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A CA-model of population dynamics of organisms living in Baikal. Verification and investigation of pollution influence*

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Abstract. The CA-model was modified in order to take into account the birthrates seasonal dependency and the influence of water streams on movements of organisms. The model was verified within the production-to-biomass and the relative average quantity criteria. A difference in the verification results and the assessments of physical values is about 20 %. The simulation of a possible pollution influence was carried out. The assessment of the critical pollution that leads to the total death of individuals and acceptable pollution with no observable influence are presented.

1. Introduction

Investigation of the population dynamics is an important problem. A realistic result of the investigation is the ability to assess limits of the anthropogenic influence on the ecosystem, which is environmentally acceptable and economically sound.

Up till now, the population dynamics has been investigated with the help of differential equations systems [1, 2]. In the models used, the number of interacting organisms groups is less or equal to 3, and the population parameters are uniformly distributed over the modeling area. The model of eight populations in lake Baikal has been proposed and investigated in [3] using ODE. So, the first restriction was overcome using the numerical simulation.

A CA-model of the population dynamics of eight organisms groups living in lake Baikal is presented in [4]. This model allows us to take into account the spatial distribution of organisms and a possible local pollution. However, the modeling area is proposed to be in the form of a square of the size much less than that of the lake. Also, the seasonal influence and water streams are neglected.

In this paper, a modified version of the CA-model [4] is presented. The area under simulation matches the lake Baikal geometry. Also, the seasonal influence on birthrates and water streams is taken into account. The data about the organisms population used in the model are taken from [3].

The model is verified by comparing the assessments of the physical values and the simulation results. The fact is, it is difficult to find data for local areas in Baikal, so the verification criteria proposed use the global assessments

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averaged for a simulated area. The production-to-biomass and quantity-to-quantity criteria were chosen for the verification. Simulation results differ from assessments of physical values in about 20 %.

The model was used to investigate a possible pollution influence. Assessments of a critical and an acceptable pollution are presented.

2. A composite CA-model

Three kinds of organisms are investigated: *macrohectopus. comephorus dybovsky* and *comephorus baikalensis.*

Each kind of organisms is divided into the age groups (letters $\{m, d, b\}$ designate the kind, and numbers $\{1, 2, 3\}$ designate the age group):

- Macrohectopus: immature m_1, puberal m_2,
- Comephorus dybovsky: one-year-old d_1, immature d_2, puberal d_3,
- Comephorus baikalensis: one-year-old b_1, immature b_2, puberal b_3.

Totaly, there are eight groups of organisms. Prey-predator and demographic relations are considered for groups.

Let us define the CA-model of the population dynamics of the Baikal organisms as

$$\aleph = \langle \Sigma, M, f, \rho \rangle,$$

where Σ is the alphabet of states, M is the cells names set, f is the global transition operator, ρ is the functional mode.

The model is a parallel composition [5] of eight CA, each being designed to simulate a concrete group of the population dynamics.

Let Q be a square mesh that covers the surface of Baikal. A set of cell names M is a union of eight pairwise disjoint subsets M_i^α:

$$M = M_1^m \cup M_2^m \cup M_1^d \cup M_2^d \cup M_3^d \cup M_1^b \cup M_2^b \cup M_3^b.$$

Assume that bijective mappings $\psi_i^\alpha : Q \mapsto M_i^\alpha$ exist for all i and α.

Let us define the *cell* as an element of the set $M \times \Sigma$. Cell states are integers $n \in \Sigma$, to characterize the density of organisms in this cell. A finite collection of cells

$$S(c) = \langle (\phi_1(c), n_1), \dots, (\phi_k(c), n_k) \rangle$$

is a *local configuration*, where $n_i \in \Sigma$ and $\phi_i : M \to M$ are naming functions, which define the names of cells, interacting with the cell named c.

In the general case, *Local transition operator* f is

$$f : \{S(c)\} \mapsto \{S(c)\}.$$

The result of applying the operator f to a cell named c is replacing the local configuration $S(c)$ by the local configuration $f(S(c))$. *Iteration* or *application of the transition operator* F is application of the local operator to all the cells.

The operator F is a sequential composition of the two operators [5]: $F = F_1 \circ F_2$, where F_1 is the movement operator, F_2 is the demographic operator.

The movement operator F_1 on its part is a sequential composition of a free movement operator F_d and a water-stream operator F_s: $F_1 = F_d \circ F_s$.

2.1. Free movement operator

Let f_z and F_z be local and global transition operators of integer diffusion according to [6]:

$$f_z : \{S_1(c)\} \mapsto \{S_1(c)\},$$

where $S_1(c)$ is a set of the neighbors of the cell named c including the cell itself. Let us define cells with the names $c, c\prime \in M_i^\alpha$ as *neighbors* if they correspond to the neighboring cells of the square mesh Q.

Application of f_z to the cell named c is performed with the following algorithm:

1. Let $(c_1, n_1), \dots, (c_k, n_k) \in S_1(c)$ be the neighbors of the cell (c, n), $k \leq 4$. Number $i \in \{1, \dots, k\}$ is randomly chosen with equal probability.
2. The new states n' and n_i' of the cells named c and c_i are obtained by the formula

$$n' = n - [\sigma \cdot n] + [\sigma \cdot n_i], \qquad n_i' = n_i + [\sigma \cdot n] - [\sigma \cdot n_i],$$

   where σ is the diffusion ratio, $0 \leq \sigma \leq 1$.

The operator F_z is asynchronously applied.

Let us define the free movement operator F_d. Let l be the physical size of a cell from the mesh Q. Let v_{cr} be a cruising speed of organisms of the kind α and the age group i. Let Δt be a physical time step equivalent to the model iteration. The maximum number of cells visited by the individual can be calculated as

$$K_i^\alpha = \frac{v_{\mathrm{cr}} \Delta t}{l}.$$

Let us define $F_d|_{M_i^\alpha}$ as a K_i^α-wise sequential application of F_z:

$$F_d|_{M_i^\alpha} = (F_z)^{K_i^\alpha}.$$

2.2. The water-stream operator

Let $\mathrm{stream}(c) : M \mapsto R^2$ be a water streams map. The two-dimensional vector $\mathrm{stream}(c) = (v_x, v_y)$ is a physical speed of water stream in the cell named c. The direction x corresponds to the West-to-East direction. The direction y corresponds to the South-to-North direction.

The local operator f_s is designed to simulate the organisms movement with water streams. The algorithm of f_s application to the cell named c is recursive. The cell named c is observed at the first level of recursion. The neighbors of the cell named c are observed at the second level of recursion. Then the neighbors of the neighbors, and so on. The number of cells observed at the k-th level does not exceed 2^k.

Let l be the size of a cell from the mesh Q. Let Δt be a time which corresponds to one iteration.

Application of f_s to a cell named c is performed with the following algorithm:

1. Find c_x^k and c_y^k the neighbors names of the cell named c^k in the directions v_x^k and v_y^k, where k is a level of recursion, c^k is the name of cell currently being observed, $(v_x^k, v_y^k) = \mathrm{stream}(c^k)$ is the stream vector in the cell named c^k.

2. Find n_x^k and n_y^k, i.e., the number of organisms of the cell named c^k moved by water streams to the cells named c_x^k and c_y^k, respectively:

$$n_x^k = \begin{cases} n^k \frac{v_x^k}{|v_x^k| + |v_y^k|}, & s_x^k > 1, \\ n^k \frac{v_x^k}{|v_x^k| + |v_y^k|} s_x^k, & s_x^k \le 1, \end{cases} \qquad n_y^k = \begin{cases} n^k \frac{v_y^k}{|v_x^k| + |v_y^k|}, & s_y^k > 1, \\ n^k \frac{v_y^k}{|v_x^k| + |v_y^k|} s_y^k, & s_y^k \le 1, \end{cases}$$

where n^k is a model number of moving organisms of the cell named c^k, n^k is calculated at $(k-1)$-th level of recursion at Step 3 if $k > 1$ and n^1 is equal to the state of the cell named c if $k = 1$, s_x^k and s_y^k are a number of cells, covered with a water stream during the time τ^k:

$$s_x^k = \frac{\tau^k v_x^k}{l}, \qquad s_y^k = \frac{\tau^k v_y^k}{l},$$

τ^k is the physical time calculated as follows:

$$\tau^1 = \Delta t, \qquad \tau^k = \tau^{k-1} - \Delta\tau^{k-1},$$

$\Delta\tau^{k-1}$ being the time spent by the water stream to cover the distance l at the previous recursion level:

$$\Delta\tau^{k-1} = \frac{l}{v_x^{k-1}} \quad \text{or} \quad \Delta\tau^{k-1} = \frac{l}{v_y^{k-1}}.$$

3. If $s_x^k > 1$, then the algorithm is recursively applied for the cell named $c^{k+1} = c_x^k$ and $n^{k+1} = n_x^k$ and $\Delta\tau^k = \frac{l}{v_x^k}$. The same checking is done for y-direction.

2.3. The demographic operator

Let f_2 be a local demographic operator:

$$f_2 : \{S_2(c)\} \mapsto \{S_2(c)\},$$

where $S_2(c)$ is a set of twins (Figure 1). Let us define cells with the names $c_i^\alpha \in M_i^\alpha$ and $c_j^\beta \in M_j^\beta$ as *twins* if they correspond to the same cell in Q, i.e., $(\psi_i^\alpha)^{-1}(c_i^\alpha) = (\psi_j^\beta)^{-1}(c_j^\beta)$:

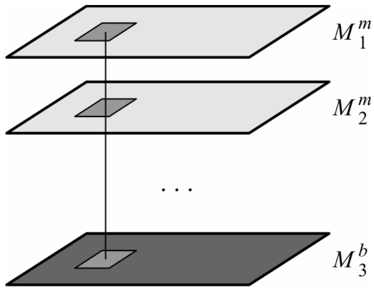

Figure 1

$$\exists i\, \exists \alpha : \quad c \in M_i^\alpha,$$
$$q = (\psi_i^\alpha)^{-1}(c), \quad q \in Q,$$
$$S_2(c) = \left\{ \begin{array}{lll} (\psi_1^m(q), n_1^m), & (\psi_2^m(q), n_2^m), & \\ (\psi_1^d(q), n_1^d), & (\psi_2^d(q), n_2^d), & (\psi_3^d(q), n_3^d), \\ (\psi_1^b(q), n_1^b), & (\psi_2^b(q), n_2^b), & (\psi_3^b(q), n_3^b) \end{array} \right\}$$

The functional mode of the local operator f_2 is synchronous.

The new state of the cell named $c_i^\alpha \in M_i^\alpha$ is given by the formula

$$(n_i^\alpha)' = n_i^\alpha + (\rho_i^\alpha n_j^\alpha - \lambda_i^\alpha n_i^\alpha - \theta_i^\alpha n_i^\alpha)\Delta t,$$

where j is the age group of organisms that generates organisms of the i-th age group, Δt is the time which corresponds to one iteration, $\rho_i^\alpha n_j^\alpha$ is an income into the group due to the birth or aging, $\lambda_i^\alpha n_i^\alpha$ is an outcome from the group due to the death, $\theta_i^\alpha n_i^\alpha$ is an outcome from the group due to aging.

The birthrates ρ_i^α, the deathrates λ_i^α and the aging rates θ_i^α are taken from [3].

1. Aging rates θ_i^α are constant,
2. Deathrates $\lambda_2^d, \lambda_3^d, \lambda_2^b, \lambda_3^b$ are constant,
3. Deathrates $\lambda_1^m, \lambda_2^m, \lambda_1^d, \lambda_1^b$ are obtained as follows:

$$\lambda_i^\alpha = a_i^\alpha + b_i^\alpha(n_2^d + n_3^d) + d_i^\alpha(n_2^b + n_3^b), \tag{1}$$

   where a_i^α, b_i^α, d_i^α are constant, b_i^α and d_i^α define deaths of preys due to eating by predators, a_i^α define deaths due to other reasons,
4. Birthrate ρ_1^m is constant,

5. Birthrates ρ_1^d, ρ_1^b are obtained as follows:

$$\rho_1^\alpha = \mu^\alpha(n_1^m + n_2^m) + \nu^\alpha(n_1^d + n_1^b),$$

where constants μ^α and ν^α depend on preferences of predators.

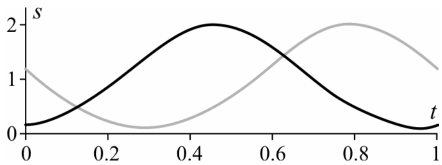

Figure 2. Graphs of $s_b(t)$ (grey) and $s_d(t)$ (black)

In order to take into account the seasonal dependency, the birthrates ρ_1^d and ρ_1^b are multiplied by the periodic functions $s_d(t)$ and $s_b(t)$ (Figure 2). Their period is equal to one year. Zero corresponds to the 1st of January.

3. The model verification

The model verification is done for the three kinds of organisms (data are summarized by the age groups).

The following criteria were chosen for the verification:

- P_α/B_α is the ratio of production to biomass for organisms of the kind α. The production P_α is the positive change of biomass per year. The production includes the born and aged organisms, but does not include dead organisms, B_α is an averaged biomass per year.
- N_α/N_β is the ratio of the number of organisms of the kind α to the number of organisms of the kind β. N_α is an averaged number of organisms per year.

$$\begin{aligned} P_m &= P_m^1 + P_m^2, & B_m &= B_m^1 + B_m^2, & N_m &= N_m^1 + N_m^2, \\ P_d &= P_d^1 + P_d^2 + P_d^3, & B_d &= B_d^1 + B_d^2 + B_d^3, & N_d &= N_d^1 + N_d^2 + N_d^3, \\ P_b &= P_b^1 + P_b^2 + P_b^3, & B_b &= B_b^1 + B_b^2 + B_b^3, & N_b &= N_b^1 + N_b^2 + N_b^3. \end{aligned}$$

The assessment of B_m is given in [7] and equals 110,000 ton. The assessment of P_m is also given in [7] and varies from 330,000 to 900,000 ton. Thus, the assessment of P_m/B_m is 3–8.

The assessment of $\dfrac{P_d + P_b}{B_d + B_b}$ is given in [8] and equals 1.24. Assessments of N_m/N_d and N_m/N_b is given in [3].

	$\frac{P_m}{B_m}$	$\frac{P_d+P_b}{B_d+B_b}$	$\frac{N_m}{N_d}$	$\frac{N_m}{N_b}$
[3, 7, 8]	3–8	1.24	6.05	21.52
Model	5.77	1.49	6.00	20.45

The verification results are shown in the table. The difference in the model results from the assessments given in [3, 7, 8] is about 20 %.

4. The pollution influence

The model allows us to take into account the influence of the possible pollution. Let $\mathrm{poll}(c) : M \mapsto R_+$ be a pollution map, i.e., a function of the cell name with a range in the positive numbers characterizing pollution intensity. Let us assume that pollution affects deathrates.

1. The new deathrates of predators $\lambda_2^d, \lambda_3^d, \lambda_2^b, \lambda_3^b$ are given as follows:

$$(\lambda_i^\alpha(c))' = \lambda_i^\alpha(1 + \mathrm{poll}(c)).$$

2. The new deathrates of preys $\lambda_1^m, \lambda_2^m, \lambda_1^d, \lambda_1^b$ are given as follows:

$$(\lambda_i^\alpha(c))' = \lambda_i^\alpha + \mathrm{poll}(c)a_i^\alpha,$$

where a_i^α is a constant deathrate from (1).

The pollution map $\mathrm{poll}(c)$ used is shown in Figure 3. The function $\mathrm{poll}(c)$ is the normal distribution density with the center in the cell c_0 south of lake Baikal multiplied by a constant. Grey color means land. White color means water. Darker color means a greater pollution intensity.

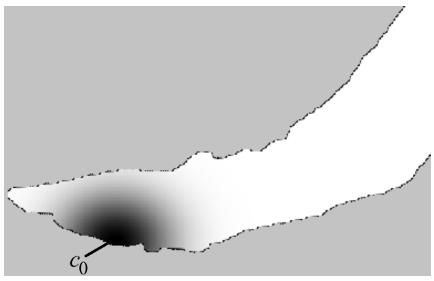

Figure 3

5. Computational experiments

In further computational experiments, the initial state is a uniform distribution of individuals and organisms' density equal to the stable state $\mathrm{poll}(c_0) = 14$ [3].

Some iterations for the simulation of the puberal macrohectopus population dynamics are presented in Figure 4. The pattern formation process is observed outside of the polluted area. Non-uniform distribution is a result of the water streams influence.

Population dynamics in the north area of Baikal (Figure 5) tends to steady annual oscillation process. Annual oscillations are the result of seasonal dependency of birthrates.

The population dynamics in the pollution epicenter c_0 is presented in Figure 6; $\mathrm{poll}(c_0) = 14$.

The population dynamics in the cell with $\mathrm{poll}(c) = 2.3$ is given in Figure 7. The number of preys is increased and the number of predators is decreased as compared to non-polluted area.

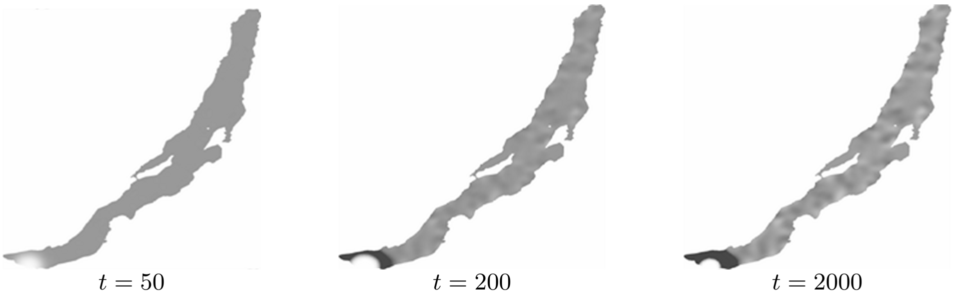

Figure 4. Some iterations for simulating the puberal macrohectopus population density. A darker color means a higher individuals density

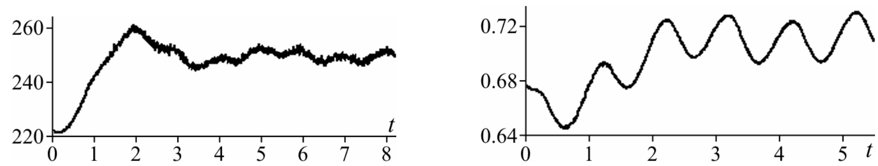

Figure 5. The population dynamics in a cell in the north area of Baikal for immature macrohectopus (left) and immature comephorus dybovsky (right)

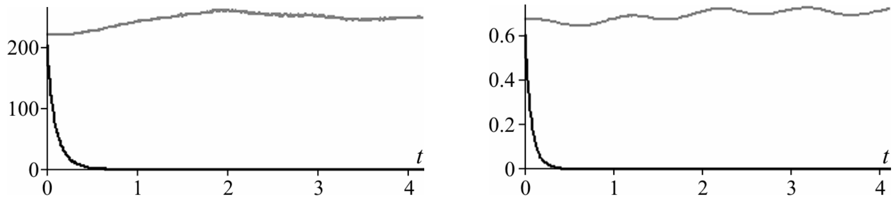

Figure 6. The population dynamics in the pollution epicenter for immature macrohectopus (left) and immature comephorus dybovsky (right). A brighter line means the same dynamics in the cell without pollution

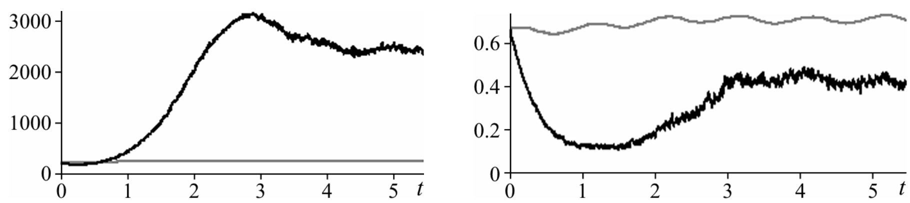

Figure 7. The population dynamics if the cell with poll$(c) = 2.3$ for m_1 (left) and d_2 (right). A brighter line means the same dynamics in the cell without pollution

Figure 8

In order to investigate the dependency of the average annual density on the pollution intensity, several points along the segment given in Figure 8 were investigated.

The distribution and density of the pollution intensity at the 2,000th iteration along the segment is presented in Figure 9.

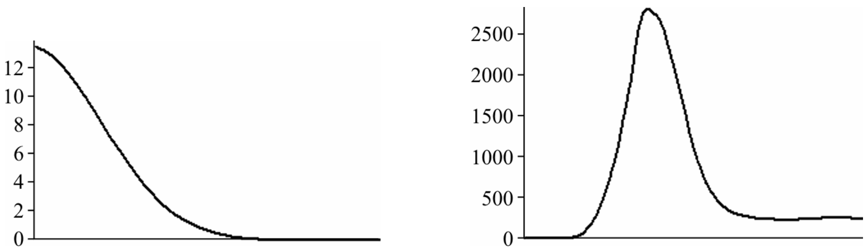

Figure 9. The pollution intensity distribution (left) and the density of immature macrohectopus at the 2,000th iteration (right) along the segment. The axis x means the distance from the South-West point in the pollution epicenter to North-East

The following conclusions might be proposed:

- If $\mathrm{poll}(c) > 10$, then the macrohectopus density at the 2,000th iteration is less than 3 % of average density in non-polluted area. That means total death of organisms.
- If $\mathrm{poll}(c) \in (0.15, 5)$, then the number of preys increases and number of predators decreases in comparison to non-polluted case. Concrete values depend on $\mathrm{poll}(c)$ intensity.
- If $\mathrm{poll}(c) < 0.03$, then pollution influence is not observable within natural annual oscillations.

6. Conclusion

The CA-model [4] of the population dynamics of eight organisms in lake Baikal was corrected and extended by taking into account the lake geometry, seasonal dependency of birthrates and water streams.

A difference in the model verification results from the assessments given in [3, 7, 8] is about 20 %.

The dependency of the model behavior on the pollution intensity is investigated. The influence of local pollution is localized in the polluted area. The assessments of the critical pollution that leads to the total death of organisms have been obtained, as well as the acceptable pollution whose influence is not observable.

References

[1] Svirezhev Y.M., Logofet D.O. Stability of Biological Communities. — Moscow: Mir, 1983.

[2] Bazykin A.D. Non-linear Dynamic of Interacting Populations. — Izhevsk: ICRC, 2003.

[3] Zorkalcev V.I., Kazazaeva A.V., Mokry I.V. Model of interactions of three pelagic organisms of Baikal // Modern Technologies. System Analysis. Modeling. — IRIIK, 2008. — No. 1. — P. 182–193.

[4] Afanasyev I.V. Population dynamic cellular-automata model for Baikal organisms // Theoretical Backgrounds of Applied Discrete Mathematics. — 2012. — No. 1. — P. 121–132.

[5] Bandman O.L. Cellular Automata Composition Techniques for Spatial Dynamics Simulation Simulating Complex Systems by Cellular Automata // Understanding Complex Systems / A.G. Hoekstra, et al., eds. — Berlin, 2010. — P. 81–115.

[6] Medvedev Y.G. Multi-particle cellular automata models for diffusion simulation // Methods and Tools of Parallel Programming Multicomputers. — 2011. — Vol. 6083. — P. 204–211.

[7] Mazepova G.F., Timoshkin O.A., Melnik N.G., Obolkina L.A., Tanichev A.I. Guide and Key to Pelagic Animals of Baikal. — Novosibirsk: Nauka, 1995.

[8] Starikov G.V. Comephorus. — Novosibirsk: Nauka, 1977.