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Simulation of complex phenomena by Cellular Automata composition*

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Abstract. Methods of Cellular Automata (CA) composition are systematically considered and analyzed. To formally define them some basic mathematical operations on cellular arrays are introduced. Trivial sequential and parallel composition types with no shared variables in cell transition functions are given in brief. The main concern is with two basic methods of composing CA with shared variables. For each method an example is given and the domain of application is determined.

1. Introduction

The problem of constructing a CA, whose evolution simulates the spatial dynamics of a complex phenomenon, which is given by some kind of a qualitative or a quantitative description, is not completely solved. Meanwhile, by the present time, a large number of CA-models of natural processes are proposed and well studied [1]. The best known and practically used are CA models of diffusion, gas–lattice, phase–separation, snow flakes formation, stripes formation, etc. In order that a CA be obtained, which simulates a complex process, such as reaction–diffusion, prey–predatory, snow–flakes formation in an active medium, etc., it seems reasonable to use a composition of CA models of its simple components. This idea has been already used in a number of methods, the best being intended for reaction-diffusion simulation [2, 3].

A similar problem in mathematical physics also exists. There is a set of typical functions and differential operators, simulating convection, diffusion, reaction, wave propagation etc., which are composed to represent complex processes. The same strategy may be used to form a composed CA of a number of simple ones. Of course, to approach the problem, a number of mathematical operations in the CA domain are needed. Some of them were introduced in [2, 4] for combining diffusion and reaction component when constructing a diffusion CA with a nonlinear reaction function. As for a systematic approach to the CA composition, to our knowledge, it has not been developed by now. Hence, it is not clear in what applications it may be fruitful, if at all it is worth to be studied.

The paper does not pretend to give an exhaustive answer to the above questions. The problem is only formally stated, types of the CA composition

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are classified and their capabilities are considered. Taking into account a poor predictability of the CA behavior, the first priority in the investigation has been given to the study of experimental case of the compositions. To attach much importance to practical use of the composition, an extended concept of CA is considered. This means that all kinds of alphabets and CA transition function are allowed, the main features of classical CA (cell interaction locality and fine-grained parallelism) being preserved. To capture all the above extensions, the Parallel Substitution Algorithm formalisms [5] are used.

Apart from Introduction and Conclusion there are three sections. The second contains definitions of the concepts under consideration. The third describes two trivial compositions with independent component evolution: sequential and parallel. In the third section two composition types with shared variables are (unidirectional and bidirectional) introduced and considered.

2. Formal statement of the problem

Cellular Automata are intended for processing spatially distributed functions, represented by *cellular arrays*, which are finite sets of pairs (u, m) called *cells*:

$$\Omega = \{(u, m) : u \in A, m \in M\}, \tag{1}$$

u being a *cell state* from the *alphabet* A, m – a *cell name* from a discrete *naming set* M. Further, a naming set is used, whose elements are integer vectors, representing coordinates of a Cartesian space of finite size. For example, in the 2D case, $M = \{(i, j) : i, j = 0, 1, \ldots, N\}$. A notion m instead of (i, j) is used for making general expressions shorter and to indicate that they are valid for any other kind of a naming set. No constraint but finiteness is imposed on A, the symbols of A are allowed to be Boolean, real or integers. To indicate to the state of a cell, named m, a notation $u(m)$ or u_m is used.

A mapping $\phi : M \to M$, called a *naming function*, is defined on M. It determines a neighboring cell location of any cell named m. It is conditioned that $\phi_0(m) = m$. In the naming set $M = \{(i, j)\}$, naming functions are usually given in the form of shifts $\phi_k = (i + a, j + b)$, a, b being integers not exceeding a fixed r, called a *radius of neighborhood*.

On cellular arrays, some mathematical operations are defined [4]. When all operands have the state alphabet in the domain of real numbers, the following ordinary cell-by-cell *arithmetical* rules may be used. Let the sign "$\diamond$" represent $+, -$ or $\times$, then

$$\begin{array}{lll} \Omega = \Omega_1 \diamond \Omega_2 : & (u, m) = (u_1 \diamond u_2, m) & \forall m \in M, \\ \Omega = c\Omega_1 : & (u, m) = (cu_1, m), & \forall m \in M. \end{array} \tag{2}$$

In the class of Boolean arrays, the *logical* cell-by-cell operations AND, OR and NOT are defined in the same way.

When operands have different alphabet types, *hybrid* cellular array operations are defined based on the requirement that the above arithmetic rules be valid for the averaged form $Av(\Omega)$ of the Boolean array The latter has the cell states $\langle v(m)\rangle$ in the real interval $(0,1) \subset \boldsymbol{R}$, being equal to mean values of the cell states in the averaging area $Av(m) \in \Omega$,

$$Av(m) = \{(v_0, \psi_0(m)), (v_1, \psi_1(m)), \ldots, (v_q, \psi_q(m))\}, \tag{3}$$

and

$$\langle v(m)\rangle = \frac{1}{q}\sum_{k=0}^{q} v_k, \qquad v_k \in \{0,1\}, \quad \langle v(m)\rangle \in (0,1). \tag{4}$$

Let $\Omega_1 = \{(u_m, m)\}$ and $\Omega_2 = \{(v_m, m)\}$ have alphabets of different types, i.e., $u_{ij} \in (0,1)$, $v_m \in \{0,1\}$, then their addition, resulting in the Boolean array $\Omega = \{(v'_{ij}, (i,j)\}$ is as follows [3, 4]:

$$\Omega = \Omega_1 \oplus \Omega_2 :$$

$$v'_m = \begin{cases} \bar{v}_m & \text{with probability } \pi_1(m) \quad \text{if } (u_m > 0 \ \&\ v_m = 0), \\ \bar{v}_m & \text{with probability } \pi_2(m) \quad \text{if } (u_m < 0 \ \&\ v_m = 1), \\ v_{ij} & \text{otherwise,} \end{cases} \tag{5}$$

where $\bar{v}$ is the logical inversion of v, the probabilities being computed as follows:

$$\pi_1(m) = \frac{u_m}{1 - \langle v_m\rangle}, \quad \pi_2(m) = \frac{|u_m|}{\langle v_m\rangle}. \tag{6}$$

The local interaction between cells in a cellular array is determined by a *weighted template*

$$Q(m) = \{(w_0, \phi_0(m)), (w_1, \phi_1(m)), \ldots, (w_n, \phi_n(m))\}. \tag{7}$$

The weight values $w_k \in \boldsymbol{R}$ in (7) form a *weight vector* $W = (w_0, \ldots, w_n)$, the naming functions – a naming vector $\Gamma = (\phi_0(m), \phi_1(m)), \ldots, \phi_n(m))$. When for all $k = 0, 1, \ldots, n$, $w_k = 1$, a template is called *unweighted*.

A naming vector puts into correspondence to each name $m \in M$ a subset of cells

$$S(m) = \{(u_0, \phi_0(m)), (u_1, \phi_1(m)), \ldots, (u_n, \phi_n(m))\}. \tag{8}$$

which is called a *local configuration*. Its cell state values form a *state vector* of the local configuration $U(S(m)) = (u_0, \ldots, u_n)$.

Each cell of Ω acts as a finite automaton with a transition rule, which is represented as a set $\Phi = \{\Phi_1, \ldots, \Phi_n\}$ of *parallel substitutions* [5]:

$$\Phi_k:\ S(m) \to S'(m), \quad \Gamma'(m) \subseteq \Gamma(m), \quad k = 0, \ldots, n, \tag{9}$$

where $\Gamma(m)$, $\Gamma'(m)$ are underlying naming templates for $S(m)$, $S'(m)$, respectively, and $S'(m)$ is the next state local configuration, whose cell states are the results of a *cell transition function* application

$$u_m(t+1) = f_k(W(m), U(m)). \tag{10}$$

There are two modes of operation of CA. *Synchronous* mode, when all cells compute their next states in parallel and transit to the next state at once on time steps $t = 0, 1, \ldots$, changing the global cellular array state, and *asynchronous* mode, when the cells execute transitions sequentially in a certain order. In any case, the transition to the next global state, $\Omega(t+1) = \Phi(\Omega(t))$, is considered to be *an iteration*. The sequence of cellular arrays,

$$\Omega(0), \Omega(1), \ldots, \Omega(t), \ldots, \Omega(T),$$

obtained by iterative application of Φ to the initial array $\Omega(0)$ is called *evolution*. The time T is a termination step. It has a finite value if a CA comes to its stable state. Otherwise, the evolution should be stopped by an external signal.

With the above notions, a CA is defined as a pair $\Theta = (\Omega, \Phi)$ together with indication to of the mode of operation. When CA functioning under certain initial conditions is of interest, the initial cellular array $\Omega(0)$ should also be given, as well as the termination condition.

3. Trivial CA composition methods

3.1. Sequential trivial composition. The common functioning of two CA is considered to be a trivial composition if their transition rules use disjoint sets of variables. So, they evolve independently, interacting only at the start and the termination.

Two CA $\Theta_1 = \langle \Omega_1, \Phi_1 \rangle$ and $\Theta_2 = \langle \Omega_1, \Phi_1 \rangle$ are said to form a *trivial sequential composition* $\Theta = \langle \Omega, \Phi \rangle$ if the following condition holds:

$$\begin{array}{ll} \Omega(0) = \Omega_1(0), & \Omega(T_1 + T_2) = \Omega_2(T_2), \\ \Omega_1(T_1) \subseteq \Omega_2(0), & M_1 \subseteq M_2. \end{array} \tag{11}$$

It should be noted that time steps of the CA are allowed to be different, and the second CA starts only after the first one terminates. The composition is useful, especially, in pipelined systems executing fine-grained parallel cellular computation or image processing.

A special case of trivial sequential composition is its periodic functioning: when the resulting state of Θ_2 serves as the initial state to Θ_1 and both CA work in cycle. The reaction–diffusion CA, based on composing CA-diffusion with reaction function [2, 3], represent this kind of composition with $T_1 = T_2 = 1$.

Example 1. Two CA $\Theta_1 = \langle \Omega_1, \Phi_1 \rangle$, and $\Theta_2 = \langle \Omega_2, \Phi_2 \rangle$ form a sequential composition, both having Boolean alphabet and identical naming sets $M_1 = M_2 = \{(i,j),\ i,j = 0,1,2,\ldots,200\}$. The initial cellular array $\Omega_1(0)$ has two vertical belts on the borders of the array with $u_1 = 1$, all other cells being in zero state.

The CA Θ_1 performs $T_1 = 100$ iterations according to the CA diffusion-transition rules proposed in [6], which are as follows. At each even iteration $(t = 0,2,4,\ldots)$, for the local configurations

$$S(i,j) = \{(a,(i,j)),(b,(i+1,j)),(c,i+1,j+1)),(d,(i,j+1))\}, \tag{12}$$

where $a,b \in \{0,1\}$, and both i and j even, are replaced by

$$\begin{aligned} S'_1(i,j) &= \{(b,(i,j)),(c,(i+1,j)),(d,i+1,j+1)),(a,(i,j+1))\} \quad \text{or} \\ S'_2(i,j) &= \{(d,(i,j)),(a,(i+1,j)),(b,i+1,j+1)),(c,(i,j+1))\} \end{aligned} \tag{13}$$

with probabilities p or $1-p$, respectively. At each odd iteration $(t = 1,3,\ldots)$ the same is done relative to the local configurations (i,j), whose both coordinates are odd. In other words, even and odd blocks of cells are in turn replaced by similar blocks but rotating clockwise or counterclockwise according to a given probability, whose value determines the diffusion coefficient D. With $p = 0.5$, $D = 1.5$ [7], any other D may be obtained by changing p.

The CA Θ_2, is a totalistic CA [1], which evolves to a stable pattern. It has a weighted template

$$Q(i,j) = \{(1,(i+k,j+l))\} \cup \{(-0.2,(i+g,j+h))\},$$

where $k,l = -1,0,1,\ g,h = -3,-2,2,3$. Its cell transition function is as follows:

$$u_{ij}(t+1) = \begin{cases} 1 & \text{if} \quad U_{ij} \times W_{ij} > 0, \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

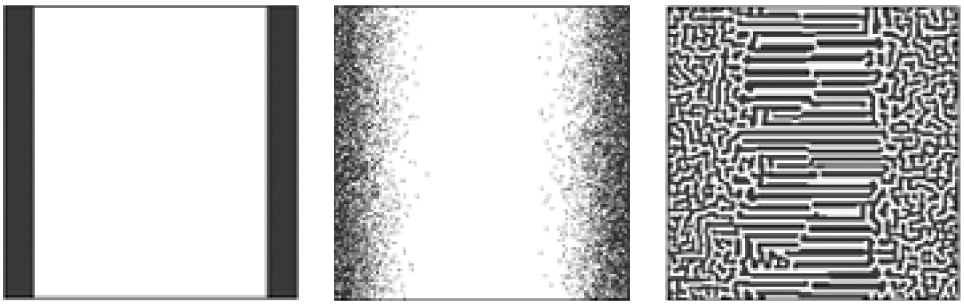
In Figure 1, three snapshots of a composed CA are shown: $\Omega(0)$, $\Omega(100)$, and $\Omega(115)$, the latter being a stable pattern.

3.2. Parallel trivial CA composition. Two CA $\Theta_1 = \langle \Omega_1, \Phi_1 \rangle$, $\Omega_1 = \{(u,m)\}$, and $\Theta_2 = \langle \Omega_2, \Phi_2 \rangle$, $\Omega_2 = \{(v,m)\}$, are said to form a *trivial parallel composition* $\Theta = \langle \Omega, \Phi \rangle$ if the following conditions are satisfied:

$$M_1 = M_2, \quad \Omega(0) = \Omega_1(0) = \Omega_2(0), \quad \Omega(T) = \Omega_1(T_1) \diamond \Omega_2(T_2). \tag{15}$$

In (15), "$\diamond$" is any cellular array rule allowed on the results of parallel operands.

In this composition, the CA may have different alphabets and different timings, hence, they need a synchronization signal to start $\diamond$ operation.



a b c

Figure 1. Sequential composition of two CA: a) initial CA $\Omega(0)$, b) the intermediate result of $\Omega(100)$, c) the composed CA result $\Omega(115)$

Example 2. A separation phase process is simulated by two different CA independently in parallel and the resulting cellular arrays are then compared by taking unsigned arithmetic difference of the final cellular arrays.

Here Θ_1 is totalistic CA with Boolean alphabet, the square local configuration

$$S(i,j) = \{(u_{i+k,j+l}, (i+k, j+l) : k, l = -3, \ldots, 0, \ldots, 3\},$$

an unweighted template, and the following cell transition function:

$$u_{ij}(t+1) = \begin{cases} 1 & \text{if} \quad s > 25 \quad \text{or} \quad s = 24, \\ 0 & \text{otherwise}, \end{cases} \tag{16}$$

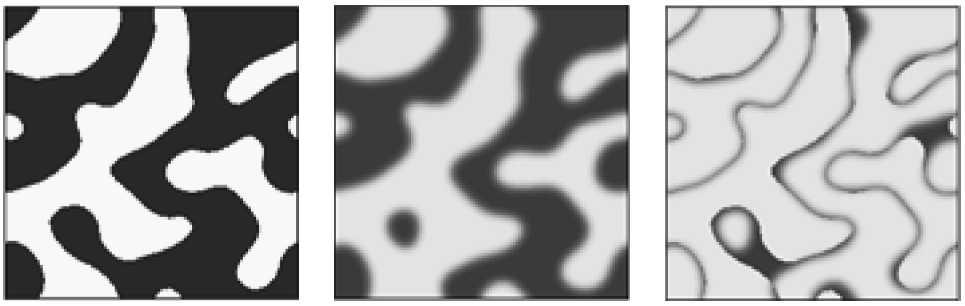
where $s = \sum_k \sum_l u_{i+k,j+l}$. The initial state $\Omega_1(0)$ is a random distribution of "ones" with density $\rho = 0,5$.

A CA Θ_2 is a finite difference explicit solution to the PDE proposed in [8], cited in [2]:

$$\frac{\partial v}{\partial t} = 0.2\frac{\partial^2 v}{\partial x^2} - 0.2(v-0.1)(v-0.5)(v-0.9). \tag{17}$$

The initial state $\Omega_2(0)$ is taken equal to the averaged form of $\Omega_1(0)$ with the averaging area of radius $r = 8$. So, $v_{ij}(0) = \rho + \Delta v$, where Δv is a deviation from the averaged value due to the stochastic character of the initial random distribution, needed for having small bifurcation.

Both CA evolve to their steady states (Figures 2a and 2b). The difference in the results (Figure 2c) shows the borders of the pattern and narrow bridges.



a b c

Figure 2. Parallel composition of two different CA, simulating the phase separation process: a) by a Boolean CA (16), b) by a finite difference solution of a PDE (17), and c) the difference in the result

4. The CA composition with shared variables

4.1. Unidirectional composition. A CA $\Theta = \langle \Omega, \Phi \rangle$ is *unidirectional* composition of $\Theta_1 = \langle \Omega_1, \Phi_1 \rangle$ and $\Theta_2 = \langle \Omega_2, \Phi_2 \rangle$ if transition rules of Θ_2 use the state variables of Θ_1, while the latter evolve independently. So, the CA Θ_1 plays the role of a context for Θ_2 and is referred to as *context CA*, Θ_2 being called *basic CA*.

To formally define the unidirectional composition, let A_1, M_1 and A_2, M_2 characterize the cellular arrays of Ω_1 and Ω_2, and f_1 and f_2 be cell transition functions of finite automata, respectively. Then the composition requires that the following conditions be met:

1. $A_1 \cap A_2 = A_0, \quad A_0 \neq \emptyset, \quad M_1 \subseteq M_2,$
2. Each local configuration $S_2(m)$ in the transition rules $\Phi_k(\Theta_2)$ (9) contains two parts:

$$S_2(m) = S_{22}(m) \cup S_{21}(m), \quad S_{22}(m) \subset \Omega_2, \quad S_{21}(m) \subset \Omega_1. \tag{18}$$

   Similarly, the weighted templates $T_2(m) = T_{22}(m) \cup T_{21}(m)$ with the naming underlying templates $\Gamma_{22}(m) \subset M_2, \quad \Gamma_{21}(m) \subset M_1$.
3. Interactions between two-component CA should be synchronized, i.e., one iteration of the first may be performed in several interactions of the second, or vice versa. A widespread mode of operation is synchronous with a common synchronizing clock.

The composition is especially useful in those cases, when the process under investigation needs to be observed with some parameters changing over time or over space, the basic CA serving for making the context CA results observable and comprehensive.

Example 3. A soliton-like 1D process [9] is simulated by a parity totalistic CA $\Theta_1 = (\Phi_1, \Omega_1)$ with $A_1 = \{0, 1\}$, $M = 0, 1, \ldots, N$. The local configuration of Φ_1) (9) is as follows:

$$S(i) = \{(u_{i-k}, i-k), \ldots, (u, i), \ldots, (u_{i+k}, i+k)\}, \quad S'(i) = (u'_i, i)$$

with $k = -r, \ldots, 0, \ldots, r$ and an unweighted template. The cell transition function

$$u'_i(t+1) = \begin{cases} 1 & \text{if} \quad \sum_{k=-r}^{r} u_{i+k} \quad \text{is even, but not 0,} \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

The transition rule is applied in the ordered asynchronous mode. This means, that (19) is applied sequentially to the cells named $0, 1, \ldots, N$. Hence, at the time of the rule execution the first $(t+1)$ items of the sum occur to be in the next states. The soliton-like behavior can be obtained only with initial patterns (referred to as "particles") satisfying certain conditions [9]. In the example, two "particles" are used: "11011" and "10001001". The first has the displacement $d = 7$ cells to the left with a period $p = 2$. The second has $d = 12$ also to the left, and $p = 6$. So, each 6 iterations the distance between the particles diminish by 9 cells. Between the 12th and the 24th iterations the particles are superimposed, and after the 30th iteration the first "particle" is ahead, as is shown in the following states of $\Omega_1(t)$:

$t = 0:$ 00000000000000000 ... 00**10001001**0000000000000000000**11011**00
$t = 6:$ 000000000 ... 00**10001001**0000000**11011**0000000000000000000
$t = 30:$ 00000000000000000000000**11011**0000**10001001** ... 00000000000000
$t = 36:$ **01101**1000000000000**10001001**0000000 ... 0000000000000000000

The basic CA Θ_2 performs the double averaging. Its alphabet $A_2 = \boldsymbol{R}$. The mode of execution is synchronous. Each iteration is divided into two steps. At each step, averaging of Boolean states is performed with a radius $r = 4$ according to (4). The boundary conditions are periodic, and the two particles run around from right to left, the speed of the first being larger that of the second one (Figure 3).

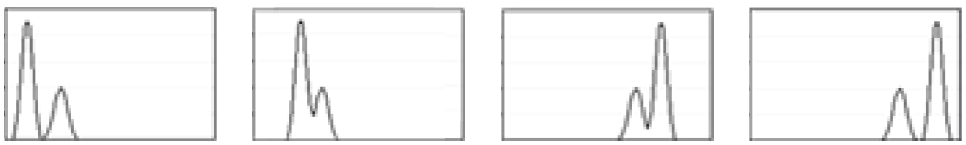


Figure 3. Unidirectional composition of two CA: a totalistic parity CA simulating a soliton-like process and an averaging CA for transforming the CA results to a comprehensive form. Three snapshots of the composed CA are shown for $t = 6, 30$ and 36

4.2. Bidirectional composition. A CA $\Theta = \langle \Omega, \Phi \rangle$ is *bidirectional* composition of $\Theta_1 = \langle \Omega_1, \Phi_1 \rangle$ and $\Theta_2 = \langle \Omega_2, \Phi_2 \rangle$ if each component of CA uses variables from both alphabets. It means that the transition rules of each component depend on local configurations in both CA components.

To be formal, let A_1, M_1 and A_2, M_2 characterize the cellular arrays of Ω_1 and Ω_2, and f_1 and f_2 be cell transition functions of Θ_1 and Θ_2, respectively. Then the bidirectional composition is characterized by the following properties:

1. $A_1 \cap A_2 = A_0, \quad A_0 \neq \emptyset, \quad M_1 = M_2,$
2. The intercell interaction structure is represented by four types of local configurations: the first type includes S_{11} and S_{22} with the naming functions $\phi_{11} : M_1 \to M_1$ and $\phi_{22} : M_2 \to M_2$, determining the neighborhoods of a cell from Ω_k in the same cellular arrays Ω_k, $k = 1, 2$; the second type of local configurations S_{12}, S_{21} with the naming functions $\phi_{12} : M_1 \to M_2$ and $\phi_{21} : M_2 \to M_1$, determining the neighborhood of a cell from Ω_k allocated in Ω_j, $j \neq k$, $k, j = 1, 2$.
3. The transition rules of Θ_1 and Θ_2 are as follows.

$$\begin{aligned} \Phi_1(m): &\quad S_{11}(m) \cup S_{12}(m) \to S_1'(m), \\ \Phi_2(m): &\quad S_{22}(m) \cup S_{21}(m) \to S_2'(m) \end{aligned} \tag{20}$$

4. Interactions between two component automata should be synchronized, i.e., one iteration of the first CA is performed during several iterations of the second one or vice versa. The most usable mode of operation is synchronous action of each iteration.

The composition is useful in all those cases when a process with two or more species is under simulation. That is the case when spatial dynamics of reaction-diffusion is investigated, where diffusion components are independent and the reaction functions depend on all or several species taking part in the process. In conventional mathematics such phenomena are usually represented by a PDE system.

Example 4. A process of localized structures appearance, interaction and oscillation, which has been studied in a number of publications on self-organizing reaction-diffusion systems [10, 11] is simulated as bidirectional composition of two CA, each component being constructed as a probabilistic (hybrid) reaction-diffusion CA [3], according to the PDE system, which is in its simplified form as follows [10]

$$\frac{\partial v}{\partial t} = d_v \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \lambda v - a v^3 - b u, \tag{21}$$

$$\frac{\partial u}{\partial t} = d_u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + v - u. \tag{22}$$

The simulation has been performed with $\lambda = 2.0$, $a = 1$, $b = 1.5$, $d_u = 0.025$ cm^2/s, $d_v = 0.0025$ cm^2/s, with a time step $\tau = 0.6$ s, and space step $h = 0.1$ cm, which yields for the normalized diffusion coefficients $D_v = \tau d_v/h^2 = 1.5$ and $D_u = \tau d_u/h^2 = 0.15$ and provides the averaged state values of both CA being in the interval $(0, 1)$. The initial conditions are random distributions of "ones" with averaged densities: $\rho_1 = 0.1$ in $\Omega_1(0)$ and $\rho_2 = 0.9$ in $\Omega_2(0)$. The alphabets and the naming sets are taken as follows: $A_1 = A_2 = \{0, 1\}$, $M_1 = M_2 = \{(i, j) : i, j = 0, 1, \ldots, N\}$, $N = 200$.

The two CA-components of the composition are almost identical. They differ only in the values d_u, d_v and in the reaction functions, which are as follows:

$$R_1(ij) = \tau(\lambda v_{ij} - a v_{ij}^3 - b u_{ij}), \quad R_2(i, j) = \tau(v_{ij} - u_{ij}). \tag{23}$$

Hence, it is convenient to denote any CA as $\Theta_k = (\Omega_k, \Phi_k)$, $k = 1, 2$.

Both CA have a diffusion part which may be realized as a Boolean CA, described in Example 1, and the reaction part, which computes real functions (23) of variables both from Ω_1 and Ω_2. The results of the diffusion and the reaction parts should be summed up, which requires performing at each t-th iteration in 2 steps. The first step is to compute two functions on each Ω_k: a diffusion iteration, applying (13) and obtaining $\Omega'_k(t) = \{(z_k, (i, j)) : z_k \in \{0, 1\}\}$, and computing the result of a reaction iteration $\Omega''_k(t) = \{(y_k, (i, j)) : y_k \in \boldsymbol{R}\}$ according to (23), where $y_k = R_k(\langle u\rangle, \langle v\rangle)$. Then the main substitution (20) of the composed CA takes the following form:

$$\Phi_k(i, j): \ S_{kk}(i, j) \cup S_{kl}(i, j) \to S'_{kk}(i, j), \quad k, l = 1, 2, \quad k \neq l, \tag{24}$$

where $S_{kk}(i, j) = (\langle z_k\rangle, (i, j))$, $S_{kl}(i, j) = (y_k, (i, j))$, $S'_k(i, j) = (x_k, (i, j))$, x_1 stands for v, x_2 stands for u and is computed as follows (5):

$$x_k(i, j) = \begin{cases} \bar{z}_k(i, j) & \text{with } \pi_{k1}(i, j) \quad \text{if } (y_k(i, j) > 0 \ \&\ x_k(i, j) = 0), \\ \bar{z}_k(i, j) & \text{with } \pi_{k2}(i, j) \quad \text{if } (y_k(i, j) < 0 \ \&\ x_k(i, j) = 1), \\ z_k(i, j) & \text{otherwise.} \end{cases} \tag{25}$$

The probabilities π_{k1}, π_{k2} according to (6) take the following form:

$$\pi_{1k}(i, j) = \frac{y_k(i, j)}{1 - \langle z_k(i, j)\rangle}, \quad \pi_{2k}(i, j) = \frac{y_k(i, j)}{\langle z_k(i, j)\rangle}. \tag{26}$$

In Figure 4, three snapshots of both CA evolutions are shown, the averaged state values being displayed in gray palette. In a few time steps

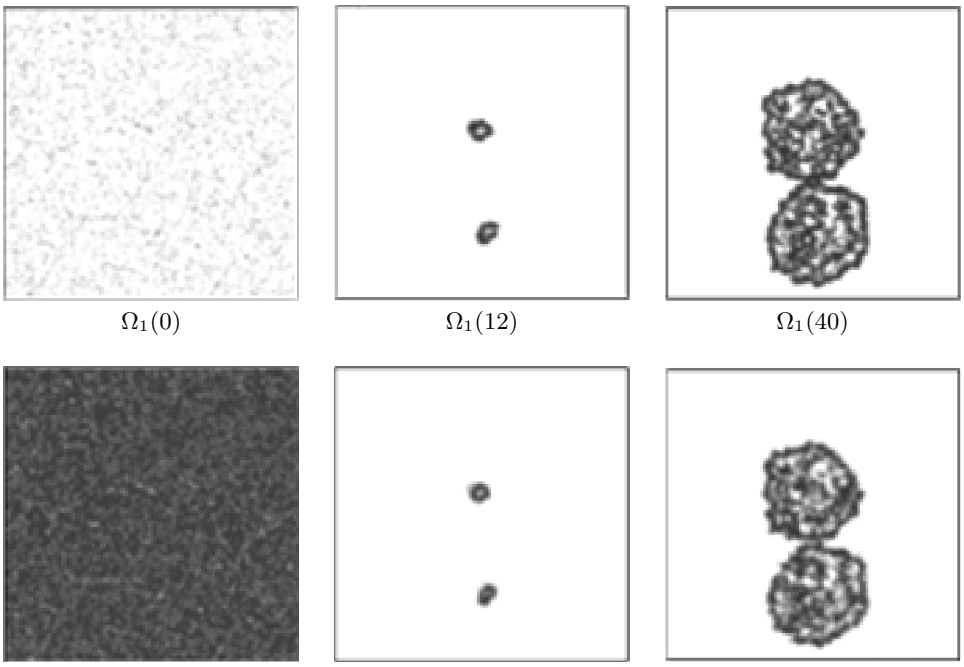


Figure 4. Bidirectional composition of two CA. Three snapshots are are given. At $t = 12$, in both CA two localized structures appear, at t=40 height become larger with oscillations inside

two dense spots appear and begin growing, while the densities outside them become equal. In the spots, the cell states oscillate with a constant frequency but a slightly changing amplitude. The spots grow gradually filling the whole cellular array with oscillating cell states.

5. Conclusion

Since the construction of transition rules for a CA, simulating complex phenomena, is a hard task and sometimes impossible, the idea of composing simple ones seems to be fruitful. So, the systematic formal approach to the CA composition technology is important, because it enables one to extend the domain of CA application for spatial dynamics simulation. In this paper, the main methods of the CA composition are shown, based on the unique formalism and illustrated by computer experiments. There is a good deal of investigations to be done in future for obtaining computational characteristics of the considered methods, such as loss of accuracy, providing stability, computational complexity, etc. Although, the true assessment may be done on the basis of practical applications.

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