Bull. Nov. Comp. Center, Num. Anal., 16 (2013), 1–6

Computer-aided simulation of the nonlinear regime of the beam-plasma interaction*

E.A. Berendeev, G.I. Dudnikova, A.A. Efimova

Abstract. The problem of an electron beam and plasma interaction is considered. The physical mechanism of the beam-plasma interaction includes a resonant excitation of plasma oscillations, the occurrence of the plasma density modulation, followed by electron scattering. For the modeling, the PIC-method is used. In order to solve this problem, a parallel algorithm of calculations is implemented.

Introduction

One of the most important problems of the plasma physics is heating of high-temperature plasma in thermonuclear devices. In this paper, the processes of establishment and nonlinear evolution of the quasi-stationary plasma turbulence raised by a powerful electron beam on the facility of the controlled thermonuclear fusion with the numerical modeling are investigated. The beam and plasma parameters were chosen close to the parameters in the experiment on the GOL-3 facility [1].Detection of suppressing the longitudinal electron heat conductivity on the facility in the course of injection of a relativistic electron beam is one of the recent important achievements in the physics of open traps.

The task of relaxation of an electron beam in plasma is a classical problem of the plasma physics, and there are numerous theoretical models describing various regimes of the beam-plasma interaction, but the research into this area remains actual [2].

Studying the influence of the beam nonlinearities on the behavior of instability in the conditions of the developed turbulence requires numerical modeling which, on the one hand, is capable of tracing turbulence evolution raised by the beam at long time periods, and on the other, provides rather a detailed description of the kinetic effects related to capture of a beam.

In this paper, the created two-dimensional numerical model based on the particles-in-cell method (PIC-method) [3, 4], focused on the research into stability and plasma heating by a warm electron beam is described. It is known that the electronic beam extending in the dense plasma, is unstable in relation to the longitudinal density modulation. By means of numerical modeling it appeared possible to reproduce various scenarios of excitement of plasma turbulence with an electron beam. The problem of the

*Supported by the RFBR under Grant 11-01-00249 and IP SB RAS 130.

two-stream instability leading to modulation of density is considered. The numerical convergence of the solution depending on calculating parameters was investigated, good compliance with available analytical decisions is obtained. Calculations are performed on the supercomputers NKS-30T and the “Lomonosov”.

1. The problem statement and the solution of the main equations

The most comprehensive investigation of the plasma processes can be carried out only through complex approach that combines both experimental studies and research computing procedures which adequately describe these processes. It is necessary to construct an adequate mathematical model of the physical processes in order to avoid simplifications and to obtain a qualitatively correct physical picture. The model uses a collisionless approximation of plasma [3, 4]. Plasma is described by the system of the Vlasov–Maxwell equations:

$$\frac{\partial f_k}{\partial t} + (\vec{v}, \vec{\nabla}) f_k + q_k \left(\vec{E} + \frac{1}{c} [\vec{v} \times \vec{H}] \right) \frac{\partial f_k}{\partial \vec{p}} = 0, \tag{1}$$

$$\begin{aligned} \operatorname{rot} \vec{H} &= \frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}, & \operatorname{div} \vec{H} &= 0, \\ \operatorname{rot} \vec{E} &= -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}, & \operatorname{div} \vec{E} &= 4\pi \rho, \end{aligned} \tag{2}$$

$$\vec{j} = \sum_k q_k \int \vec{v} f_k(\vec{p}, \vec{r}, t)\, d\vec{p}, \qquad \rho = \sum_k q_k \int f_k(\vec{p}, \vec{r}, t)\, d\vec{p}, \tag{3}$$

where f_k is the particle distribution function of the kind k (either electrons or ions), $\vec{H}$ is the magnetic field, $\vec{E}$ is the electric field, c is the speed of light, v is the electric charge density, w is the electric current density and q_k is the charge of a particle of the kind k.

Equation (1) is the collisionless kinetic Vlasov equation, equations (2) are the system of Maxwell's equations, equations (3) define the current and charge densities by the particle distribution functions. It is assumed that all magnitudes depend on the spatial Cartesian coordinates (x, y), i.e., the two-dimensional non-stationary problem is solved.

For transition to a dimensionless form of the main variables, the following parameters were used: the speed of light c, the electron mass m_e, the time $t = \omega_{\mathrm{pe}}^{-1}$, where ω_{pe} is the plasma electron frequency.

At the initial time in the rectangular box of the domain of solution $x \in [0, l_x]$, $y \in [0, l_y]$, there is plasma consisting of a hydrogen electron and an ion and an electron beam.

The plasma density is set as follows: $n_0 = 10^{15}$ cm^{-3}, $\omega_{\rm pe} = 5.6 \cdot 10^{11}$ s^{-1}, the beam density $n_{\rm b} = 2 \cdot 10^{12}$ cm^{-3}, the plasma electron density $n_{\rm e} = 1 - n_{\rm b}$, the plasma electron temperature $T_{\rm e} = 51$ eV, and the ion temperature $T_{\rm i} = 0$.

The initial distribution of the beam electron velocities is the Maxwell's function:

$$f(v) = \frac{1}{\Delta v \sqrt{2\pi}} \exp\left(-\frac{(v - v_0)^2}{2\Delta v^2}\right),$$

where Δv is the particle velocity dispersion ($T_b = \Delta v^2$), v_0 is the mean velocity, $v_0 = 0.2$. All particles in the computational domain have a uniform distribution.

The boundary conditions are periodic, i.e., $F|_{x=0} = F|_{x=l_x}$, $F|_{y=0} = F|_{y=l_y}$, where F is any of the following quantities $\vec{E}$, $\vec{H}$, f_k, $\vec{j}$, ρ.

The PIC-method [3, 4] is used for solving the Vlasov equation. In this method, plasma is simulated by a set of separate particles, each characterizing the motion of many physical particles. The characteristics of the Vlasov equation describe trajectories of the particle motion. The equations of these characteristics are as follows

$$\frac{d\boldsymbol{p}_{i,e}}{dt} = q_k(\boldsymbol{E} + [\boldsymbol{v}_{i,e}, \boldsymbol{B}]), \quad \frac{d\boldsymbol{r}_{i,e}}{dt} = \boldsymbol{v}_{i,e}, \quad \boldsymbol{p}_{i,e} = \frac{\boldsymbol{v}_{i,e}}{\sqrt{1 - \boldsymbol{v}_{i,e}^2}}.$$

To solve these equations, the lip-frog scheme is used. The scheme for solving the problem in one step is divided into two stages. At the first (Lagrangian) phase, the velocities and coordinates of particles are calculated with the lip-frog scheme. Here, we use components of the current density $j^{m+1/2}$ and the charge density ρ^{m+1}. At the second (Euler) step with the Langdon–Lazinsky scheme [5], Maxwell's equations are solved, i.e., the values of $H^{m+1/2}$ and E^{m+1} grid nodes are determined. The values of the electric and the magnetic fields acting on each particle are calculated using bilinear interpolation. The solution of the basic equations is described in detail in [6].

2. The computer-aided simulation results

Let us consider the results of the solution to the two-stream instability problem. There are two electron beams that occupy all the space of the area. The initial distribution of the electrons is spatially homogeneous and is a superposition of two counter Maxwell's streams:

$$f_0(v) = a_1 \exp\left[-\frac{(\boldsymbol{v} - \boldsymbol{v}_0)^2}{2\sigma_1^2}\right] + a_2 \exp\left[-\frac{(\boldsymbol{v} + \boldsymbol{v}_0)^2}{2\sigma_2^2}\right],$$

where σ_1^2 and σ_2^2 are the dispersions.

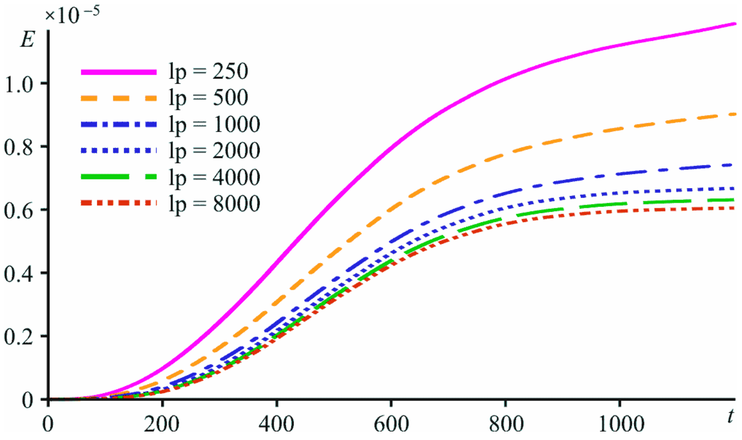

Figure 1

With time, the beams begin to interact and gradually transform to a beam of electrons with Maxwell's distribution function of the particle velocity. The two-stream instability is increasing during of this interaction. Figure 1 shows graphs of the electric field time history for a number of particles in a cell (lp). At a certain time interval, there is an increase in the amplitude of the electric field. The electric field in this case is described by an exponential function. Figure 1 shows that the increasing number of particles in a cell provides the convergence of the solution. Numerical and analytical values of the growth rate of the amplitude of the field were found. A good agreement with available theoretical solutions was obtained.

The evolution of the charge density of an electron beam with the development of plasma turbulence has been studied for this problem. The parameters for calculations are the following: the size of the calculation is $l_x = l_y = 2.5136$, the grid size is 256×256 cells, the number of particles in a cell is 250.

To solve this problem, the efficient scalable parallel algorithm with the mixed Euler–Lagrangian decomposition [7] was developed. We have to use supercomputers because, first, a sufficiently fine grid for reproducing of the resonant interaction of the relativistic electronic beam with plasma, and, secondly, a large number of model particles to simulate the instability arising in the future is required. The calculations were performed on the supercomputer "Lomonosov" (Moscow State University; processors Intel Xeon 5570 2932 MHz, Cache 8 Mb) and on the supercomputer "NCC-30T" (ICM&MG SB RAS; processors Intel Xeon E5540 2530 MHz, Cache 8 Mb).

The algorithm developed makes possible to perform calculations on the grids of 1024×1024 cells, thus the total number of model particles is 5,242,880,000. On the supercomputer "Lomonosov" about 8192 processor cores were used. The program is implemented in Fortran 90 with MPI.

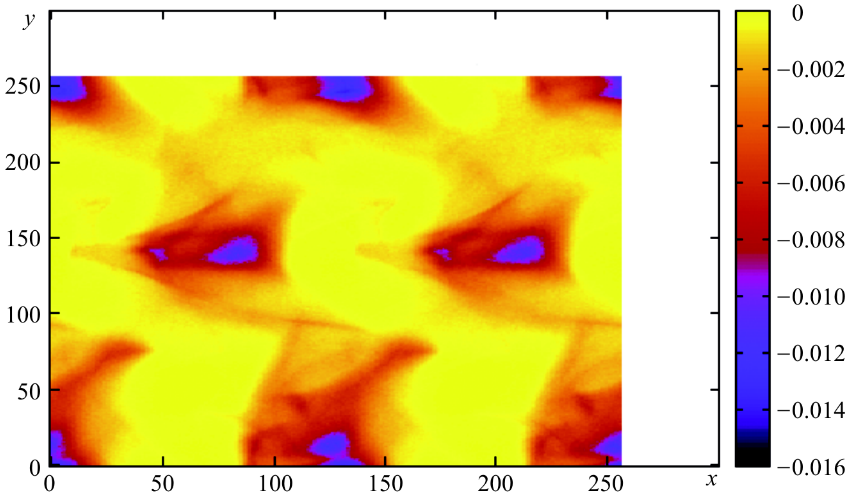

Figure 2. The electron beam charge density

At the initial time, the charge of the beam electrons is uniformly distributed over the area. When interacting with plasma as a result of resonant oscillations, the density modulation areas are formed. This effect is seen from Figure 2, which shows a graph of the charge density at the time $t = 76\,w_{\mathrm{pe}}^{-1}$. As expected, this is due to modulation of the plasma density and the resonant excitation of longitudinal oscillations which suppress the heat conductivity at the end of the facility when injecting a relativistic electron beam (REB).

3. Conclusion

We have developed a model and a program allowing the simulation of the evolution and instability of a warm low-density electron beam in plasma. The behavior of the solution with various calculation parameters was investigated. It was appeared possible to reproduce the modulation effect of the plasma density and its resonant oscillations. The considered problem is quite resource-consuming, however a scalable parallel algorithm proposed provides a uniform loading of cores and allows carrying out complex calculations.

References

[1] Koidan V.S., Arzhannikov A.V., Astrelin V.T., et al. Progress in multimirror trap GOL-3 // Fusion Engineering and Design. — 2005. — Vol. 47, No. 1T. — P. 35–42.

[2] Timofeev I.V., Lotov K.V. Relaxation of a relativistic electron beam in plasma in the trapping regime // Physics of Plasmas. — 2006. — Vol. 13, No. 6.

[3] Birdsall C.K., Langdon A.B. Plasma Physics via Computer Simulation. — Bristol: Hilger, 1991.

[4] Berezin Y.A., Vshivkov V.A. Particles Method in the Dynamics of a Rarefied Plasma. — Novosibirsk: Nauka, 1980.

[5] Langdon A.B., Lasinski B.F. Electromagnetic and relativistic plasma simulation models // Meth. Comput. Phys. — 1976. — Vol. 16. — P. 327–366.

[6] Efimova A.A. Simulation of an electron beam and plasma interaction. // Bull. Novosibirsk Comp. Center. Ser. Numerical Analysis. — Novosibirsk, 2009. — Iss. 14. — P. 1–12.

[7] Berendeev E.A., Efimova A.A. Effective parallel calculations realization for modeling big problems of the plasma physics by the PIC-method // Vestnik UGATU. — Ufa, 2013. — Vol. 17, No. 2 (55). — P. 112–116.