

3. The influence of the HPPRP parameters on the sound wave propagation velocity

3.1. The HPPRP characteristics. In terms of the cellular automata modeling, the sound wave propagation is represented by a 2D cellular array W with $M \times N$ size cells. The first several array rows form a source for generation of the initial momentum. The source cells are the HPP cells. The rest of the array cells are the HPPRP cells, including the HPP cells. Each cell is assigned to an automaton. The automaton transition table has 2^{r+4} states. The source cells generate the moving particles with some probability within one iteration. The initial states of the HPPRP cells are generated according to a set of probabilities of the presence of the rest particles in the initial particle distribution. Further, a set of probabilities will be denoted by $P_{rp} = \langle p_r, p_{r-1}, \ldots, p_1 \rangle$, where p_k is the probability that the k-th rest particle exists in the initial particle distribution. The collision and the propagation rules make up the automaton transition rules. For such rules, condition (1) must be satisfied.

The boundary conditions are as follows. The left boundary (the column W_0) is the wall this one collision rule (the bounce-back rule: a moving

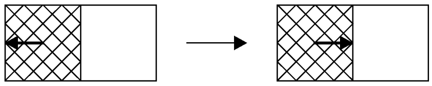

Figure 6. Bounce-back rule

particle colliding with a wall cell simply reverses its momentum (Figure 6). The right boundary (the column W_{N-1}) is open ($W_{N-1} = W_{N-2}$). The upper and the lower boundary conditions are periodical. Further an array of the HPPRP cells will be called by *the HPPRP medium*.

The medium cell with the coordinates (i, j) will be given by the moving particle density d_{ij} and the particle density $\hat{d}_{ij}$. Let $v_{ij} = (v_4(ij), v_3(ij), v_2(ij), v_1(ij))$ and $m_{ij} = (m_r(ij), m_{r-1}(ij), \ldots, m_1(ij))$ are the velocity vector and the mass vector of the cell with the coordinates (i, j), respectively. Then

$$d_{ij} = \sum_{k=1}^{4} v_k(ij), \qquad \hat{d}_{ij} = \sum_{k=1}^{4} v_k(ij) + \sum_{k=1}^{r} 2^k m_k(ij).$$

For the HPP medium $d_{ij} = \hat{d}_{ij}$. The wave propagation process in this medium will be given by the following parameters:

- averaged density of the moving particles $\langle \rho^0 \rangle$,
- averaged density of the medium $\langle \rho \rangle$, and
- averaged sound wave propagation velocity $\langle v_s \rangle$.

Further $\langle \rho \rangle$ and $\langle v_s \rangle$ will be called *the model density of a medium* and *the model velocity*, respectively. At the site of action of the initial momentum to the medium $\langle \rho \rangle = \langle \rho^1 \rangle + \langle \rho^2 \rangle$, where $\langle \rho^1 \rangle$ is the uniform background density, $\langle \rho^2 \rangle$ is the initial momentum density. For the given averaging radius r, averaged values of the density of the moving particles ρ^0 and the model density of the medium ρ are defined as

$$\langle \rho_j^0 \rangle = \frac{1}{(2r+1)M} \sum_{l=-r}^{r} \sum_{i=0}^{M-1} d_{ij}, \qquad j = 0, 1, \ldots, N-1,$$

$$\langle \rho_j \rangle = \frac{1}{(2r+1)M} \sum_{l=-r}^{r} \sum_{i=0}^{M-1} \hat{d}_{ij}, j = 0, 1, \ldots, N-1.$$

The table presents averaged values of the background density $\langle \rho^1 \rangle$ for all media on condition that the probability of the presence of all the rest particles in the initial distribution is equal to 0.5. Obviously, that for HPP medium $\rho_j^0 = \rho_j^1$.

LGA model	$\langle\rho^1\rangle$
HPP	2
HPP1rp ($m_1 = 2$)	3
HPP2rp ($m_2 = 4$, $m_1 = 2$)	5
HPP3rp ($m_3 = 8$, $m_2 = 4$, $m_1 = 2$)	9
HPP4rp ($m_4 = 16$, $m_3 = 8$, $m_2 = 4$, $m_1 = 2$)	17

3.2. The model velocity dependence of the HPPRP medium parameters. In this paper, the model sound velocity for all HPPRP media are experimentally obtained. As discussed above, the sound wave propagation contains is represented by the 2D cellular array. In our experiment, the array 200×800 cells. The first twenty array rows form a source for generation of the initial momentum. Each source cell generates three moving particles with probability 0.95 within one iteration. (The states of all source cells are equal in value and are defined as vector velocity $\boldsymbol{v} = (0, 1, 1, 1)$.) The initial states of the rest of the array cells are generated according to a set of probabilities of the presence of the rest particles in the initial particle distribution. In our experiment, the presence of the rest particles probabilities have the same value, namely, $p_4 = p_3 = p_2 = p_1 = 0.5$.

So, in the HPPRP medium, a wave propagates under the influence of the initial momentum. As discussed above, the wave propagation process in a medium will be given by a change in model density $\langle\rho\rangle$. Figure 7 shows a change in the density function in the HPP2rp medium every 100 iterations (in our experiments, $r = 10$). The model sound velocity is defined as the number of array cells for which a maximum value of the media density travels in a unit time. Figure 7 shows the sound velocity dependence of the moving particle density for the media with the following sets of the presence of the rest particles probability in the initial particle distribution.

The velocity curves are symmetric about the density $\langle\rho\rangle = 2$, regardless of the probability of the rest particles presence. In all HPPRP media, the

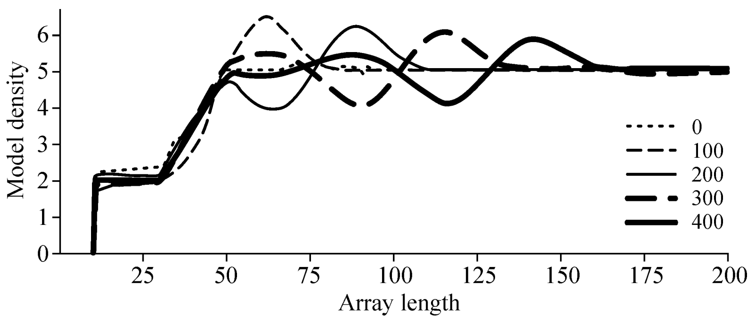

Figure 7. A change in density function in the HPP2rp medium

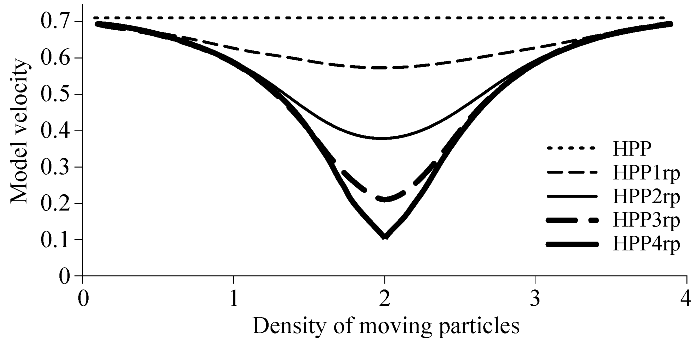

Figure 8. The model velocity dependence of density of the moving particles

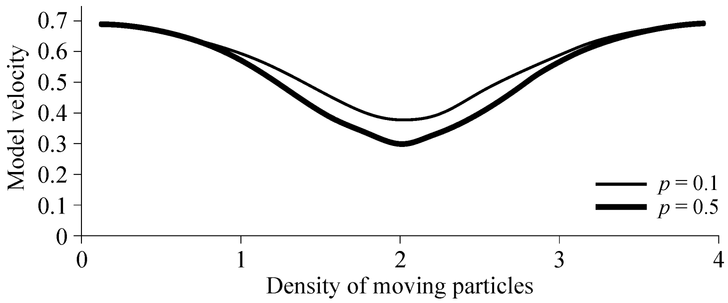

Figure 9. The model velocity dependence of density in the HPP2rp media for different values of the probability of a mass 4 rest particles presence

sound waves attain a maximum velocity and the greatest difference in magnitude for the density $\langle \rho \rangle = 2$. The model sound velocity can be increased (reduced) in the following ways.

A change in the rest particle initial distribution. At a given density of the moving particles, the sound wave velocity can be increased (reduced) due to enhancement (reduction) of the probability of the rest particles presence. For the given HPPRP media, the sound wave propagates with the greatest velocity in such HPPRP media wherein the rest particles with a maximum mass have a minimal probability in the initial particle distribution (Figure 9).

A change in the collision rules. The model sound wave velocity can be increased (reduced) at the cost of reduction (enhancement) the probability creating of the rest particle from two (four) moving particles. Indeed, the reduction of the probability creating the rest particle means that if only one moving particle do not take part in collision, then at the next iteration, this moving particle either collides with another particle or both travel in the direction of the wave propagation. The simulation supports that for the given HPPRP medium, the sound wave reaches a maximum value in media with such a set of the collision rules, wherein the probability creating the

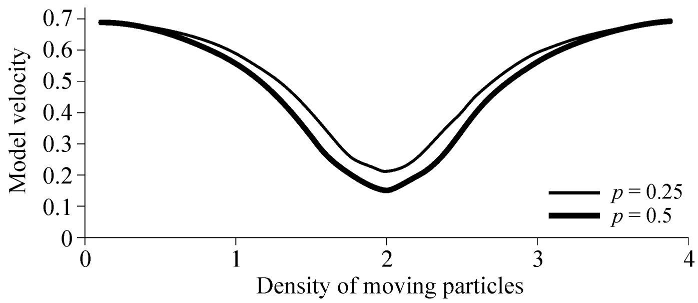

Figure 10. The model velocity dependence of density in the HPP3rp medium for different values of the probability of creating the rest particle

rest particle falls with a rise of its mass. It has been found experimentally that with reduction of the probability creating a mass 8 rest particle from 0.5 to 0.25 in the HPP3rp (Figure 10), the sound wave velocity reaches the model velocity v_m computed from the following formula [2]:

$$v_m^2 = \frac{2d(d-1)}{4d(d-1) + \sum_{k=1}^{r} m_k^2 p_k d_k (1-d_k)},$$

where d is the density of the moving particles in a cell with the coordinates (i,j) $(d = \langle \rho_j \rangle)$, m_k is the mass of the k-th rest particle, p_k is the probability of the presence of the k-th rest particles, $d_k = d^{m_k}/(d^{m_k} + (1-d)^{m_k})$ is the density of the k-th rest particle in a cell.

3.3. Correlation between the model and the physical sound wave velocity. In order for the sound wave propagation to be simulated in an inhomogeneous medium consisting of two materials, each material must be assigned to the HPPRP medium by the physical velocity v_{ph}, with which the sound wave travels in the given material and vice versa. For this purpose, a scale coefficient for velocity conversion (from the physical velocity to the model velocity and vice versa) must be defined. Let the sound wave propagate in the solid materials (2000–6320 m/s). The sound wave reaches a maximum physical velocity in aluminium, maximum model velocity (cells/iteration) in the HPP medium. As a result, aluminium is assigned to the HPP medium. In the general case, any HPPRP medium can be used as basic medium under the following condition: the ratio between the physical velocity of sound in a material does not exceed the ratio between the model velocity corresponding to those and materials. Further, the HPP medium will be considered as a basic one, then the scale coefficient is defined as

$$\mu_{al} = v_{ph}/v_m = 8.94 \cdot 10^3.$$

For example, a sound propagates in nickel with the velocity equal to 5400 m/s. According to the scale coefficient, the model sound velocity is

0.634. The sound wave propagates with such a velocity in all HPPRP media, but for different density of moving particles and the probability of the presence of the rest particles in the initial particle distribution (see Figure 8). Which of the all models is preferred? This is determined by a task. Such a medium is most often chosen, which has the density close to the equilibrium state. If this is not obtained, then the probability of the presence of the rest particles or (and) the probability creating of the rest particles need to be changed. For the example in question, nickel corresponds to the HPP1rp medium with the probability $p_1 = 0.25$.

4. The simulation of sound wave propagation in inhomogeneous media

In experiments, an inhomogeneous medium is formed of two solid materials: aluminium (a light medium) and ebonite (heavy medium). According to the scale coefficient, ebonite ($v_{ph} = 2500$ m/s) is assigned to the media with a model velocity equals 0.3. The sound wave propagates with such a model velocity in the HPP2rp medium with the following parameters: the density of the moving particles $\rho = 2$, the probabilities of the presence of the rest particles are $p_1 = 0.5$, $p_2 = 0.44$, the collision rules are equiprobable. As mentioned above, aluminium is assigned to the HPP medium. The sound wave propagation in inhomogeneous media is represented as a 2D cellular array W with the size of 200×800 cells. The array is comprised of three subarraies: a source, a subarray generated by the HPPRP cells, and a subarray generated by the HPP2rp cells. The demarcation line between two media does not require additional boundary conditions and collision rules.

Example 1 (The simulation of the sound wave propagation in aluminium-ebonite medium). In the experiment, the demarcation line between two media runs the length of the 250-th column of the array. The sound wave reaches the demarcation line at the 300-th iteration (Figure 11), and then propagates in the heavy medium (ebonite) with the velocity equal in magnitude to the sound wave model velocity in the media corresponding to ebonite (Figure 12).

Example 2 (The simulation of the sound wave propagation in ebonite-aluminium medium). In the experiment, the demarcation line between two media runs the length of the 180-th column of the array. The sound wave reaches the demarcation line at the 500-th iteration (Figure 13), and then passes into the light media (aluminium). The simulation has shown that the sound wave velocity in aluminium (ebonite) pre-demarcation line (Figure 12) and post-demarcation line (Figure 14) strongly coincide.

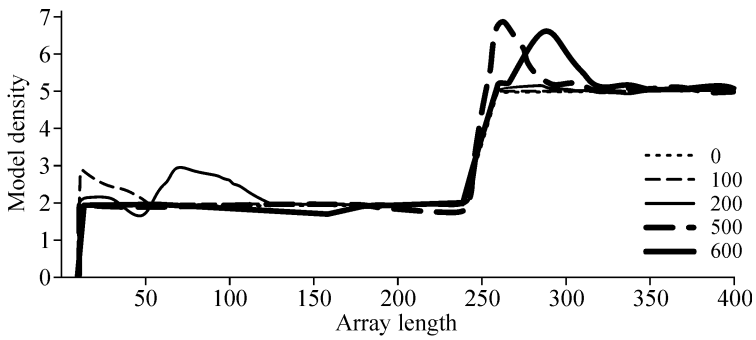

Figure 11. The sound wave propagation in aluminium-ebonite medium

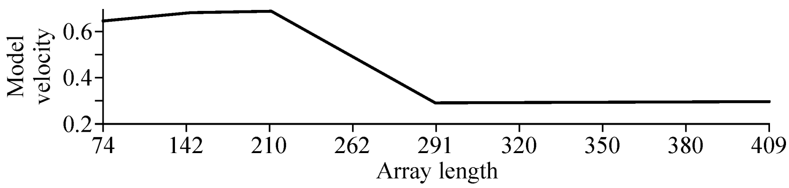

Figure 12. A change in the model wave velocity in aluminium-ebonite medium

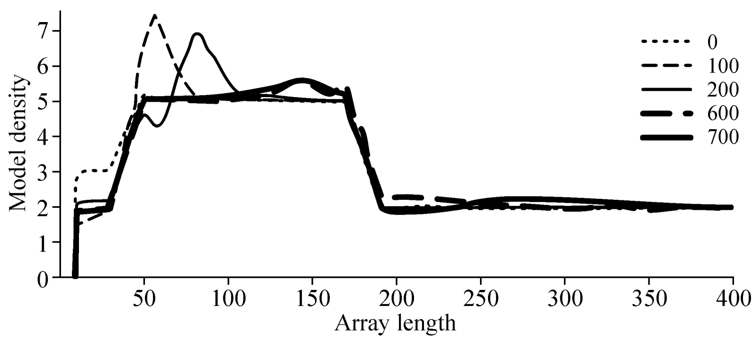

Figure 13. The sound wave propagation in ebonite-aluminium medium

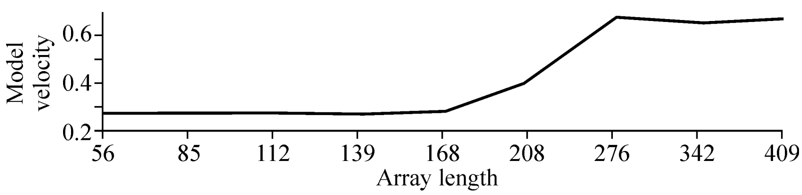

Figure 14. A change in the model wave velocity in ebonite-aluminium medium

5. Conclusion

In this paper, the model wave velocity dependence of the LGA model parameters (the number of the rest particles, density of the moving particles, the probability of the presence of the rest particles, and the probability of creating the rest particles) are experimentally investigated. Two LGA models on a 2D lattice with four neighbors (the HPP, and the HPPRP) are used. The technique for evaluation the HPPRP medium and its parameters by physical velocity of the sound wave propagation in a given medium is given. The sound wave propagation in inhomogeneous media formed of two solid materials (aluminium and ebonite) is simulated. The demarcation line between two media does not require additional boundary conditions and collision rules. The simulation has shown that the sound wave velocity in the light and in the heavy media, pre-demarcation line and post-demarcation line coincide very closely.

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