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About one combined inverse problem for the equations of porous media and Maxwell's equations*

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Let us consider the propagation of magnetoacoustic waves in the porous media saturated with electrolyte in the case when the loss of energy occurs due to electroconductivity, intercomponental friction coefficient and electrokinetic effect. Linearized equations describing electromagnetic quasi-stationary effects in the porous media saturated with electrolyte look like [1, 2]:

$$
\begin{aligned}
&\frac{\partial^2 \boldsymbol{u}}{\partial t^2} - c_t^2 \Delta \boldsymbol{u} - a_1 \nabla \operatorname{div} \boldsymbol{u} + a_2 \nabla \operatorname{div} \boldsymbol{v} + \\
&\qquad \frac{\rho_l^2}{\rho_s} \bar{\chi} \Big(\frac{\partial \boldsymbol{u}}{\partial t} - \frac{\partial \boldsymbol{v}}{\partial t} \Big) + \frac{\alpha c_e \rho_l}{4\pi\sigma\rho_s} \operatorname{rot} \frac{\partial \boldsymbol{B}}{\partial t} = 0, \\
&\frac{\partial^2 \boldsymbol{v}}{\partial t^2} + a_3 \nabla \operatorname{div} \boldsymbol{u} - a_4 \nabla \operatorname{div} \boldsymbol{v} - \rho_l \bar{\chi} \Big(\frac{\partial \boldsymbol{u}}{\partial t} - \frac{\partial \boldsymbol{v}}{\partial t} \Big) - \\
&\qquad \frac{\alpha c_e}{4\pi\sigma} \operatorname{rot} \frac{\partial \boldsymbol{B}}{\partial t} + \frac{\boldsymbol{B}_0}{4\pi\rho_l} \wedge \operatorname{rot} \frac{\partial \boldsymbol{B}}{\partial t} = 0, \\
&\frac{\partial \boldsymbol{B}}{\partial t} = \operatorname{rot} \Big[-\frac{\alpha c_e}{4\pi\sigma} \operatorname{rot} \boldsymbol{B} + \frac{\alpha c_e \rho_l}{\sigma} (\boldsymbol{u} - \boldsymbol{v}) + \boldsymbol{v} \wedge \boldsymbol{B}_0 \Big],
\end{aligned}
\tag{1}
$$

where $\boldsymbol{u}$ is the velocity of motion of an enclosing matrix, $\boldsymbol{v}$ is the velocity of a conducting liquid, $\boldsymbol{B}$ is a magnetic field, $\sigma = \sigma_l + \sigma_s$ ($\rho = \rho_l + \rho_s$) is the electroconductivity (density) of a medium, σ_s (ρ_s) is the electroconductivity (partial density) of an enclosing matrix, σ_l (ρ_l) is the electroconductivity (partial density) of a liquid, c_e is the speed of light, α is the electrokinetic constant, $\bar{\chi} = \chi^{\partial} - \alpha^2/\sigma$, χ^{∂} is the friction coefficient, a_k ($k = 1, 2, 3, 4$) are coefficients of functions of two velocities of longitudinal waves and one velocity of a transverse wave of a porous saturated medium [1].

In the one-dimensional case, the axis x coincides with the direction of propagation of transverse waves and is parallel to the direction of the magnetic field. Let us assume that the electroconductivity of the enclosing matrix is absent, the velocities $\boldsymbol{u} = (0, u_y, u_z)$, $\boldsymbol{v} = (0, v_y, v_z)$, and the magnetic field $\boldsymbol{B} = (0, B_y, B_z)$ are perpendicular to $\boldsymbol{B}_0$. Let a medium rest at $t < 0$, and on the boundary $x = 0$ the following conditions are given:

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$$\mu\frac{\partial u_y}{\partial x}\Big|_{x=0} = f_y(t), \qquad \mu\frac{\partial u_z}{\partial x}\Big|_{x=0} = f_z(t),$$
$$B_y\big|_{x=0} = B_y^0(t), \qquad B_z\big|_{x=0} = B_z^0(t), \tag{2}$$

where $\mu = \rho_s c_t^2$.

Let us now consider steady-state oscillations with a frequency ω of a homogeneous half-space $x > 0$. Let the velocities and the magnetic field look like

$$(u_y, u_z, v_y, v_z, B_y, B_z) = (u_y(x), u_z(x), v_y(x), v_z(x), B_y(x), B_z(x))e^{-i\omega t}.$$

Within the limits of the statement proposed, after separation of a time factor with respect to amplitudes we will obtain a boundary value problem. The bounded solution of the obtained boundary value problem looks like

$$u_y = \frac{\omega_*^2(i\frac{\omega}{\omega_*}B_z^0(\omega) - B_y^0(\omega))}{\beta_1\Lambda_2^* - \beta_2\Lambda_1^*}[\beta_2 e^{-\beta_1 x} - \beta_1 e^{-\beta_2 x}] +$$
$$\frac{\Lambda_1^* e^{-\beta_2 x} - \Lambda_2^* e^{-\beta_1 x}}{\beta_1\Lambda_2^* - \beta_2\Lambda_1^*}F_y,$$
$$u_z = \frac{\omega_*^2(i\frac{\omega}{\omega_*}B_y^0(\omega) + B_z^0(\omega))}{\beta_1\Lambda_2^* - \beta_2\Lambda_1^*}[\beta_1 e^{-\beta_2 x} - \beta_2 e^{-\beta_1 x}] +$$
$$\frac{\Lambda_1^* e^{-\beta_2 x} - \Lambda_2^* e^{-\beta_1 x}}{\beta_1\Lambda_2^* - \beta_2\Lambda_1^*}F_z,$$
$$B_y = \frac{\beta_1\Lambda_2^* e^{-\beta_2 x} - \beta_2\Lambda_1^* e^{-\beta_1 x}}{\beta_1\Lambda_2^* - \beta_2\Lambda_1^*}B_y^0(\omega) +$$
$$\frac{F_y + i\frac{\omega}{\omega_*}F_z}{\omega^2 - \omega_*^2}\frac{\Lambda_1^*\Lambda_2^*}{\beta_1\Lambda_2^* - \beta_2\Lambda_1^*}(e^{-\beta_1 x} - e^{-\beta_2 x}),$$
$$B_z = \frac{\beta_1\Lambda_2^* e^{-\beta_2 x} - \beta_2\Lambda_1^* e^{-\beta_1 x}}{\beta_1\Lambda_2^* - \beta_2\Lambda_1^*}B_z^0(\omega) +$$
$$\frac{F_y + i\frac{\omega}{\omega_*}F_z}{\omega^2 - \omega_*^2}\frac{\Lambda_1^*\Lambda_2^*}{\beta_1\Lambda_2^* - \beta_2\Lambda_1^*}(e^{-\beta_2 x} - e^{-\beta_1 x}),$$

where

$$\Lambda_k^* = \frac{\Lambda(\beta_k)}{\beta_k}, \quad F_{y,z} = -i\frac{\omega}{\mu}f_{y,z}(\omega), \quad \omega_* = \sigma\frac{B_0}{c_e}\frac{\bar{\chi}}{\alpha},$$
$$\Lambda(\beta) = \frac{4\pi\rho_s c_t^2\omega_*^2(1+i\lambda)}{\lambda B_0, \omega}\left[\beta^2 + \frac{\omega^2}{c_t^2}\Big(1 + \frac{i\lambda}{1+i\lambda}\frac{\rho_l}{\rho_s}\Big)\right],$$
$$D = \frac{c_e^2}{4\pi\sigma} + \frac{i}{\omega}\frac{B_0^2}{4\pi\rho_l(1+i\lambda)}\Big(1 + \frac{\omega^2}{\omega_*^2}\lambda^2\Big), \qquad \lambda = \frac{\bar{\chi}\rho_l}{\omega},$$

β_k, $k = 1, 2$, are characteristic roots with positive real parts of the differential operator

$$\left[\frac{d^2}{dx^2}+\frac{\omega^2}{c_t^2}\Big(1+\frac{i\lambda}{1+i\lambda}\frac{\rho_l}{\rho_s}\Big)\right]\left[\frac{d^2}{dx^2}+i\frac{\omega}{D}\right]B_{y,z}=$$
$$i\frac{\omega}{D}\frac{\lambda^2B_0^2}{4\pi\rho_s c_t^2(1+i\lambda)^2}\Big(1-\frac{\omega^2}{\omega_*^2}\Big)\frac{d^2B_{y,z}}{dx^2}.$$

The velocity of the conducting liquid is found from

$$v_y=\frac{i}{\omega}\frac{B_0}{4\pi\rho_l(1+i\lambda)}\left[\frac{dB_y}{dx}-\lambda\frac{\omega}{\omega_*}\frac{dB_z}{dx}\right]+\frac{i\lambda}{1+i\lambda}u_y,$$
$$v_z=\frac{i}{\omega}\frac{B_0}{4\pi\rho_l(1+i\lambda)}\left[\frac{dB_z}{dx}+\lambda\frac{\omega}{\omega_*}\frac{dB_y}{dx}\right]+\frac{i\lambda}{1+i\lambda}u_z.$$

Components of the electric field are calculated by the formulas

$$\begin{aligned}E_y&=\frac{i}{c_e}\frac{\omega}{\omega_*}\Big\{i\frac{M_1\beta_2e^{-\beta_1x}-M_2\beta_1e^{-\beta_2x}}{\beta_1\Lambda_2^*-\beta_2\Lambda_1^*}\frac{\omega_*}{\omega}(\omega^2-\omega_*^2)B_z^0(\omega)+\\&\qquad\frac{\Lambda_1^*M_2e^{-\beta_2x}-\Lambda_2^*M_1e^{-\beta_1x}}{\beta_1\Lambda_2^*-\beta_2\Lambda_1^*}\Big(F_y+i\frac{\omega_*}{\omega}F_z\Big)\Big\},\\E_z&=\frac{1}{c_e}\Big\{\frac{M_1\beta_2e^{-\beta_1x}-M_2\beta_1e^{-\beta_2x}}{\beta_1\Lambda_2^*-\beta_2\Lambda_1^*}(\omega^2-\omega_*^2)B_y^0(\omega)+\\&\qquad\frac{\Lambda_1^*M_2e^{-\beta_2x}-\Lambda_2^*M_1e^{-\beta_1x}}{\beta_1\Lambda_2^*-\beta_2\Lambda_1^*}\Big(F_y+i\frac{\omega}{\omega_*}F_z\Big)\Big\},\end{aligned}\tag{3}$$

where

$$M_k=\frac{\Lambda(\beta_k)}{\omega^2-\omega_*^2}D+\frac{i\lambda}{1+i\lambda}B_0,\quad k=1,2.$$

Let us consider the case when magnetoacoustic waves are excited by a seismic source. For the sake of definiteness we consider that $B_y^0=B_z^0=0$, $f_y=0$, $f_z\neq 0$. The case when magnetoacoustic waves are excited by a magnetic source were considered in [1]. From (3), we will obtain

$$E_y=-\frac{1}{c_e}\frac{\Lambda_1^*M_2e^{-\beta_2x}-\Lambda_2^*M_1e^{-\beta_1x}}{\beta_1\Lambda_2^*-\beta_2\Lambda_1^*}F_z,$$
$$E_z=i\frac{1}{c_e}\frac{\omega}{\omega_*}\frac{\Lambda_1^*M_2e^{-\beta_2x}-\Lambda_2^*M_1e^{-\beta_1x}}{\beta_1\Lambda_2^*-\beta_2\Lambda_1^*}F_z.$$

Hence, there is a connection between the components

$$E_z=-i\frac{\omega}{\omega_*}E_y.$$

This relation allows us to formulate a method of measuring the kinetic coefficients $\alpha/\bar{\chi}$. For measuring $\alpha/\bar{\chi}$, we will place the sensor measuring two orthogonal components of the vector of the electric field $\boldsymbol{E} = (E_x, E_y)$ on the boundary $x = 0$. We gain the equality of the modules $|E_x|$ and $|E_z|$ by a change in the frequency of an external exciting pulse signal. The equalities of the modules of electric fields are achieved with the frequency of excitation

$$\omega = \omega_* = \sigma \frac{B_0}{c_e} \frac{\bar{\chi}}{\alpha}.$$

Knowing the external magnetic field B_0, elecrtocunductivity $\sigma = \sigma_l + \sigma_s$ and characteristic frequency ω_*, we will obtain the formula for defining of the combination the electrokinetic coefficient and the friction coefficient

$$\frac{\bar{\chi}}{\alpha} = \frac{\sigma}{\omega_*} \frac{B_0}{c_e}.$$

References

[1] Dorovsky V.N., Dorovsky S.V. An electromagnetoacoustic method of measuring electric conductivity and ζ-potential // Russian Geology and Geophysics. — 2009. — No. 6. — P. 564–570.

[2] Dorovsky V.N., Imomnazarov Kh.Kh. A mathematical model for the movement of a conducting liquid through a conducting porous medium // Math. Comput. Modelling. — 1994. — Vol. 20, No. 7. — P. 91–97.