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Reduction of coloured Petri nets based on resource bisimulation

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A pair consisting of a place and a token in a coloured Petri net is considered as an elementary resource for this net, and a resource is a multiset of elementary resources. Two resources are bisimilar, if replacement of one by another in any marking doesn't change the net behaviour. Due to this fact, bisimilar resources can be merged. The paper presents an algorithm for computing resource bisimulation for coloured Petri nets and describes some ways of net reduction based on merging bisimilar resources.

1. Introduction

Coloured Petri nets (CPN) [2] is a class of high-level Petri nets, widely used for modelling and analysis of concurrent and distributed systems. In this paper CPN are studied with respect to bisimulation equivalence.

A notion of bisimulation equivalence has been introduced by R. Milner and D. Park. It captures an observable behaviour of a system. As a rule, bisimulation equivalence is a relation on sets of states. Two states are bisimilar, if they are undistinguishable modulo system behaviour. For ordinary Petri nets, the state (marking) bisimulation is undecidable [3]. To overcome this, a weaker place bisimulation has been introduced for ordinary Petri nets in [1]. A place bisimulation is a relation on sets of places. Two places are bisimilar, if replacement of a token in one place by a token in another place in all markings doesn't change the system behaviour. Hence, bisimilar places can be merged without changing the behaviour of a net.

In this paper a similar approach is developed for high-level CPN. Since in CPN a colour of a token must be taken into account, we consider not places but elementary resources — pairs of places and coloured tokens. A resource is a multiset of elementary resources. For CPN a resource bisimulation is defined. When elementary resources are considered, it corresponds to place bisimulation for ordinary Petri nets. Otherwise it gives a generalization of place bisimulation for ordinary Petri nets to multisets of places, which allows us to obtain additional net reductions.

We describe a simple algorithm of computing an approximation of the largest bisimulation.

The paper is organized as follows. In Section 2 we recall basic definitions and notations on CPN and bisimulations and give definitions of an elementary resource and a resource for CPN. In Section 3 resource bisimulations are studied and an algorithm for computing the maximal bisimulation of finite resources is presented. Section 4 contains an algorithm for CPN reductions based on resource bisimulations and some examples.

2. Basic definitions

A multiset M over a non-empty set X is a function $M : X \to Nat$ where Nat is a set of non-negative integers. The non-negative integers $\{M(x) \mid x \in X\}$ are the coefficients of the multiset. As usually, we define

$(M_1 + M_2)(x) = M_1(x) + M_2(x)$,
$(M_1 - M_2)(x) = M_1(x) - M_2(x)$ if $M_1(x) > M_2(x)$ and
$(M_1 - M_2)(x) = 0$ if $M_1(x) \leq M_2(x)$,
$(M_1 \cup M_2)(x) = max(M_1(x), M_2(x))$,
$(M_1 \cap M_2)(x) = min(M_1(x), M_2(x))$.

The power of a multiset M over X is defined as $|M| = \sum_{x \in X} M(x)$. By X_{MS} we denote the set of all multisets over X.

Let us suppose $\mathcal{L}$ to be a language of typed expressions and $\mathcal{U}$ its finite model. Expressions in $\mathcal{L}$ are built from variables and constants using the only operation of a multiset addition. Elements of $\mathcal{U}$ are coloured tokens. A type is defined as a set of colours, and the type of the sum of two multisets — as the union of their types. The type of an element e is denoted by $Type(e)$, the type of an expression $\theta \in \mathcal{L}$ — by $Type(\theta)$. $Var(b)$ denotes the set of all variables of the expression b. A binding of variables in an expression is defined in the usual way.

A labelled (unmarked) coloured Petri net (CPN) is a tuple $\mathcal{N} = (\Omega, N, Type, W, A, l)$, where Ω is a finite nonempty set of types; $N = (P, T, F)$ is a net, where P are places, T — transitions, F — a flow relation; $Type : P \to \Omega$ is a type function; $W : F \to \mathcal{L}$ is an arc expression function, where for all $p \in P,\ t, u \in T$ s.t. $(t,p), (p,u) \in F$ we have $Type(W(t,p)) = Type(W(p,u)) = Type(p)$; A is an alphabet of labels; $l : T \to A$ is a labelling function for transitions.

A marking of a net $\mathcal{N}$ is a function $M : P \to \mathcal{U}_{MS}$ s.t. $Type(M(p)) = Type(p)$. It puts a multiset of tokens of the appropriate type in every place. A marked CPN is a pair $(\mathcal{N}, M_0)$ of a net and its initial marking.

For a transition $t \in T$ $\quad Var(t)$ denotes a set of all variables in an arc expression adjacent to t.

A binding of a transition t is a function b defined on $Var(t)$, s.t. $\forall v \in Var(t) \quad b(v) \in Type(v)$. A binded transition $t[b]$ is enabled in a marking M if $\forall p \in P\ W(p,t)[b] \subseteq M(p)$.

If $t[b]$ is enabled in M, it may fire changing M to another marking M', s.t. $\forall p \in P,\ M'(p) = M(p) - W(p,t)[b] + W(t,p)[b]$ (written a $M \xrightarrow{t[b]} M'$).

$Y(t)$ denotes a set of all possible bindings of $t \in T$. $\mathcal{T}(\mathcal{N}) = \{t[b] \mid t \in T, b \in Y(t)\}$ is a set of all binded transitions of $\mathcal{N}$.

An *elementary resource* is a pair $(p,d) \in P \times \mathcal{U}$, where $d \in Type(p)$, i.e. it's a place with one coloured token. A *resource* is a multiset of elementary resources.

Recall that a marking maps each place to a multiset of coloured tokens and can be considered as a set of pairs of the form (a place, a multiset of coloured tokens). It is easy to see that actually resources and markings are the same mathematical objects represented in a slightly different form or encoding. In this sense, every marking is a resource, and every resource is a marking. However, we distinguish between these notions, because we give them different substantative interpretations. We mean that a resource represents a part of markings which provides this or that kind of the net behaviour.

We denote the set of all resources of CPN $\mathcal{N}$ by $\mathbf{M}(\mathcal{N})$, the set of all its elementary resources by $\mathbf{M}_1(\mathcal{N})$. We define a precondition of $t[b]$ to be a resource ${}^\circ t[b] = \sum_{p \in P} W(p,t)[b]$, and a postcondition of $t[b]$ to be a resource $t[b]^\circ = \sum_{p \in P} W(t,p)[b]$.

We say that a symmetric relation R on the set of markings of CPN $\mathcal{N}$ satisfies the *transfer property* iff for all $(M_1, M_2) \in R$, for every step $M_1 \xrightarrow{t[b]} M_1'$ there exists an imitating step $M_2 \xrightarrow{u[c]} M_2'$ with $(M_1', M_2') \in R$ and $l(t) = l(u)$.

A symmetric relation R on the set of markings of CPN $\mathcal{N}$ which satisfies the transfer property is called a (marking) bisimulation for $\mathcal{N}$ (written as $R : \mathcal{N} \underline{\leftrightarrow} \mathcal{N}$).

3. Resource bisimulation

Given a relation $B \subseteq \mathbf{M}(\mathcal{N}) \times \mathbf{M}(\mathcal{N})$, we define the relation $\overline{B} \subseteq \mathbf{M}(\mathcal{N}) \times \mathbf{M}(\mathcal{N})$ by

$$(D_1, G_1), \ldots, (D_n, G_n) \in B \ \Rightarrow\ (D_1 + \ldots + D_n, G_1 + \ldots + G_n) \in \overline{B}.$$

So, two markings are related by $\overline{B}$ if their tokens can be partitioned into pairs satisfying B.

A relation $B \subseteq \mathbf{M}(\mathcal{N}) \times \mathbf{M}(\mathcal{N})$ is called *a resource bisimulation over* $\mathcal{N}$ iff $\overline{B}$ is a marking bisimulation (written as $B : \mathcal{N} \approx \mathcal{N}$).

A resource bisimulation detects the possibility of replacement of one multiset of tokens by another in all CPN markings, so that this replacement doesn't influence the net behaviour. The resource bisimulation equivalence is stronger than the marking bisimulation. Bisimilarity of two resources implies bisimilarity of them as markings, but the converse is not true.

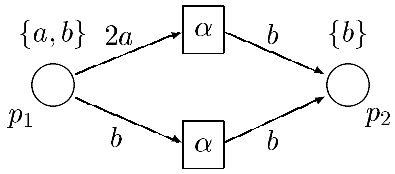


Figure 1. An example of bisimilar resources ($(p_1, b) \approx 2 \cdot (p_1, a)$)

Figure 1 represents two bisimilar resources. The first contains one elementary resource (p_1, b), and the second contains two copies of the same elementary resource (p_1, a).

Theorem 1. *Given a CPN $\mathcal{N}$, there exists the maximal resource bisimulation over $\mathcal{N}$.*

The proof is straightforward. The sum of two bisimulations is also a bisimulation. Therefore, the sum of all bisimulations is the maximal bisimulation.

Now we define an analog of the weak transfer property ([1]) for CPN resource bisimulation. We say that a relation $B \subseteq \mathbf{M}(\mathcal{N}) \times \mathbf{M}(\mathcal{N})$ satisfies the *weak transfer property* iff, for all $(D, G) \in B$ and for all $t[b] \in \mathcal{T}(\mathcal{N})$, s.t. $D \cap {}^\circ t[b] \neq \emptyset$, there exists a binded transition $u[c] \in \mathcal{T}(\mathcal{N})$, s.t. $l(t) = l(u)$ and, writing M_1 for ${}^\circ t[b] \cup D$ and M_2 for ${}^\circ t[b] - D + G$, we have $M_1 \xrightarrow{t[b]} {M_1}'$ and $M_2 \xrightarrow{u[c]} {M_2}'$ with $({M_1}', {M_2}') \in \overline{B}$.

Theorem 2. *A reflexive and symmetric relation B satisfies the weak transfer property iff B is a reflexive and symmetric resource bisimulation.*

Proof. ($\Rightarrow$) Assume the converse. Let B be not a resource bisimulation. Then $\overline{B}$ is not a marking bisimulation, so it doesn't satisfy the transfer property. Therefore $\exists (M_1, M_2) \in \overline{B}$, $t[b] \in \mathcal{T}(\mathcal{N})$, s.t. $M_1 \xrightarrow{t[b]} {M_1}'$ cannot be imitated starting from the marking $M_1 - {}^\circ t[b] + t[b]^\circ$.

However, M_1 and M_2 can be decomposed into pairs, belonging to B:

$$M_1 = D_1 + \ldots + D_n,\ M_2 = G_1 + \ldots + G_n, \text{ where } (D_i, G_i) \in B.$$

Here the pair (D_1, G_1) satisfies the weak transfer property. It means that there exists a transition $u_1[c_1]$, s.t. $l(u_1) = l(t)$ and ${}^\circ u_1[c_1] \subseteq F_1 = M_1 - D_1 + G_1$, where markings $F_1 - {}^\circ u_1[c_1] + u_1[c_1]^\circ$ and $M_1 - {}^\circ t[b] + t[b]^\circ$ are bisimilar w.r.t. $\overline{B}$. If $D_1 \cap {}^\circ t[b] = \emptyset$, we can choose $u_1[c_1] = t[b]$.

Similarly, there exists a binded transition $u_2[c_2]$ imitating $u_1[c_1]$ (and, therefore, $t[b]$) with a precondition contained in $F_2 = F_1 - D_2 + G_2 = M_1 - D_1 + G_1 - D_2 + G_2$ and a postcondition bisimilar to $F_1 - {}^\circ u_1[c_1] + u_1[c_1]^\circ$.

Repeating this reasoning for the n-th time, we obtain a binded transition $u_n[c_n]$ with the precondition ${}^\circ u_n[c_n] \subseteq F_n = F_{n-1} - D_n + G_n$.

It is clear that $F_n = M_1 - D_1 + G_1 - \ldots - D_n + G_n = (D_1 + \ldots + D_n) - D_1 + G_1 - \ldots - D_n + G_n = M_2$.

Since the relation $\overline{B}$ is transitive, markings $F_n - {}^\circ u_n[c_n] + u_n[c_n]^\circ$ and $M_1 - {}^\circ t[b] + t[b]^\circ$ are bisimilar w.r.t. $\overline{B}$.

So, we've got a binded transition $u_n[c_n]$ with the same (as for $t[b]$) label, transforming M_2 to some bisimilar to M_1' marking. Hence $u_n[c_n]$ imitates $t[b]$. This contradicts our assumption.

($\Leftarrow$) It follows from the fact that the weak transfer property is a special kind of the transfer property restricted to pairs (M_1, M_2) and steps $M_1 \xrightarrow{t[b]} {M_1}'$, where M_1 and M_2 differ only in one resource. $\square$

Corollary. *The maximal resource bisimulation coincides with the maximal reflexive and symmetric relation B satisfying the weak transfer property.*

Note that for a net with at least one place its maximal resource bisimulation is infinite. The point is that, if a net contains at least one elementary resource r, then an infinite relation $B = Id(\mathbf{M}_1(\mathcal{N})) \cup \{(2\cdot r, 2\cdot r), (3\cdot r, 3\cdot r), \ldots\}$ is a resource bisimulation. Since the maximal bisimulation contains all others, it is also infinite.

By Res_p we denote the set of all resources R, s.t. $\mid R \mid\leq p$, i.e. containing no more than p elementary resources (taking copies into account).

Theorem 3. *Given a CPN $\mathcal{N}$ and a positive integer p, there exists the maximal resource bisimulation on Res_p (written as $B(\mathcal{N}, p)$).*

The proof is straightforward.

Since Res_p is finite, we can use the weak transfer property to compute $B(\mathcal{N}, p)$.

An algorithm for computing $B(\mathcal{N}, p)$**.**

input: *a labelled unmarked CPN $\mathcal{N}$, a positive integer p*

output: *the relation $B(\mathcal{N}, p)$*

step 1: *Set $B = Res_p \times Res_p$*

step 2: *Check whether B satisfies the weak transfer property:*

- *If it's true then B is $B(\mathcal{N}, p)$.*
- *Otherwise, there is a $t[b] \in \mathcal{T}(\mathcal{N})$, $D, G \in Res_p$ with $D \cap {}^{\circ}t[b] \neq \emptyset$ and $(D, G) \in B$, s.t. $t[b]$ cannot be imitated by ${}^{\circ}t[b] - D + G$. Then remove pairs (D, G) and (G, D) from B and go back to step 2.*

Since Res_p is finite, the algorithm makes a limited number of steps. The output is the maximal relation, because no element of $B(\mathcal{N}, p)$ can be removed from B (since $B \subseteq B(\mathcal{N}, p)$, these elements always satisfy the weak transfer property in B). The time complexity of the algorithm is $O(S^2 * |\mathcal{T}(\mathcal{N})|^2 * |\mathbf{M}_1(\mathcal{N})|^{2*p})$, where $S = \max\limits_{t\in T}\{|{}^{\bullet}t|, |t^{\bullet}|\}$.

Replacing Res_p by $\mathbf{M}_1(\mathcal{N})$, we obtain the algorithm for computing the maximal elementary resource bisimulation. Elementary resource bisimulations in CPN correspond to place bisimulations for ordinary Petri nets [1] (where all tokens are of the same colour).

By applying the resource bisimulation to ordinary Petri nets, we obtain *a bisimulation of multisets of places.* For example, this allows us to determine that two tokens in one place are equivalent to three tokens in another place (for all markings). This equivalence is weaker than place bisimulation and in some cases allows us to derive additional reductions.

It is easy to show that the linear combination of resource bisimulations (with respect to the operation of multiset addition) is also a resource bisimulation. So, a finite set of resource bisimulations generates infinite resource bisimulations. The question whether for a given CPN there exists a finite basis of finite resource bisimulations generating all resource bisimulations is a subject of further investigations.

4. Reduction

The resource bisimulation can be used for CPN reduction, since bisimilar resources can be merged. However, this "merging" must be more subtle than place merging for ordinary Petri nets. Bisimilar resources may have different size and structure. Moreover, they may intersect. Therefore we can't just "merge" two resources, but are to replace one of them by another. Since we want to reduce the net, we'd like this replacement to decrease the number of elementary resources. For example, if we have two bisimilar resources D and G, where $G \subset D$, it makes sense to replace D by G, but not vice versa.

It is not always possible to replace one resource by another. We give a sufficient condition for that.

Let D and G be two bisimilar resources, and M_0 be an initial marking. A resource D can be replaced by a resource G, if

1. $\forall t[b] \quad (D \subseteq {}^\circ t[b] \ \vee\ D \cap {}^\circ t[b] = \emptyset) \quad \wedge \quad (D \subseteq t[b]^\circ \ \vee\ D \cap t[b]^\circ = \emptyset)$,
2. $M_0 = k * D + X$, where $k \in Nat$ and $X \cap D = \emptyset$.

The first condition is a rather strong restriction. It means that the resource D must be indivisible. For any transition, it either transfers all tokens in D (probably, several instances of D), or none of them.

The second condition guarantees that the resource D is entirely contained in the initial marking. This is necessary, because each of its instances must be replaced by an instance of G.

It is obvious that elementary resources satisfy both conditions.

Suppose D and M_0 satisfy conditions 1 and 2. Then the following transformation of the net will not change its behaviour (modulo bisimulation):

Resource replacement algorithm:

input: *a labelled marked CPN* $(\mathcal{N}, M_0)$

output: *the labelled marked CPN* $(\mathcal{N}', M_0')$ *with bisimilar behaviour*

step 1: *Adding imitating transitions.*

For all $t[b] \in \mathcal{T}(\mathcal{N})$ *s.t.* $D \subseteq {}^\circ t[b] \vee D \subseteq t[b]^\circ$ *add a new binded transition* $t'[b']$ *s.t.*

1. if ${}^\circ t[b] = m * D + Pre$ *with* $D \cap Pre = \emptyset$*, then* ${}^\circ t'[b'] = m * G + Pre$,

2. if $t[b]^\circ = n * D + Post$ *with* $D \cap Post = \emptyset$*, then* $t'[b']^\circ = n * G + Post$,

3. $l(t') = l(t)$.

step 2: *Removing* D*.*

Delete from the net all elementary resources contained in D *and replace types of places by new types* $Type'(p) = Type(p) \setminus Type(D)$.

Remove all arcs, corresponding to binded transitions $t[b] \in \mathcal{T}(\mathcal{N})$*, s.t.* $D \subseteq {}^\circ t[b] \vee D \subseteq t[b]^\circ$ *(note that transitions imitating them were added at step 1).*

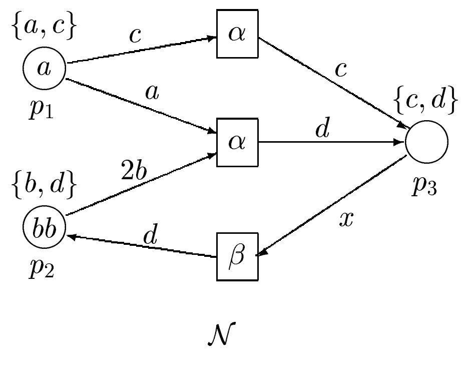
step 3: *Changing the initial marking.*

If $M_0 = k * D + X$*, where* $k \in Nat$ *and* $X \cap D = \emptyset$*, then* $M_0' = k * G + X$.

Note that this algorithm is nondeterministic. Since we deal with high-level net, there are many possible ways to add a binded transition (as well as there are many possible ways to build a high-level Petri net equivalent to the given ordinary Petri net). Choosing the best approach is a nontrivial problem.

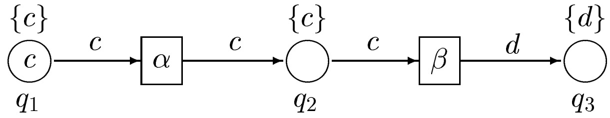
For example, adding a separate transition with constants on adjacent arcs and, hence, with the only possible binding seems to be the simplest solution. But for some nets this reduction would not be the best.

Now we give a small example of a CPN reduction.



$$B = Id(\mathbf{M}_1(\mathcal{N})) \cup \{\ ((p_3, c), (p_3, d)),\ \ ((p_1, a) + 2\cdot(p_2, b), (p_1, c))\ \}$$

Figure 2a. An example of a CPN reduction — a net and some resource bisimulation



$\mathcal{N}'$

$$(q_1, c) = \{(p_1, c),\ (p_1, a) + 2{\cdot}(p_2, b)\}$$
$$(q_2, c) = \{(p_3, c),\ (p_3, d)\}$$
$$(q_3, d) = \{(p_1, d)\}$$

Figure 2b. An example of a CPN reduction — the result

Thus, in the net reductions elementary resources in CPN are similar to places in ordinary Petri nets. Hence the complexity of reduction algorithms depends on the size of $\mathbf{M}_1(\mathcal{N})$. Our algorithm reduces the set of elementary resources, but in some cases it can add new transitions.

5. Conclusion

Resource bisimulation is a generalization of the ordinary Petri net place bisimulation for the case of high-level CPN. It allows us to compute equivalences weaker than marking bisimulation. These equivalences can be used for simplifying reductions of CPN.

This approach can be easily applied to other classes of high-level Petri nets, such as predicate/transition nets, or algebraic nets.

Directions for further research could be considered, such as whether it is possible to represent the maximal resource bisimulation by a finite basis (and to compute this basis). Also it is interesting to apply our approach to restricted sets of "relevant" markings introduced in [4].

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