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The fast method for a rough tsunami amplitude estimation*

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Abstract. The method for estimating a tsunami height using the wave front kinematics computation has been developed and tested. This method is based on an orthogonal advance of computational points located along a moving tsunami wave front line. Precise algorithms for determining these points movement direction and an addition of new ones have been proposed. This method was tested in an area with a constant depth. Then in the areas with a parabolic and sloping bottom topography the obtained result of wave front propagation was compared to exact analytical solutions, which are delivered to such depth models. The method proposed makes possible to compute not only tsunami travel times but wave rays as well. Tsunami amplitudes can be estimated by the wave-ray divergence and a change in depth along the wave route. The wave amplitude estimation was tested against the results of the shallow-water numerical modeling of tsunami propagation using the MOST software. A difference in results between the two methods on the model (slope-like) bathymetry does not exceed a few percent. The advantage of the method proposed is its rapidness and low computer costs.

The tsunami wave characteristics such as the length-to-depth ratio can be considered to be long waves. The propagation process of this type of waves can be correctly described by a system of differential shallow-water equations. This has been checked by practice again and again. In the one-dimensional case without friction and Coriolis power, the linearized shallow-water equations can be written down as

$$\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = 0, \tag{1}$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial (Du)}{\partial x} = 0. \tag{2}$$

Here u is the horizontal flow velocity, η is the water surface displacement above the mean water level, g is the gravity acceleration, and D is depth. From the shallow-water equation it follows that the tsunami propagation velocity does not depend on its length and is expressed by the so-called Lagrange formula [1]

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$$c = \sqrt{g(D + \eta)}. \tag{3}$$

This formula plays the key role for the long-wave (tsunami) kinematics. Here it is necessary to note that a tsunami front and a crest are propagating with different velocities and the wave crest is step-by-step approaching the front. A tsunami wave will break when its crest reaches the wave front. When a tsunami wave propagates in deep water, this effect (the amplitude nonlinearity) is difficult to detect even when passing the whole Pacific Ocean. However a wave can break after a long cruise along a shallow shelf or when tsunami is approaching the coast. Further, the wave propagation velocity will be mentioned as the wave front propagation velocity which, according to (3), does not depend on the wave parameters but only on the water depth:

$$c = \sqrt{gD}. \tag{4}$$

This fact makes possible to obtain in advance a lot of peculiarities of the behavior of wave above the uneven bottom.

Properties of the shallow-water differential equations can help in obtaining some wave parameters estimations which can be used for a rough tsunami height determination. In the long-wave horizontal flow the velocity is constant at all depth levels from surface to bottom. We, also, take into account the fact that in sufficiently deep water (exceeding 100 m) the tsunami wave height usually does not exceed 2–3 m. This means that the water depth is much greater than the wave amplitude. First, let us derive an approximate formula for the horizontal flow velocity when η meters high tsunami wave propagates in D meters deep ocean. This formula can be obtained directly from equations (1), (2) and Lagrange formula (4). Let a wave be a harmonic function

$$\eta = a\cos(kx - bt), \tag{5}$$

which describes a wave of the amplitude a propagating along the axis OX with the velocity $c = b/k$. Placing the expression for wave amplitude (5) into equation (1) we have

$$\frac{\partial u}{\partial t} = gka\sin(kx - bt). \tag{6}$$

Taking integrals with respect to the time of the both parts of equation (6) we can determine how the horizontal flow velocity depends on the wave amplitude and water depth

$$\begin{aligned} u &= \int -\frac{gk}{b} a\sin(kx - bt)\, d(kx - bt) = \frac{g}{c} a\cos(kx - bt) \\ &= \frac{g}{\sqrt{gD}}\eta = \eta\sqrt{\frac{g}{D}}. \end{aligned} \tag{7}$$

Thus in a harmonic wave of the type of (5) the water flow velocity is determined by formula (7). Due to linearity of the tsunami propagation process any wave can be presented as superposition of waves with various frequencies. Because of this, formula (7) is valid for any long wave being a solution to the system of linear shallow-water differential equations (1), (2). For quasilinear shallow-water equations the velocities of a wave front and a wave crest are different (3). So, the formula for the horizontal flow velocity in propagating tsunami wave will be as follows:

$$u = \eta\sqrt{\frac{g}{D+\eta}}, \tag{8}$$

where η is the surface elevation, D is the depth and g is the acceleration of the gravity.

Now let us express the kinetic energy of propagating one-dimensional long wave with allowance for (8)

$$E_K = \int\limits_0^L \frac{\rho u^2}{2}(D+\eta)\,dx = \int\limits_0^L \frac{\rho\eta^2}{2}\frac{g}{(D+\eta)}(D+\eta)\,dx = \int\limits_0^L \frac{\rho\eta^2 g}{2}\,dx. \tag{9}$$

Here L is a wave length and ρ is the fluid density. Let us also write down an expression for the wave potential energy assuming that potential energy of quiet water is equal to zero:

$$E_P = \int\limits_0^L \frac{\rho g\eta^2}{2}\,dx. \tag{10}$$

The comparison of functions to be integrated in (9) and (10) shows their full identity. This means that in any length segment of a propagating wave the kinetic energy is equal to the potential one.

Using (7) it is possible to find an approximate formula for the one-dimensional wave height when it propagates above the uneven bottom. Let a one-dimensional wave have the profile $\eta_1(x)$ $(x = \overline{0, L_1})$ when it propagates in the ocean locality where depth is equal to D_1. Then this wave arrived at the depth D_2. Its length has been changed as follows:

$$L_2 = L_1\frac{\sqrt{gD_2}}{\sqrt{gD_1}}. \tag{11}$$

This follows from the stability of the one-dim wave period through the whole propagation process and the Lagrange formula (4). Let the new wave profile be described by the function $\eta_2(y)$ $(y = \overline{0, L_2})$. Due to the whole energy (including the kinetic energy) conservation in a propagating wave, we can write down the integral equality of the kinetic energy for the two different depth values D_1 and D_2, which are approximately equal to the water layer thickness $D_1 + \eta_1$ and $D_2 + \eta_2$:

$$\int_0^{L_1} \frac{\rho(v_1(x))^2}{2} D_1\,dx \approx \int_0^{L_2} \frac{\rho(v_2(y))^2}{2} D_2\,dy. \tag{12}$$

Here we made use of the constancy of the horizontal flow velocity from bottom to surface in a propagating tsunami wave. If both parts of equation (12) are to be integrated along the wave period which is constant along the whole propagation process, then (12) will be transformed to

$$\int_0^{T} \frac{\rho(v_1(t))^2}{2} D_1\,d(t\sqrt{gD_1}) = \int_0^{T} \frac{\rho(v_2(t))^2}{2} D_2\,d(t\sqrt{gD_2}).$$

Taking into account the horizontal flow velocity dependence on the amplitude and depth (7), the following equation can be written down:

$$\int_0^{T} \frac{\rho\eta_1^2(t)\sqrt{g/D_1})^2}{2} D_1\sqrt{gD_1}\,dt = \int_0^{T} \frac{\rho\eta_2^2(t)(\sqrt{g/D_2})^2}{2} D_2\sqrt{gD_2}\,dt. \tag{13}$$

For a sufficiently deep ocean ($D > 200$ m), the wave propagation process is quasilinear. So, as was noticed earlier, integral equation (13) turns into the approximate equality of expressions under the integral sign. Then after a certain simplification we have

$$\frac{\eta_1^2(x)}{\sqrt{D_2}} \approx \frac{\eta_2^2(x)}{\sqrt{D_1}}. \tag{14}$$

The final formula for the tsunami amplitude can be written as

$$\eta_2(x) \approx \eta_1(x)\sqrt[4]{\frac{D_1}{D_2}}. \tag{15}$$

Thus, during one-dimensional tsunami propagation from a deep ocean to a shallow shelf its amplitude will grow up to the fourth root of the initial and destination depth ratio (formula (15)). If a tsunami wave is not flat, then in addition to the depth-change factor the wave amplitude will also change due to its refraction (transformation of a wave-front line).

Let us consider a simple case when a round-shaped wave propagates in the area with a constant depth. According to the Lagrange formula (4) the wave-front line is a circle of a constantly increasing radius, but the wavelength remains constant. Let us once again use the energy conservation law in order to estimate the wave height decreasing rate. If at one instant of time a radius of the wave-front line is equal to R_1 and later it becomes to be equal to R_2, then the length of the circle-shaped wave fronts will be equal to $L_1 = 2\pi R_1$ and $L_2 = 2\pi R_2$. The tsunami wave parameters are the same along the whole front line. So, the constancy of the total potential energy of a wave can be written as

$$E_P = \int_0^{L_1}\int_0^{\lambda_1} \frac{\rho g \eta_1^2}{2}\, d\lambda\, dl = \int_0^{\lambda_1} \frac{\rho g \eta_1^2}{2} L_1\, d\lambda$$
$$= \int_0^{L_2}\int_0^{\lambda_1} \frac{\rho g \eta_2^2}{2}\, d\lambda\, dl = \int_0^{\lambda_1} \frac{\rho g \eta_1^2}{2} L_2\, d\lambda. \tag{16}$$

Here λ_1 is the wave length which is constant during the whole propagation process. In the linear (quasilinear) case, the integral equality (16) means the equality of sub-integral functions

$$\frac{\rho g \eta_1^2}{2} 2\pi R_1 = \frac{\rho g \eta_2^2}{2} 2\pi R_2, \quad \eta_2 = \eta_1 \sqrt{\frac{R_1}{R_2}}. \tag{17}$$

Therefore, due to the cylindrical divergence the wave height is decreasing inversely-proportional to the square root of the circle-shaped front radius (formula (17)).

In the general case, the perturbation kinematics in various media is described by the eikonal equation. In the two-dimensional space it can be written as

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \frac{1}{v^2(x,y)}, \tag{18}$$

where $v(x, y)$ gives the velocity distribution in a medium. If the function $f(x, y)$ is a solution of the eikonal equation (18), then the wave-front location at the time instant T is described by the equation $f(x, y) = T$, and the equation $f(x, y) = 0$ gives the initial location of perturbation sources or the initial wave front position (the tsunami source boundary). For example, let the solution function be of the form $f(x, y) = x^2 + y^2$.

In this case, the equation $f(x, y) = 0$ sets a source in the beginning of the origin of coordinates ($x = 0$, $y = 0$). If inside the circle of radius R with the center at a point $(0, 0)$ the propagation velocity is constant and equal to V_0, then after the time period $T = R/V_0$ the wave front that initially consists of one point will become a circle of radius R.

In the paper [2], where properties of the eikonal equation are studied, it is shown that for any propagation velocity distribution in a medium all the points on a wave front are moving in the orthogonal direction to the frontal line. The velocity is determined as a property of a medium. As far as tsunami waves are concerned, their propagation velocity is dependent only on a depth and is determined by Lagrange formula (4). The definition of a wave ray as a line, which is always orthogonal to the wave frontal line, is also given [2]. These properties of a wave front and a ray are the basis for the numerical method of the step-by-step wave frontal line advancement that will now be described.

Let us consider the rectangular computational domain where the wave propagation velocity $c(x, y)$ is known at any point. In the case of tsunami modeling this velocity can be obtained from a depth value using formula (4). Let also set the initial wave-front being a curve somewhere in the computational domain. As a rule, these curves are closed and convex. For example it can be a circle or an ellipse. Its smoothness is not required because in the numerical implementation the initial wave front is presented by a limited number of computational points located along this curve. We will assume that the area bounded by this curve completely consists of perturbation sources. Due to this fact a tsunami wave must propagate in the off-normal direction to the initial front line. Before carrying out computations, it is necessary to set a time step which determines a time difference between every computed location of the wave frontal line. It is obvious that a smaller time step will give us a better approximation accuracy of an actual wave-front.

Thus, we have a limited number of computational points (x_i, y_i) $(i = 1, \ldots, N)$ which are located along the closed initial wave frontal line. The point (x_N, y_N) is the neighboring point to (x_1, y_1). In order to obtain reliable results of modeling it is better a time step be taken inversely-proportional to a maximum of a propagation velocity gradient in the whole domain. We need to determine the next position of all the wave-front computational points (x_i, y_i) $(i = 1, \ldots, N)$ after one time step. First, it is necessary to determine the moving direction for all these points. This direction must be orthogonal to the wave-front line. Instead a smooth curve we have a broken line coming through N computational points (x_i, y_i). In this case, the orthogonal direction vector can be built by various ways. In the numerical implementation of this method the movement direction for the point (x_i, y_i) is determined as the outer-normal to the circle which passes the three computational points (x_{i-1}, y_{i-1}), (x_i, y_i) and (x_{i+1}, y_{i+1}). So if we will move the point (x_i, y_i) in this direction within the distance $c(x_i, j_i) \cdot \Delta t$, then this location will present the new position of the point with the index equal to i at the time instant $t = \Delta t$. If a wave-front is a closed line then for the computational points with indices 1 and N one of the neighboring points must be (x_N, y_N) or (x_1, y_1) respectively. When all the computational points move, we will build the wave-front location at the time instant $t = \Delta t$. If we repeat this procedure as many times as is necessary for a tsunami to reach the computational domain boundaries, the kinematic picture of the wave propagation (tsunami isochrones) will be built. The source was bounded by the initial wave-front line. If the initial front is not a closed line, then the direction of the edge points movement is determined as being orthogonal to the segment which connects this point and its neighbor. This direction can also be determined as a normal to the circle which passes through 3 edge computational points of the wave-front. In this method all computational points are moving along wave rays [2]. This is a basic factor for the tsunami amplitude estimation algorithm that will be described below.

Now let us carry out a few tests of the method on some problems with known exact analytical solutions. The first test includes the propagation of an initially round-shaped wave-front in a domain with constant depth. In this case, theoretically the wave frontal line will be a circle of radius

$$R = R_0 + t\sqrt{gD}, \tag{19}$$

where R_0 is the initial frontal line radius, D is the depth and t is the time after beginning of propagation. In the numerical experiment, 50 computational points of the initial front were equidistantly located along the circle of radius 50 km (a small circle in Figure 1).

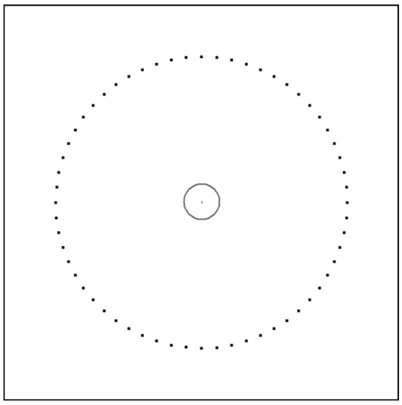

Figure 1. The computational points locations of the initially round-shaped wave front at 500 seconds after the beginning of propagation

Due to the concept of this method (normally directed to a wave-front line advancement of the wave frontal points), the resulting location of the wave-front is absolutely identical to the exact solution. The next problem is about the propagation of the initially round-shaped wave-front above the parabolic bottom. This means that the depth increases from zero value at the lower boundary of the domain proportionally to the squared distance to this boundary

$$D(x, y) = 9 \cdot 10^{-9} \cdot y^2. \tag{20}$$

At the upper boundary the depth is equal to 9000 m. In this case, the frontal line is always a circle but its center is moving off the shore [3]. The comparison of the numerical results (black dots) and the analytical solution (grey circles) of this problem is presented in Figure 2. Here in 1000×1000 km computational domain the initial wavefront consists of 40 computational points being located along the circle of radius equal to 50 km and centered at a point $(500, 300)$ (the small circle in Figure 2). The large circle here visualizes the exact solution (the wave front) for the time instant 3000 s. No difference can be seen in Figure 2 between the numerical and the exact solutions.

One more test is about the behavior of non-closed wavefront line above the parabolic bottom with a cylindrical symmetry. Let in 1000×1000 km computational domain a depth be proportional to the squared distance to

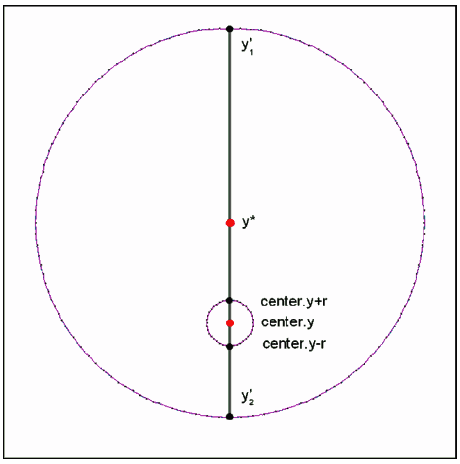

Figure 2. The numerical and exact solution comparison in the domain with parabolic bottom topography

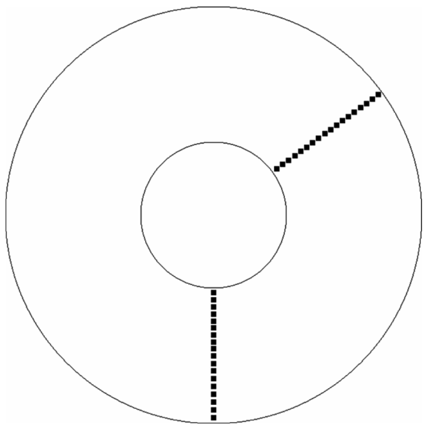

Figure 3. The rotation of the wavefront segment above the parabolic bottom topography with cylindrical symmetry

the central point $(500, 500)$. The initial wavefront is being a segment of the straight line directed down from the central point, where the depth is equal to zero. Theoretically, this segment of the wavefront will rotate around the central point of the domain like a clock arrow always being straight and having a fixed length. Results of the numerical computation suggest such a behavior of the wavefront (Figure 3). Here the initial front was taken as a segment of the straight line between the circles having the radii R_1, R_2 and directed vertically down from the central point.

All these tests deal with the wavefront only. As was noted earlier, each computational point of the wavefront moves along the wave ray. It is also possible to compare the computational point traces with the exact solutions that are known for some types of the bottom topography [4]. For the parabolic bottom topography where depth is proportional to the squared distance to one of the domain boundaries, a wave ray is being an arc of a circle. In the numerical experiment, the depth in 1200×1200 km computational domain is defined by formula (20). The initial wavefront consists of 50 computational points located along the circle of 50 km radius. The circle center was posed at a point $(450, 500)$ (Figure 4).

Let us consider the movement trajectory of the wavefront point initially having the coordinates $x = 500$ km, $y = 500$ km. This computational point at the first time step will advance in the horizontal direction because of the same its vertical position as a center of the initially round-shaped wave front. Theoretically, this ray must move along the circle of a radius of 500 km and will arrive at the coast (the lower boundary of the domain) at

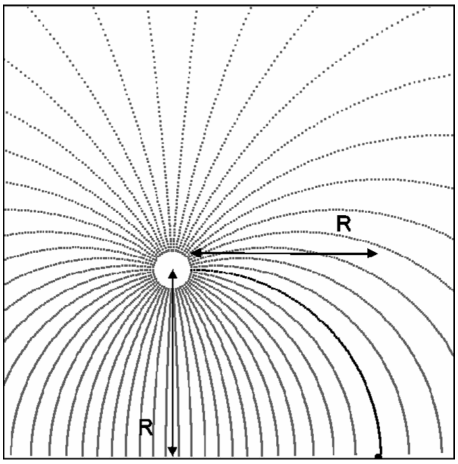

Figure 4

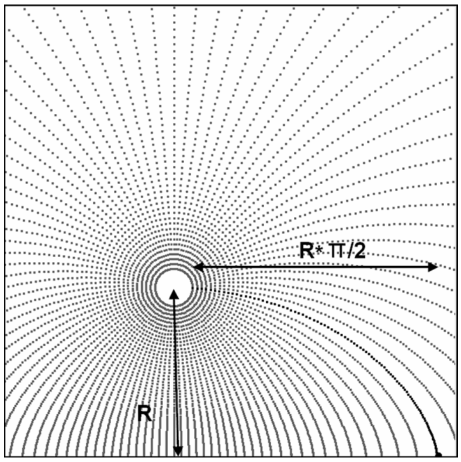

Figure 5

the point $(1000, 0)$. This point is marked as a big black dot in the bottom of Figure 4. Such a ray being numerically computed is also drawn here by the black color. The computed wave ray looks very similar to the circle arc and a difference between the theoretical and the actual coast arrival points is less than 1 percent of the arc radius. This error can be reduced by using a smaller time step in computations. In the second problem, used for testing, the bottom topography presents a uniform slope, where the depth linearly increases from the lower boundary of the domain $D = 0.009y$. Here the dimension of the computational domain was 1200×1200 km and the initial wave-front consists of 100 points located along the circle of a radius of 50 km having the center at the point $(450, 450)$. Above such a kind of the bottom topography any wave ray has the cycloid arc shape [4]. Due to the above-said, the wave ray, which is horizontally directed at a point $(500, 450)$, must theoretically approach the coastline (the lower boundary) at the point having the coordinates $x = 450 + 450\pi/2$, $y = 0$.

As a result of the numerical experiment, the wavefront computational point that is initially located at a point $(500, 450)$ moves along the trajectory which is very close to the cycloid arc (Figure 5). The computed ray approaches the lower boundary of the domain at a point $(1156, 0)$, which has approximately the same location as the theoretical one. All the tests show that the method proposed is sufficiently precise for the numerical simulation of tsunami wave fronts and wave rays.

If a computational domain is large, then sooner or later the neighboring computational points of the wavefront will move away from each other widely spaced. In this case some bottom irregularity can occurs between these points, and kinematics of a wavefront will not be correct. So, we need to add new computational points of the wavefront if a distance between the neighboring points will exceed the value we have set. It is necessary to do this correctly without spoiling the general curvature of a front line at this locale. For example, if a new point is posed towards the center of the segment which connects two computational points of the wave-front, then this segment of the frontal line will move slower than an actual one. Figure 6 schematically shows the procedure of adding new frontal points which is implemented in the computer program.

Let a distance between two computational points P_i and P_{i+1} of the wave-front exceed a maximum available value that we have set. Now we need to pose a new computational point somewhere between these two points. Let us also take into account the other two points P_{i-1} and P_{i+2} that are the nearest neighbors to the points P_i and P_{i+1}. Let us build an orthogonal line coming through the center of the segment $[P_i, P_{i+1}]$ (Figure 6) and two circles which pass through the points P_{i-1}, P_i, P_{i+1} and P_i, P_{i+1}, P_{i+2}. If any set of these points locates on a straight line, then this line must be taken instead of a circle. If the first circle intersects the orthogonal line at a point $\boldsymbol{A}$ and the second circle at a point $\boldsymbol{B}$, then a new computational point of the wave-front $\boldsymbol{C}$ must be posed towards the center of the segment $[\boldsymbol{A}, \boldsymbol{B}]$ (see Figure 6). This algorithm was tested on model problem with a parabolic bottom topography. The results of this numerical experiment are presented in Figure 2. The wave-front initially consists of 40 computational points, and in the course of computations their number has increased up to 200 (see Figure 2).

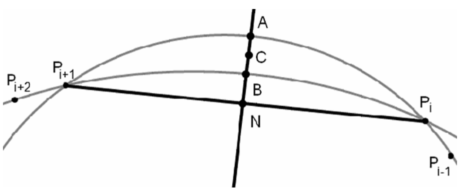

Figure 6. The scheme of the algorithm for adding a new computational point of a wavefront

Thus, the proposed numerical method makes possible to determine not only the wave-front location at different instants of time, but also to build the wave-ray trajectories. Two wave rays being traces of two neighboring computational points of the wave-front construct the so-called ray tube inside which the total amount of wave energy is constant. In other words, there is no energy transfer between the ray tubes. This allows us to use formulas (15) and (17) for estimating a wave height inside a ray tube. If we know the tube width in the beginning of wave propagation and later when a wave-front segment moves along the ray tube, then according to (17) it is possible to calculate the wave height decreasing coefficient due to the wave rays divergence. Knowing, also, depth values all around the computational domain, one can determine the coefficient of the tsunami amplitude change due to a wave transition from one depth to another. If we multiply both these coefficients and the initial wave height, then we can obtain a rough estimation of the tsunami height at any point of the computational domain, where the wave-front was determined using this method. This formula can be written down as

$$\eta_2 = \eta_1 \sqrt{\frac{L_1}{L_2}} \sqrt[4]{\frac{D_1}{D_2}}. \tag{21}$$

Here L_1 is the ray tube initial width that can be presented as a distance between the neighboring computational points of the initial wavefront, D_1 being the depth value, L_2 is the ray tube width at the destination site, where the depth value is equal to D_2. As already was noticed, sometimes a new computational point of the wavefront is added. This procedure divides the width of the ray tube by 2. We count a number of such additions for each ray tube and finally use this number in the formula for the wave height estimation

$$\eta_2 = \eta_1 \sqrt{\frac{L_1}{L_2 \cdot 2^N}} \sqrt[4]{\frac{D_1}{D_2}}. \tag{22}$$

Here N is the number of new computational point additions for this one ray tube. If in the course of the wave segment propagation along the ray tube we eliminate computational points (due to their too "tight" closeness) M times, then formula (22) will be modified to

$$\eta_2 = \eta_1 \sqrt{\frac{L_1}{L_2 \cdot 2^{N-M}}} \sqrt[4]{\frac{D_1}{D_2}}. \tag{23}$$

So, the method for a wave front and ray kinematics which allows estimating the tsunami wave amplitude without modeling of dynamic flow parameters has been developed and realized.

Let us compare the wave height maximum distribution obtained by the method described against the results of numerical tsunami modeling within the shallow-water model using MOST algorithm [5]. Let us consider the following problem: in 1000×1000 km area, the central-symmetric tsunami source is fixed. It is centered at a point $(500, 300)$. The vertical profile of this source is defined by the following expression:

$$\eta(x) = 1 + \cos\frac{\pi R}{R_0}, \quad R = [0, R_0]. \tag{24}$$

Here R is a distance to its center. The initial source radius R_0 is equal to 50 km. It is seen from formula (24) that at the source center the initial water surface displacement is equal to 200 cm. The depth is varying only along the ordinate axis. The depth linearly increases from zero at the shoreline ($y = 0$) up to 2000 m at a distance of 200 km from the coast. Then there is a 200 km wide terrace with a constant depth (2000 m). Then from $y = 400$ km to the upper boundary of the domain a depth linearly increases from 2000 up to 8000 meters. The tsunami source is located above the bottom terrace (the coordinates of its center are $x = 500$ km, $y = 300$ km). Figure 7 shows

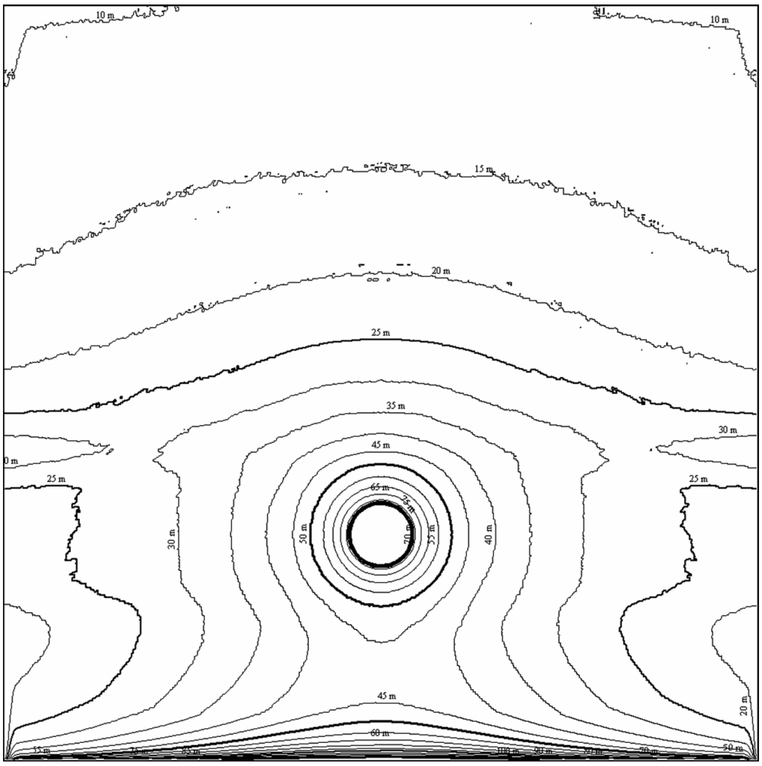

Figure 7. The distribution of wave-height maxima as a result of tsunami numerical simulation within the shallow-water model

isolines of the wave height maxima obtained by the numerical modeling of tsunami propagation within the shallow-water model.

Due to the depth constancy above the bottom terrace these isolines are circles when tsunami propagates in this zone. For example, the wave height is equal to 50 cm at a distance of 96 km from the source center. So, if the initial wave front is a circle of a radius of 96 km with the center at a point $(500, 300)$, then the numerical simulation of initially 50 cm high tsunami must give the wave height distribution similar to the one obtained by the MOST algorithm (see Figure 7). The results of such a modeling are presented in Figure 8 as trajectories of wave-front point movement. Here the wave amplitude maxima that were estimated by formula (23) are visualized by the grey color brightness according to the color legend shown in Figure 8. It is very clearly seen that above the bottom terrace the wave rays are straight lines.

And finally, for the quantitative comparison of the results that were obtained by these two different methods, the isolines of the height maxima distribution and the corresponding wave-height value locations along the wave rays are simultaneously presented in Figure 9. Here the isolines were drawn by black color within 5 cm intervals and as a maximum limited by 0.5 m. Short grey lines indicate to the locations on the ray trace where formula (23) gives approximately the same wave height within 5 mm difference against the isoline levels. The comparison of these 2 distributions confirms their approximate similarity. The spatial difference between these two sets of tsunami height isolines does not exceed a few kilometers.

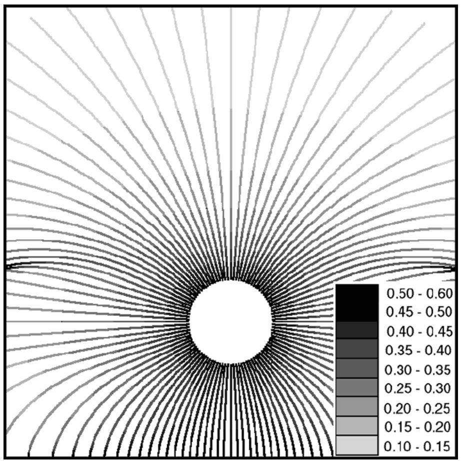

Figure 8. The wave-ray traces and the wave height change along the each one

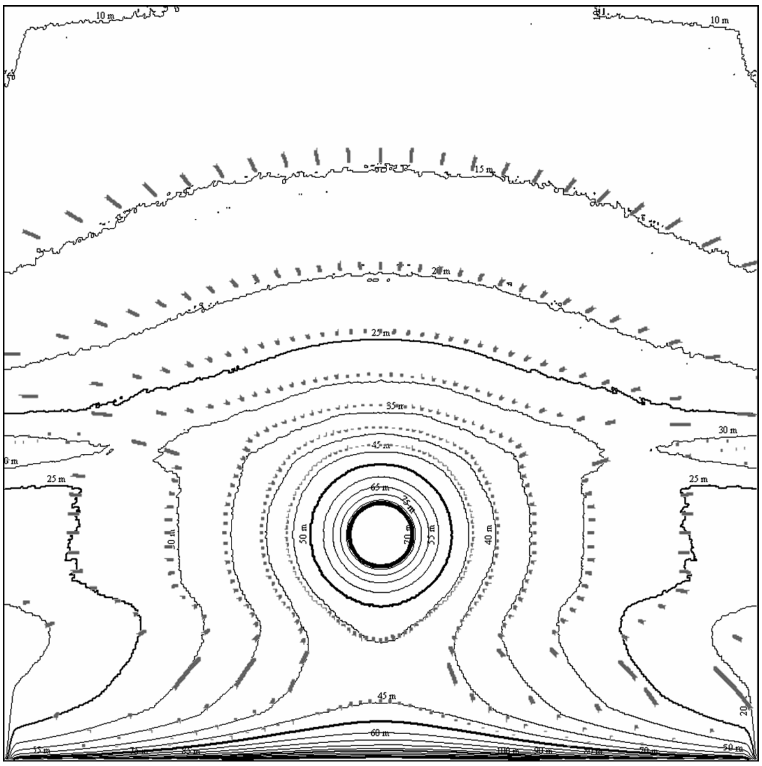

Figure 9. The comparison of maximum wave height distribution obtained by two different methods

Conclusion

The new method for the tsunami wave height estimation using the wavefront kinematics computation was developed and tested. In comparison with the numerical methods for solving the shallow-water differential equations, the method proposed requires much less computer resources and time.

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